STATISTICS AND PROBABILITY

1. (a) The probability that Moses wins a game is 2/3. If he plays 6 games, what is
   (i) the expected number of games won ?
   (ii) the chance of winning at least two games ?

   (b) A machine manufacturing nails makes approximately 15% that are outside set tolerance limits. If a random sample of 200 nails is taken, find the chance that
   (i) more than 21 nails will be outside the tolerance limits,
   (ii) between 20 and 30 nails inclusive, will be outside the tolerance limits.

2. The table below shows marks obtained by some students:

Marks	25 -<29	-<35	-<40	-<50	-<55	-<60	-<70	-<75	-<80
Frequency	6	12	27	30	18	14	9	4	5

   (a) Estimate the
   (i) variance
   (ii) mode

   (b) Construct an Ogive and use it to determine the;
   (i) 68^{th} percentile
   (ii) number of students who scored above 47%

3. (a) Events A, B and C are such that P(A) = x, P(B) = y and P(C) = $x + y$. If P(AuB) = 0.6 and P(B/A) = 0.2,
   (i) Show that $4x + 5y = 3$.
   (ii) Given that B and C are mutually exclusive and that P(BuC) = 0.9, determine another equation in x and y.
   (iii) Hence find the values of x and y. Deduce whether A and B are independent events

   (b) The events A and B are independent with P(A) = $1/2$ and P(AuB) = $\frac{2}{3}$. Find;
   (i) P(B) (ii) P(A/B) (iii) $P(B^I/A)$

4. The weekly demand for petrol in thousands of units in a house is a continuous random variable x with a probability density function of the form;

$$f(x) = \begin{cases} ax^2(d - x); 0 \le x \le 1 \\ 0 \cdots elsewehere \end{cases}$$

   a) Given that the average demand per week is 600 units, determine the values of a and d
   b) Find P(0.9 < x < 1)

5. (a) A biased tetrahedral die has its faces numbered 1, 2, 3, 4. If this die is tossed, the probability of the face that it lands on, is inversely proportional to the square of the number on the face. If x is the random variable the number on the face the die lands on, determine;
   (i) the probability distribution for x. (b) Var(3x) (c) $P\left[\left|(x-2)\right| \le 1\right]$

(b) The probability distribution of a discrete random variable X is given by;

P(X=x) = kx; x =1,2,3,………,n.   Where k is a constant

Given that E(x) = 5, find the;

(i) value k and n
(ii) Var(2x+3)

6. The continuous random variable X has a probability function

$$f(x)=\begin{cases} k(x+2); -1 < x < 0 \\ 2k; 0 \le x \le 1 \\ k\dfrac{(5-x)}{2}; 1 < x \le 3 \\ 0, elswhere \end{cases}$$

where k is a constant.

a) Sketch the p.d.f and hence find the value of constant k
b) Find the cumulative distribution function, F(x).
c) Calculate $P(0.5 < x < 2 / x > 1)$

7. The weights of cabbages in the S.1 garden is normally distributed with a mean of 250g and a standard deviation of 6g
(a) If a cabbage is selected at random, find the probability that it weighs;
(i) between 248 and 257.4g
(ii) less than 246.1g
(b) If 10% of the cabbages weigh more than y g, determine y

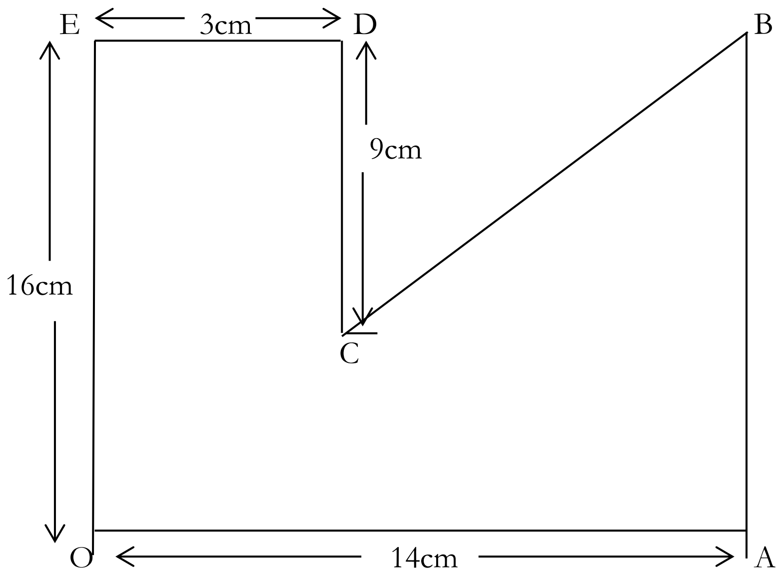
8. (a)In a certain country, on average one student in five has blue eyes. For a random selection of *n* students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of *n*.

(b) John plays two games of squash. The probability that he wins his first game is 0.3. If he wins his first game, the probability that he wins his second game is 0.6. If he loses his first game, the probability that he wins his second game is 0.15. Given that he wins his second game, find the probability that he won his first game

(c)Dan has a pack of 15 cards. 10 cards have a picture of a robot on them and 5 cards have a picture of an aeroplane on them. Emma has a pack of cards. 7 cards have a picture of a robot on them and $x - 3$ cards have a picture of an aeroplane on them. One card is taken at random from Dan's pack and one card is taken at random from Emma's pack. The probability that both cards have pictures of robots on them is 7/18. Write down an equation in terms of x and hence find the value of x.

MECHANICS

1. (a) Find the centre of gravity of the lamina shown below.



(b) If the lamina is suspended from O, find the angle that OB makes with the vertical.

2. a). The impulse of a force of (6**i**+2**j**+**k**)N acts for 1.5seconds. If this force acts on a body of mass 2kg with an initial velocity of (-4**i**+3**j**-2**k**)m/s, find the velocity at the end of the period.

   b). A vertical spring supports a body which stretches it 50cm when in equilibrium. The body is then pulled down and released. Show that the body executes simple harmonic motion with period $\frac{\pi\sqrt{5}}{5}$ seconds

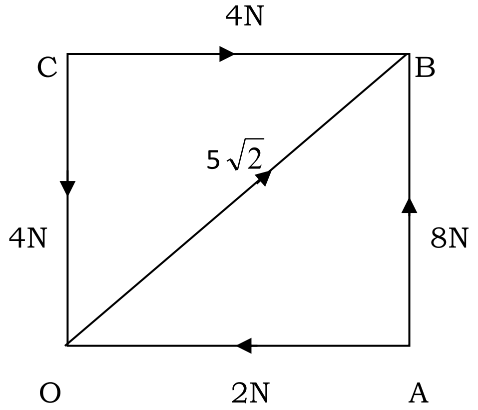
3. (a) A car of mass one tone has an engine which exerts a constant tractive force of 900N. The car moves up a hill of inclination of 1 in 10. There is a constant frictional resistance of 0.4 KN to its motion. If the initial speed of the car is 108km/hr, how far has it then travelled before it comes to rest? (g=10m/s^2)

   (b) Three forces **i**+5 **j**-4 **k**, -3 **j**+7 **k** and 3 **i**+2 **k** , act on a body from a point A(4,-2,3) to point B(8,4,5). Find the work done on the body.

4. A smooth bead of mass 0.2 kg is threaded on a smooth circular wire of radius r metres which is held in a vertical plane. If the bead is projected from the lowest point on the circle with speed $\sqrt{3rg}$. Find the;
   (a) speed of the bead when it has gone one sixth of the way round the circle.
   (b) force exerted on the bead by the wire at this point.

5. (a) A body of mass 2kg is placed on a rough plane inclined at an angle of 30^0 to the horizontal. The coefficient of friction between the body and the plane is 0.25. Find the least force needed to prevent the body from slipping down the plane if this force acts upwards at an angle of 30^0 to the line of greatest slope.

(b)



OABC is a square in which OA =2cm. Taking OA and OC as the x and y axes respectively, find the:

(i) equation of the line of action of the resultant force.

(ii) distance from C of the point where the line in (i) above crosses the side C

6. Two uniform rods AB and BC each of length **2b** but of weights **W** and **5W** respectively are freely jointed at B and stand inclined at 90° to each other in a vertical plane on a smooth floor with ends A and C connected by a rope. Show that the;

(i) tension in the rope is $\frac{3}{2}W$

(ii) reaction at the hinge B is $\frac{W}{2}\sqrt{13}$

7. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:

(i) acceleration of the system.

(ii) tension in the string.

(iii) distance moved by the moveable pulley in 1.5 s.

8. a) A particle is projected with speed u at an elevation θ from a point O on level ground. Show that the equation of trajectory is;

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta).$$

b) A particle is projected with speed $10\sqrt{g}$ ms^{-1} from a point O on the ground at an elevation θ. If the particle must clear a vertical tower of height 40m and at a horizontal distance of 40m from O, prove that $2 \leq \tan\theta \leq 3$

NUMERICAL ANALYSIS

9. (a)Locate graphically the smallest positive real root of $\sin x - \ln x = 0$

   (b) Use the Newton Raphson method to approximate this root of the equation in (a) above correct to 3d.p.

10. The mock examination and average final examination marks of a certain school are given in the following table.

Mock Marks	28	34	36	42	52	54	60
AV. Final Marks	54	66	68	70	76	66	74

(a) (i) Plot the marks on the scatter diagram and comment on the relationship between the two marks.

(ii) Draw a line of best fit and use it to predict the average final mark of a student whose mock mark is 50.

(b) Calculate the rank correlation coefficient between the marks and comment on your result.

11. If the numbers x and y are approximations with errors Δx and Δy respectively. Show that the maximum absolute error in the approximations of x^2y is given by $|2xy\Delta x| + |x^2\Delta y|$ Hence find the limits within which the true value of x^2y lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$

12. (a) Two sides of a triangle *PQR* are p and q are such that $\angle PRQ = \alpha$.

(i) Find the maximum possible error in the area of this triangle.

(ii) Hence find the percentage error made in the area if p = 4.5cm, q = 8.4cm and a = 30º.

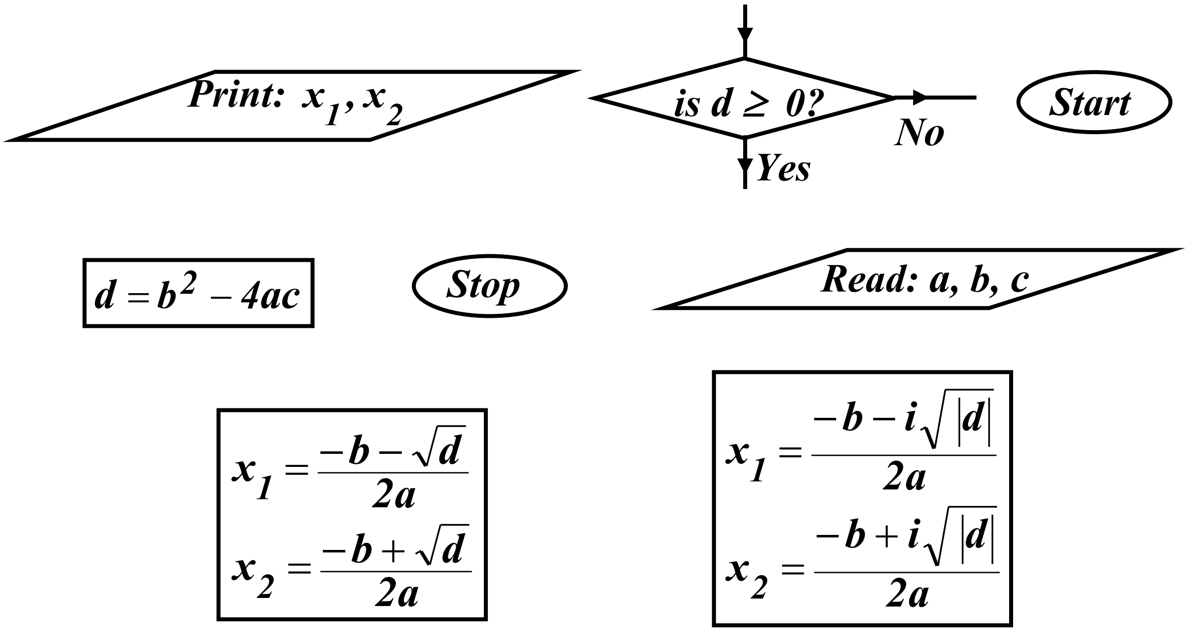
(b) Find the range within which $\frac{3.679}{2} - \frac{7.0}{5.48}$ lies.

13. (a) Use the trapezium rule with six ordinates to estimate $\int_1^3 x^2 \ln x \, dx$. Give your answer correct to 3 decimal places.

(b) Hence find the percentage error made in your estimate and suggest how it can be reduced

14. The quadratic formula method for solving the equation

$ax^2 + bx + c = 0$ is described by the following parts of the flowchart:



(i) By rearranging the given parts, draw a flow chart that shows the algorithm for the described method

(ii) By performing a dry run for the flow chart using $x^2 - 4x + 13 = 0,$ copy and complete the table below:

d	x_1	x_2
- - - -	- - - -	- - - -