

P425/2

APPLIED MATHEMATICS

Paper 2

Jul./Aug. 2023

3 hours



WAKISO-KAMPALA TEACHERS' ASSOCIATION (WAKATA)

WAKATA MOCK EXAMINATIONS 2023

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five questions from section B.

*Any additional question(s) answered will **not** be marked.*

All necessary working must be clearly shown.

Begin each answer on a fresh sheet of paper.

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

Assume where necessary, acceleration due to gravity $g = 9.8\text{ms}^{-2}$.

Graph paper is provided.

Neat work is a must!!

SECTION A: (40 MARKS)

Answer all questions in this section.

- Two forces $(i + j + k)N$ and $(i + 2j + 3k)N$ act on a particle and displace it from $(2,3,4)$ to point $(5,4,3)$. If the displacement is in meters, calculate the work done. (05 marks)
- Two hunters A and B hit a bird in succession. The probability of A and B scoring a hit correctly are 0.3 and 0.2, respectively. The second hunter will hit the bird only if the first misses the target. Find the probability that the bird is hit by the second hunter. (05 marks)

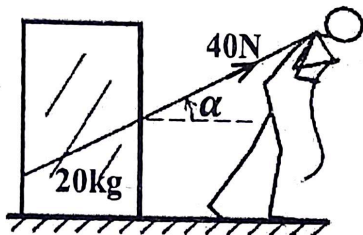
3. Given that $y = \sqrt{(2^x + x)}$.

- (a) Complete the table below, giving the values of y to 3 decimal places. (02 marks)

x	0	0.2	0.4	0.6	0.8	1
y	1	1.161	1.311	1.455	1.594	1.732

- (b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(2^x + x)} dx$ correct to three decimal places. (03 marks)

4. The diagram below shows a wooden block of mass 20kg being pulled in a straight line along a rough horizontal floor using a handle attached to the box.



The handle is inclined at an angle α to the floor, where $\tan \alpha = \frac{3}{4}$ and coefficient of friction between the box and the floor is 0.14. Find the acceleration of the block. (05 marks)

5. The discrete random variable X has the following probability distribution.

x	a	b	c
$P(X = x)$	$a \log_{36} a$	$b \log_{36} b$	$c \log_{36} c$

Where a, b and c are distinct integers ($a < b < c$). Find the values of a, b , and c .

6. Given the function, $y = \cos x^2 - x + 3$.

- Show that the function has a root α in the interval $\{2.5, 3\}$ (02 marks)
- Use linear interpolation once on the interval $\{2.5, 3\}$ to find an approximation for α giving your answer correct to two decimal places. (03 marks)

6. A boat is moving with constant velocity. At noon, it is at a point with position vector $(3i - 4j)km$ with respect to a fixed origin O . At 14:30 on the same day, it is at a point with the same day, it is at a point with position vector $(3i - 4j)km$. Find the position vector of the boat at 17:00 on the same day. (05 marks)

7. Average IQ of a group of 800 children is 98. The standard deviation is 8. Assuming normality find the expected number of children having IQ between 100 and 120. (05 marks)

SECTION B: (60MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. The table below shows the weight of 50 farm animals whose average weight is 62.8kg.

Weight (kg)	$0 < 20$	$20 - < 40$	$40 - < 60$	$60 - < 80$	$80 - < 100$	$100 - < 120$
Number of animals	5	x	10	y	7	8

(a) Find the value of x and y . (06 marks)

(b) Draw a cumulative frequency curve for the data. Use your curve to estimate the:
 (i) median weight of the animals. (06 marks)
 (ii) number of animals below 70kg.

10. A particle, P moves on the x - axis. At time, t , seconds the velocity of P is Vms^{-1} in the direction of x increasing, where V is given by;

$$V = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leq t \leq 4 \\ 16 - 2t, & t > 4 \end{cases}$$

When $t = 0$, P is at the origin O. Find the; (03 marks)

- (a) greatest speed of P in the interval $0 \leq t \leq 4$. (02 marks)
- (b) distance of P from O when $t = 4s$. (07 marks)
- (c) total distance travelled by P in the first 10s of its motion. (03 marks)

11. (a) Show that the equation $xe^x - 1 = 0$ has a root between 0 and 1. (03 marks)
 (b) Use the Newton - Raphson method to find the approximate root of $xe^x - 1 = 0$, correct to three decimal places. (09 marks)

12. A continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} a + bx; & 0 \leq x \leq 5 \\ 0, & \text{Otherwise} \end{cases}$$

where a and b are constants. (03 marks)

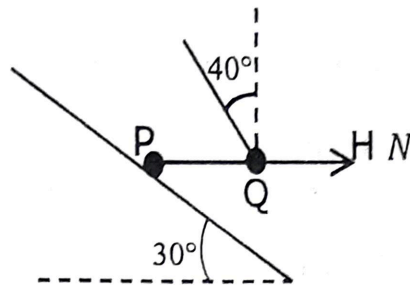
(a) Show that $10a + 25b = 2$ (06 marks)

(b) Given that $E(x) = \frac{35}{12}$, find the value of a and b . (03 marks)

(c) Find to three significant figures, the median of X .

Turn Over

13. The diagram below shows a particle Q lying on a slope inclined at 30° to the horizontal. P is attached to one end of a taut light inextensible string which passes through a small ring Q of mass, $m\text{ kg}$. The portion PQ of the string is horizontal and the other portion of the string is inclined at 40° to the vertical. A horizontal force of magnitude $H\text{ N}$, acting away from P , is applied to Q .



The tension in the string is 6.4 N , and the string is in the vertical plane containing the line of greatest slope on which P lies. Both P and Q are in equilibrium.

- (a) Find the value of m and H . (04 marks)
- (b) Given that the weight of P is 32 N , and that P is in limiting equilibrium, show that the coefficient of friction between P and the slope is 0.879 . (05 marks)
- (c) Determine whether P remains in equilibrium when Q and the string are removed. (03 marks)
14. (a) If $Q = \frac{X^n}{Y^m}$ and e_x is the error in the measurement of X , e_y is the error in the measurement of Y ; show that absolute error e_q in the measurement of Q is

$$\pm \left[\frac{X^n}{Y^m} \right] \left[n \left| \frac{e_x}{X} \right| + m \left| \frac{e_y}{Y} \right| \right] \quad \text{P(1+700) (05 marks)}$$

- (b) A teacher earns P million shillings and invests all in bank at a rate of 8% compound interest for n years.
- (i) Construct a flow chart that shows the teacher's investment.
- (ii) Given that the teacher invested 2 million shillings on 1^{st} January 2023, how much money will be on his account on 1^{st} January 2027? (07 marks)
15. The heights of girls in a certain secondary school are normally distributed with mean $\mu\text{ cm}$ and standard deviation $\sigma\text{ cm}$. It is known that 30% of the girls are taller than 172 cm and 5% are shorter than 154 cm .
- (a) Show that $\mu = 154 + 1.6449\sigma$ (04 marks)
- (b) Find the value of μ and σ . (05 marks)
- (c) A girl is chosen at random from the school, what is the probability that she is taller than 160 cm ? (03 marks)
16. Particles A and B of masses 0.4 kg and $m\text{ kg}$ respectively are joined by a light inextensible string which passes over a smooth pulley. The particles are released from rest at the same height above a horizontal surface; the string taut and portions of the string not in contact with the pulley are vertical. B begins to descend with acceleration 2.45 m s^{-2} and reaches the surface 0.3 s after being released. Subsequently, B remains at rest and A never reaches the pulley. Calculate the;
- (a) tension in the string while B in motion. (02 marks)
- (b) momentum lost by B when it reaches the surface. (05 marks)
- (c) greatest height of A above the surface. (05 marks)