P425/1
PURE
MATHEMATICS
Paper 1
July/Aug. 2024
3 hours.

ACEITEKA JOINT MOCK EXAMINATIONS 2024

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- Answer **all** the **eight** questions in section A and only **five** questions in section B.
- Indicate the five questions attempted in section B in the table aside.
- Additional question(s) answered will **not** be marked.
- **All** working **must** be shown clearly.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Question		Mark
Section A		
Section B		
Total		

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: Solve for x: $64x^{\frac{2}{3}} + x^{\frac{-2}{3}} = 20$. **[5 Marks]**

Qn 2: Evaluate $\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}}\, dx$ by using a suitable substitution. **[5 Marks]**

Qn 3: Show that: $-\sqrt{5} \leq \cos x + 2\sin x \leq \sqrt{5}$. **[5 Marks]**

Qn 4: Given that w and z are two complex numbers, solve the simultaneous equations:

$$3z + w = 9 + 11i$$
$$iw - z = -8 - 2i$$

[5 Marks]

Qn 5: Using Maclaurin's theorem, expand $(x+1)\sin^{-1} x$ up to the term in x^2. **[5 Marks]**

Qn 6: The equation of a curve is given by $3y^2 - 18y + 2x + 37 = 0$.

(a). Show that the curve is a parabola. **[3 Marks]**

(b). Find the equation of the directrix. **[2 Marks]**

Qn 7: Given that points $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$ and $D(-2,6,4)$ are vertices of a parallelogram, find the area of the parallelogram $ABCD$. **[5 Marks]**

Qn 8: The area bounded by the curves $y^2 = 32x$ and $y = x^3$ is rotated about the x-axis through one revolution. Show that the volume of the solid so formed is $\frac{320\pi}{7}$ cubic units. **[5 Marks]**

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Section B (60 Marks)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

Question 9:

(a). Find the ratio of the coefficient of x^7 to that of x^8 in the expansion of $\left(3x + \frac{2}{3}\right)^{17}$. **[5 Marks]**

(b). (i). Expand $(1 + x)^{-2}$ in descending powers of x including the term x^{-4}.

(ii). If $x = 9$, find the % error in using the first two terms of the expansion in (b)(i) above. **[7 Marks]**

Question 10:

(a). Express $\frac{x^3+9x^2+28x+28}{(x+3)^2}$ into partial fractions.

(b). Hence or otherwise show that:

$$\int_0^1 \frac{x^3 + 9x^2 + 28x + 28}{(x + 3)^2}\, dx = \frac{1}{3}\left(10 + \ln\frac{4}{3}\right)$$

[12 Marks]

Question 11:

(a). Determine the equation of the plane equidistant from the points $A(1, 3, 5)$ and $B(2, -4, 4)$. **[4 Marks]**

(b). (i). Find the coordinates of the point, P, in which the plane $4x + 5y + 6z = 87$ intersects the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z+4}{2}$.

(ii). Calculate the angle between the line and the plane in (b)(i) above. **[8 Marks]**

Question 12:

(a). Given that $\left(\frac{x}{2x-1}\right) + \left(\frac{x}{2x-1}\right)^2 + \left(\frac{x}{2x-1}\right)^3 + \cdots = \frac{3}{4}$, find the value of x. **[3 Marks]**

(b). Given that the factors $(x - 1)$ and $(x + 1)$ are factors of the polynomial, $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$, find the values of the constants a and b. Hence, find the set for real values of x for which $f(x) > 0$. **[9 Marks]**

Question 13:

(a). Show that the equation of the circle passing through the points $(-2, -4)$, $(3, 1)$ and $(-2, 0)$ is $(x - 1)^2 + (y + 2)^2 = 13$. **[7 Marks]**

(b). With reference to the circle in (a) above, show that the tangent at point $(3, 1)$ is parallel to the diameter that passes through the point $(-2, 0)$. **[5 Marks]**

Question 14:

(a). Using calculus of small changes, show that $\cos 44.6° = \frac{\sqrt{2}}{2}\left(\frac{900+2\pi}{900}\right)$

[5 Marks]

(b). The pressure in an engine cylinder is given by:

$$P = 8000[1 - \sin(2\pi t - 3)] \text{ N m}^{-2}$$

At what time does this reach a maximum and what is the maximum pressure?

[7 Marks]

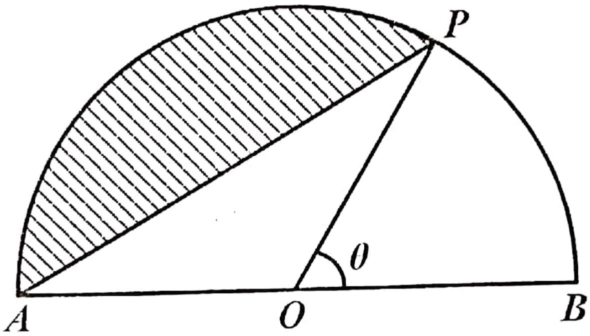
Question 15:

(a). Solve the equation: $\frac{4\sin^2\theta}{\operatorname{cosec}\theta} + \frac{3}{\operatorname{cosec}^2\theta \sec\theta} = \sin^2\theta$,

for $0° \leq \theta \leq 360°$.

[6 Marks]

(b). The diagram shows a semicircle APB on AB as diameter.



The midpoint of AB is O. The point, P, on the semicircle is such that the area of the sector POB is equal to twice the area of the shaded segment. Given that ange POB is θ radians,

show that: $3\theta = 2(\pi - \sin\theta)$.

[6 Marks]

Question 16:

(a). Solve the differential equation $(x+2)^2\frac{dy}{dx} + 3y^2 = 0$ given that $y(0) = -2$.

[4 Marks]

(b). A girl returning from a milling point is carrying mealie-meal in a cylindrical container. The container has a hole at its base and the mealie-meal trickles out through this hole. It is estimated that the rate of reduction of mealie-meal is proportional to the mass, M, of mealie-meal remaining in the container after a time, t.

(i). Form a differential equation connecting M, t and the constant of proportionality, k.

(ii). Find the general solution of the differential equation in (b)(i) above and show that it reduces to $M = M_0e^{-kt}$; where M_0 is the initial mass of mealie-meal.

(iii). The girl takes 2 hours to walk from the milling point to her home. Given that after one hour, ten percent of the mealie-meal is lost, calculate the percentage of the mealie-meal is the container when she arrives home.

[8 Marks]

END