P425/
PURE MATHEMATICS
Paper 1
July, 2024
3 Hours

RESOURCEFUL MOCKS-2024

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and any **five** from section **B.***

Any additional question (s) answered will not be marked

*All necessary working **must** be shown clearly*

Begin each answer on a fresh sheet of paper

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1. Solve the simultaneous equations:

   $x^2 + xy + 4y^2 = 6$ and $3x^2 + 8y^2 = 14$ *(05 marks)*

2. One side of a rectangle is three times the other. If the perimeter increases by 2%. What is the percentage increase in the area? *(05 marks)*

3. The points A and B have position vectors **a** and **b** respectively relative to the origin, where $a = 2i + j - 3k$ and $b = -4i + sj + tk$. Find the possible values of s and t if $|AB| = 7$ and $s = 2t$ *(05 marks)*

4. The first three terms of a G.P are $2x - 1, x + 1$ and $x - 1 \ (x \neq 0)$. Find the value of x and the sum to infinity of the G.P. *(05 marks)*

5. If $t = \tan\theta$ and $sec2\theta + \tan 2\theta = k$, prove that $t = \frac{k-1}{k+1}$ *(05 marks)*

6. Show that: $\int_0^1 \frac{1}{(1+x^2)^2} dx = \frac{\pi}{8} + \frac{1}{4}$ *(05 marks)*

7. If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$, show that $m = \frac{a}{c}$. *(05 marks)*

8. Find the value of k for which the lines $3x + 4y - k = 0$ and $12x - 5y + 29 = 0$ are equidistant from the point $(1, 3)$. *(05 marks)*

SECTION B(60 MARKS)

9. (a) If $cosecA - cotA = q$,then show that $\frac{q^2-1}{q^2+1} + \cos A = 0$ *(05 marks)*

   (b) Solve the equation; $3tan^3\theta - 3tan^2\theta = \tan\theta - 1$ for $0 \leq \theta \leq 2\pi$ *(07 marks)*

10. (a) Find the cartesian equation of the plane passing through the points $P(1, 0, -2), Q(3, -1, 1)$ and parallel to the line $r = 3i + (2\alpha - 1)j + (5 - \alpha)k$ *(07 marks)*

    (b) Find the length of the perpendicular drawn from a point $(2, 3, -4)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ *(05 marks)*

11. (a) The expression $f(x) = 6x^2 + x + 7$ leaves the same remainder when divided by $x - a$ and $x + 2a$ .Find the value of a for which $a > 0$.

(05 marks)

(b) The polynomial $P(x) = \alpha x^3 - \mu x^2 + \beta x + 2$ gives a remainder -60 when divided by $x + 2$ and $f(3) = 35$. Given that $2x - 1$ is a factor of the polynomial. Find the values of α, μ and β .Hence evaluate $P(x) = 0$

(07 marks)

12. (a) The point $(2, 1)$ lies on the curve $Ax^2 + By^2 = 11$ where A and B are constants. If the gradient of the curve at that point is 6. Find the values of A and B. *(05 marks)*

(b) A rectangular box without a lid is made from a thin cardboard. The sides of the base are $2xcm$ and $3xcm$ and the height of the box is hcm. If the total surface area is $200cm^2$, show that $h = \frac{20}{x} - \frac{3x}{5}\,cm$. And hence find the dimensions of the box to give maximum volume. *(07 marks)*

13. (a) Find the values of x and y in: $\frac{x}{1+i} + \frac{y}{2-i} = 2 + 4i$ *(06 marks)*

(b) Given that P is represented by $|Z - 2| = 2|Z + 1|$. Show that the locus of P is a circle and hence state it's radius and centre. *(06 marks)*

14. (a) Express $f(x) = \frac{32}{x^3 - 16x}$ into partial fractions. Hence find $\int f(x)\, dx$

(07 marks)

(b) Show that :$\int_2^4 x\ln x dx = 14\ln 2 - 3$ *(05 marks)*

15. (a) Find the length of the tangent to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ from the point $(5, 7)$. *(05 marks)*

(b) Prove that the circles $x^2 + y^2 - 10x - 7y + 31 = 0$ and $x^2 + y^2 + 2x + 2y - 23 = 0$ touch each other externally. *(07 marks)*

16. (a) Solve the differential equation;

$(1 + x^2)\frac{dy}{dx} = 1 + y^2$ for $y = 3$ and $x = 2$ *(05 marks)*

(b) At time, t hours, the rate of decay of a radioactive element is directly proportional to it's current mass.

(i) Show that $N = N_0 e^{-kt}$ where N is the mass of the radioactive element and N_0 is the original mass.

(ii) If the mass reduces to half the original mass in 4 hours, find the time it takes for the mass of the element to reach $\frac{1}{8}$ of the original mass.

(07 marks)

END