P425/1
PURE MATHEMATICS
AUGUST - 2022
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2022

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all the eight** questions in section A and any **five** from section B.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. By using the Binomial theorem, expand $(1+3x)^{\frac{1}{3}}$ up to the fourth term. Hence by substituting $x = \frac{1}{125}$, evaluate $\sqrt[3]{2}$ to 3 significant figures. (05 marks)

2. Solve the equation $\cot\theta + \tan\theta = 2\cos ec^2\theta$ for $0^0 \leq \theta \leq 360^0$. (05 marks)

3. Find $\int \sin(\sqrt{x})\, dx$ (05 marks)

4. Find the equation of a line through point (2 , 3) which makes an angle of 135^0 with the line $4x - 3y + 5 = 0$. (05 marks)

5. Show that the vector $2\boldsymbol{i} + b\boldsymbol{j} + 5\boldsymbol{k}$ is normal to the plane

$$\boldsymbol{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \boldsymbol{s}\begin{pmatrix}2\\1\\-2\end{pmatrix} + \boldsymbol{t}\begin{pmatrix}1\\-2\\2\end{pmatrix}$$

Hence determine the equation of the plane in the form $\boldsymbol{r}\,.\boldsymbol{n} = \boldsymbol{d}$. (05 marks)

6. Solve the simultaneous equations

$$\log_2 x + \log_2 y = 3$$
$$\log_4 x - \log_4 y = -\tfrac{1}{2}$$

(05 marks)

7. Given that $x^2 + 4xy + 3y^2 = 5$, show that $\frac{d^2y}{dx^2} = \frac{5}{(2x+3y)^3}$ (05 marks)

8. Find the equation of the tangent to the curve $(y-2)^2 = x$ which is parallel to the line $x - 2y - 4 = 0$. (05 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks

9. (a) Calculate the square roots of $15 + 8i$ (06 marks)

(b) The loci of C_1 and C_2 are given by $|z-3| = 3$ and $\text{Arg }(z-1) = \frac{\pi}{4}$ sketch on the same argand diagram the loci of C_1 and C_2. (06 marks)

10. (a) Integrate with respect to x

$$\frac{\sqrt{16-x^2}}{x^2}$$

(06 marks)

(b) By using substitution $x = 2sint$, show that

$$\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}}\, dx = \frac{\pi}{2} + \sqrt{3} - 1$$

(06 marks)

Turn over

$\tan(\theta+60) \tan(\theta-60)$

11. Prove that $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = \frac{\tan^2\theta - 3}{1-3\tan^2\theta}$. Hence solve the equation $\tan(\theta + 60^0)\ \tan(\theta - 60^0) = 4\sec^2\theta - 3$, for $0^0 < \theta < 360^0$ (12 marks)

12. The lines L_1 and L_2 have equations.

$$L_1 : \boldsymbol{r} = (1 + 2t)\boldsymbol{i} + 2t\boldsymbol{j} - (4 + 3t)\boldsymbol{k}$$
$$L_2 : \boldsymbol{r} = (4 + as)\boldsymbol{i} + (6 + 4s)\boldsymbol{j} + (2 + 9s)\boldsymbol{k}$$

Respectively, where a is a constant

(a) Find the acute angle between L_1 and the $x - axis$. (05 marks)

$a \cdot b = |a||b| \cos\theta$

(b) Given that point A has position vector $2\boldsymbol{i} - 2\boldsymbol{j} + b\boldsymbol{k}$ and that the line L_2 passes through point A, determine the

(i) values of a and b

(ii) perpendicular distance of A from the line L_1. (07 marks)

13. (a) The equation of the normal to the parabola $y^2 = 8x$ at the point $P(2t^2, 4t)$, $(t \neq 0)$ is given by $y + xt = 4t + 2t^3$.

Given that this normal meets the curve again at Q, show that the length of

$$\overline{PQ} = \frac{8\left(1+t^2\right)^{\frac{3}{2}}}{t^2}$$ (07 marks)

(b) The line through the point of focus, S (2, 0) parallel to PQ meets the tangent at P to the parabola at point M. Given that 5SM = PQ, prove that $t^2 = 4$. (05 marks)

14. (a) If α and β are the roots of the equation $x^2 + Px + q = 0$; Express

(i) $\alpha^3 + \beta^3$ and

(ii) $(\alpha - \beta^2)(\beta - \alpha^2)$, in terms of p and q. Deduce that the condition for one root of the equation to be the square of the other is

$p^3 - 3pq + q^2 + q = 0$ (06 marks)

(b) The prices of three items are in a Geometric progression (G.P). If the total prices for these three items is shs 8400 and the most expensive item priced at shs 4800, find the prices of the other two items. (06 marks)

15. (a) The equation of the curve is given by $x^3 + y^3 = 3xy$

Find the gradient of the tangent to the curve at $\left(\frac{3}{2}, \frac{3}{2}\right)$ (05 marks)

(b) An open box is to be made from a rectangular sheet measuring 16cm by 10cm by cutting squares of side $x$cm from each corner and turning up the edges. Calculate the value of x, so that the volume of the box is maximum. (07 marks)

16. (a) Solve the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \ln x \ , \quad x > 0 \ \text{ given that } y = 2 \text{ and } x = 1 \qquad \text{(05 marks)}$$

(b) At 2.23pm, the temperature of water in a kettle boiled at 100^0C and that of the surrounding 21^0C. At 2.33pm the temperature of water in the kettle had dropped to 84^0C. If the rate of cooling of the water was directly proportional to the difference between its temperature θ and that of the surroundings,

(i) write a differential equation to represent the rate of cooling of water in the kettle. (01 mark)

(ii) solve the differential equation using the given conditions. (04 marks)

(iii) find the temperature of the water at 2.44pm (02 marks)