OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2022

ALGEBRA

1. (a) Without using mathematical tables or calculators, find the value of $\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}$.

(b) Given that $2x^2+7x-4$, x^2+3x-4, and $7x^2+ax-8$ have a common factor, find the;

(i) Factors of $2x^2+7x-4$ and x^2+3x-4.

(ii) Value of a in $7x^2+ax-8$.

(c) A man deposits shs 150,000 at the beginning of every year in a microfinance bank with the understanding that at the end of seven years, he is paid back his money with 5% per annum compound interest. How much does he receive?

(d) Given that the roots of the equation; $x^2-bx+c=0$ are $\sqrt{\propto}$ and $\sqrt{\beta}$. Show that;

(i) $\propto+\beta=b^2-2c$

(ii) $\propto^2+\beta^2=(b^2-2c-\sqrt{2}c)(b^2-2c+\sqrt{2}c)$

2. (a) Find the solution set for which $\log_2 x-\log_x 4\leq 1$.

(b) Solve the simultaneous equations; $2a-3b+c=10$,

$a+4b+2c+3=0$,

$5a-2b-c=7$

(c) A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10?

(d) Given that the equations $y^3-2y+4=0$ and $y^2+y+c=0$ have a common root, show that $c^3+4c^2+14c+20=0$.

3. (a) Simplify $(2+5i)^2+5\left(\frac{7+2i}{3-4i}\right)-i(4-6i)$ expressing your answer in the form $a+bi$.

(b) The roots of the equation $3x^2+2x-5=0$ are $\propto$ and β. Find the value of $\propto^4+\beta^4$.

(c) Solve the equation $\sqrt{x+5}+\sqrt{x+21}=\sqrt{6x+40}$.

(d) Given that $\log_5 21=m$ and $\log_9 75=n$, show that $\log_5 7=\frac{1}{2n-1}(2nm-m-2)$

4. (a) Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $\sqrt[3]{24}$ correct to three s.f.

(b) In the expansion of $(1+ax)^n$, the first three terms are $1-\frac{5}{2}x+\frac{75}{8}x^2$. Find n and a , state the range of values for which the expansion is valid.

(c) Determine the term independent of x in the binomial expansion of $\left(\frac{3}{x^2} - 2x\right)^6$.

(d) Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$

ANALYSIS

5. (a) If $y = tan^{-1}\left(\frac{ax-b}{bx+a}\right)$,show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(b) Differentiate $cos(x^2e^x)$ with respect to x.

(c) Differentiate cos^2x with respect to x from first principles.

(d) Given that $x = \frac{3t-1}{t}$ and $y = \frac{t^2+4}{t}$,

(i) Show that $\frac{d^2y}{dx^2} = 2t^3$

(ii) Determine the area of the largest rectangular piece of land that can be enclosed by $100m$ of fencing if part of an already existing wall is used.

6. (a) Solve the differential equation $\frac{dy}{dx} + \frac{2xy}{x^2+1} - x = 0$

(b) The gradient of a certain curve is given by kx. If the curve passes through the point (2,3) and the tangent at this point makes an angle of $tan^{-1}(6)$ with the positive direction of the x-axis, find the equation of the curve.

(c) A research to investigate the effect of a certain chemical on a virus infection, the research revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time. Initially , the population was p_o. At $t\ months$ later, it was found to be p.

(i) Form a differential equation connecting p and t

(ii) Given that the virus population reduced to one third of the initial population in $4\ months$, solve the equation in c(i) above.

7. (a) Partialise fully, $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Hence $\int f(x)dx$ from 4 to 5.

(b)(i) On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$

(ii) Find the area enclosed by the two curves in b(i) above

(iii) Determine the volume generated when the area enclosed by the two curves in b(ii) above is rotated about the x-axis.

8. (a) Find the integral $\int xcos^2x\, dx$

(b)Find the area enclosed between the curve $y = x(x-1)(x-2)$ from $x = 0\ to\ x = 2$

(c) A conical vessel whose height is $10metres$ and radius of the base $5m$ is being filled with water at a uniform rate of $1.5m^3min^{-1}$. Find the rate at which the level of the water in the vessel is rising when the depth is $4m$

(d) Find the area enclosed by the curve $y = x - \frac{1}{x}$, the $x - axis$ and the line $x = 2$.

TRIGONOMETRY

9. (a) Solve the equation $3cos4\theta + 7cos2\theta = 0$ for $0^0 \leq \theta \leq 180^0$

(b) Express $10sinxcosx + 12\ cos2x$ in the form $Rsin(2x{+}\propto)$. Hence find the maximum value of $10sinxcosx + 12\ cos2x$.

(c) Prove that $\frac{cos11^0+sin11^0}{cos11^0-sin11^0} = tan56^0$

(d) In any triangle ABC, prove that $SinB + SinC - SinA = 4cos\,{}^{A}/_{2}\,Sin\,{}^{B}/_{2}\,Sin\,{}^{C}/_{2}$.

10. (a) Find the values of x lying between -180^0 and 180^0 that satisfy the equation $10sin^2x + 10sinxcosx = cos^2x + 2$

(b) If $\frac{Sin16\theta cos2\theta - cos6\theta sin12\theta}{cos4\theta cos2\theta + sin6\theta sin8\theta} = tanm\theta$, where m is a constant, find the value of m.

(c) Find all the solutions to $2sin3\theta = 1$ for θ between 0^0 and 360^0. Hence find the solutions for $8x^3 - 6x + 1 = 0$

(d)Show that in any triangle ABC, $\frac{a^2-b^2}{c^2} = \frac{Sin(A-B)}{Sin(A+B)}$

11. (a) Using the t-formulae or otherwise prove that $1 + sec2\theta = tan2\theta cot\theta$

(b) Given that $y = \frac{sinx-2sin2x+sin3x}{sinx+2sin2x+sin3x}$;

(i) Prove that $y + tan^2\left(\frac{x}{2}\right) = 0$,

(ii) And hence express the exact value of $tan^2 15^0$ in the form $p + q\sqrt{r}$ where p, q and r are integers.

(iii) Hence, find the value of x in the range $0^0 \leq x \leq 360^0$ for which $2y + sec^2\left(\frac{x}{2}\right) = 0$

12. (a)Given that $45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2 + \sqrt{2}}$

(b) Given that $P = 2cos2x + 3cos4x$ and $q = 2sin2x + 3sin4x$;

(i) Find the greatest and least value of $p^2 + q^2$.

(ii) Given that $p^2 + q^2 = 19$, find x for $0^0 \leq x \leq 90^0$

(iii) Using the result in (ii) above, and without using a calculator or Mathematical tables, show that $pq = \frac{-5\sqrt{3}}{4}$.

VECTORS

13. (a) Line A is intersection of two planes whose equations are $3x - y + z = 2$ *and* $x - 5y + 2z = 6$. Find the Cartesian equation of the line

(b) Given that line B is perpendicular to the plane $3x - y + z = 2$ and passes through the point C(1,1,0), find the;

(i) Cartesian equation of the line B

(ii) Angle between the line B and line A in (a) above.

(c) The point $A(2,-1,0)$, B(-2, 5, -4) and C are on a straight line such that $3\overrightarrow{AB} = 2\overrightarrow{AC}$. Find the coordinates of C.

14. (a)The points P, Q, R have position vectors $2\boldsymbol{a} - 5\boldsymbol{b}, 5\boldsymbol{a} - \boldsymbol{b}, and\ 11\boldsymbol{a} + 7\boldsymbol{b}$ respectively. Show that P, Q and R are collinear and state the ratio $PQ: QR$.

(b) Calculate the perpendicular distance from the point $(1, -2, 3)$ from the line with equation $\boldsymbol{r} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})$.

(c) Determine the equation of a plane through the point $(1, -3, 2)$ and contains the vectors $\boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{k}$ and$-\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

15. (a) The point $C(a, 4, 5)$ divides the line joining $A(1,2,3)$ and B (6,7,8) in the ratio $\lambda: 3$. Find a andλ

(b) Find the equation of a plane which contains the line $\frac{x-1}{2} = \frac{y+4}{-3} = \frac{z+1}{-1}$ and passes through the point $(2, 3, -1)$

(c) Show that the lines L_1, with vector equation $\boldsymbol{r} = \binom{2}{5} + \lambda\binom{2}{-3}$ and L_2, vector equation $r = \binom{3}{-3} + \mu\binom{3}{2}$ are perpendicular and find the position vector of their point of intersection.

16. (a) Find the length of the perpendicular distance from $A(4,3,5)$ to the plane $6x - y + 2z = 14$

(b) Find the foot of the perpendicular drawn from the point $(2, -1, 5)$ to the line $\frac{x-11}{10} = \frac{y+2}{-2} = \frac{z+5}{-11}$.

(c) Find the angle between the plane $x - 2y + z = 20$ and the line $\frac{2-x}{-3} = \frac{y+1}{4} = \frac{2-z}{-12}$

(d) $PQRS$ is a quadrilateral $P(1, -2), Q(4, -1), R(5,2)$ and $S(2,1)$. Show that the quadrilateral is a rhombus.

COORDINATE GEOMETRY

17. (a) Find the locus of a point which moves such that the ratio of its distance from the point $A(2,4)$ to its distance from the point $B(-5,3)$ is $2:3$

(b) A point P moves such that its distance from two points $A(-2,0)$ and $B(8,6)$ is in the ratio $AP:PB = 3:2$. Show that the locus of P is a circle.

(c) Find the locus of the point $P(x,y)$ which moves such that its distance from the point $A(5,3)$ is twice its distance from $x = 2$

(d) Find the centroid of the triangle whose sides are given by the equations $x + y = 11$, $y = x - 1$ and $3y = x - 3$.

18. (a) find the equation of the circle whose end diameter is the line joining the points $A(1,3)$ and $B(-2,5)$.

(b) A circle whose centre is in the first quadrant touches the x- and y-axes and the line $8x - 15y = 20$.

Find the;

(i) Equation of the circle

(ii) Point at which the circle touches the y-axis

(c) If the x-axis and y-axis are tangents to the circle$x^2 + y^2 + 2gx + 2fy + c = 0$, Show that $c = g^2 = f^2$

19. (a) ABCD is a square inscribed in a circle $x^2 + y^2 - 4x - 3y = 36$, Find the length of diagonals and the area of the square

(b) Find the coordinates of the foot of the perpendicular from the point (2,-6) to the line $3y - x + 2 = 0$.

(c) A circle touches both x-axis and the line $4x - 3y + 4 = 0$. Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$

(d) ABCD is a rhombus such that the coordinates $A(-3,-4)$ and $C(5,4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2 obtain the coordinates of B and D, Prove that the area of the rhombus is $21\frac{1}{3}sq.Units$.

20. (a) Show that the equation $y^2 - 4y = 4x$ represents a parabola; hence determine the focus, vertex and directrix.

(b) A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find;

(i) The equation of the tangent

(ii) The equation of the line L parallel to the normal at $T(t^2, 2t)$ and passes through (1,0)

(iii) The point of intersection, X, of the line L and the tangent

(c) A point $P(x,y)$ is equidistant from X and T in (b) above, Show that the locus of P is $t^3 + 3t - 2(xt + y) = 0$

(d) Find the equations of the tangents to the parabola $y^2 = 6x$ which pass through the point (10,-8)

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