EDEN INTERNATIONAL SCHOOL

END OF TERM I EXAMINATIONS ,2018

P425 MATHEMATICS PAPER ONE

(PURE MATHEMATICS)

TIME ; 3 Hours

Instructions to candidates

- Attempt **all** questions in **section** A and only **fivequestions** in **section** B.
- Begin each question on a fresh page
- Non-programmable calculators may be used.

SECTION A

1. If α and β are the roots of $x^2 + bx + c = 0$, express $(\alpha - 2\beta)(\beta - 2\alpha)$ in terms of a, b and c. Hence deduce the condition for one of the roots to be twice the other. (05 marks)
2. Show that ,$\frac{d}{dx}(\sec x\, tan^n x) = \sec x\,\{ntan^{n-1}x + (n+1)tan^{n+1}x\}$.
   (05 marks)
3. If $sinA = \frac{5}{13}$, $sinB = \frac{8}{17}$, where A and B are acute, find the values of
   a) $\cos(A + B)$, b) $\sin(A - B)$ (05 marks)
4. An arithmetic progression(A.P) has thirteen terms whose sum is 143. The third term is 5. Find the first term and the common difference. (05 marks)
5. Evaluate$\int_0^{\frac{\pi}{2}} x^2 sinx\ dx$ (05 marks)
6. Obtain the first four terms of the expansion of $\frac{1+x}{1-x}$(05 marks)
7. Solve the equation : $\log_{25} 4x^2 = \log_5(3 - x^2)$ (05 marks)
8. Given two points with position vectors $\boldsymbol{a} = 2i - 3j + k$and $\boldsymbol{b} = i + j - k$
   Find the;
   i. Angle between $\boldsymbol{a}$ and $\boldsymbol{b}$
   ii. Vector equation of a line passing through the points. (05 marks)

SECTION B

9. (a) The sum of n terms of a certain series is $3n^2 + 10n$ for all values of n. Obtain the expression for the nth term and show that a series is an arithmetic progression (A.P)
   (05 marks)

   (b) The eighth term of an arithmetic progression is twice the third term , and the the sum of the first eight terms is 39.Find the first three terms of the progression and show that its sum to n terms is$\frac{3}{8}n(n + 5)$ (07 marks)

10. (a) Solve the equation for the value of x from 0^0 to 360^0 inclusive
$\cos(x + 20^0) = \cos(x - 70^0)$ (06 marks)
(b) prove the following identities
i. $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$ (03 marks)
ii. $\cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$ (03 marks)

11. (a) If the roots of the equation $2x^2 - 4x + 1 = 0$ are α and β, find the equation whose roots are $\alpha - 2$ and $\beta - 2$. (05 marks)
(b) Find the first four terms of the expansion of $(1 - 8x)^{\frac{1}{2}}$ in the ascending powers of x. Hence by putting $x = \frac{1}{100}$, evaluate $\sqrt{23}$ correct to five significant figures. (07 marks)

12. (a) Given a complex number
$Z = \frac{(3i+1)(i-2)^2}{i-3}$, determine;
i. Z in the form $a + bi$, where a and b are constants
ii. Arg(Z) (06 marks)
(b) Find the values of x and y in
$\frac{x}{(2+3i)} - \frac{y}{(3-2i)} = \frac{(6+2i)}{(1+8i)}$ (06 marks)

13. (a) Find the gradient of the curve that is given parametrically as $x = a\cos\theta$ and $y = b\sin\theta$ at the point where $\theta = \frac{\pi}{4}$. (06 marks)
(b) Use Maclaurin's theorem to expand $\frac{1}{\sqrt{(1+x)}}$ up to the term in x^3. (06 marks)

14. Express $f(x) = \frac{5x^2-71}{(x+5)(x-4)}$ in partial fractions .Hence $\int f(x)\,dx$. (12 marks)

15. (a)Find the angle of intersection between the lines
$x = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}$ (06 marks)
(b)Find the vector equation of the line of intersection of the two planes
$2x + 3y - z = 4$ and $x - y + 2z = 5$ (06 marks)

16. Evaluate the following
a) $\int_0^1 xe^{3x}dx$ (06 marks)
b) $\int \frac{dx}{x^2\sqrt{(1-x^2)}}$ (06 marks)

-END-

P425/1

PURE MATHEMATICS

PAPER 1

16th. 06. 2015

3 hours

RS-MOCK EXAMINATIONS 2015

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

All *working* **must** *be shown clearly*

Begin each question on a fresh page

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *questions in this section*

1. Solve for x in : $(\log_x 4)(\log_8 x) = \log_{27} x$. *(05marks)*

2. The lines $y = m_1 x + 1$ and $y = m_2 x + 1$ intersect in a point A. The acute angle between the lines is 45^0, where $m_1 > m_2$, and the x intercept of the line $y = m_2 x + 1$ is 2 .

   (i) State the co-ordinates of point A.

   (ii) Find the values of m_1 and m_2. *(05 marks)*

3. Prove the identity: $\dfrac{\sin\alpha + \sin(\beta - \lambda)}{\cos(\beta - \lambda) + \cos\alpha} = \cot\lambda$; where $\alpha + \beta + \lambda = 180^0$ *(05 marks)*

4. Show that $\int \dfrac{3^x}{3^x + 3}dx = \log_3 A(3^x + 3)$; where A is a constant. *(05 marks)*

5. The parametric equations of a line are: $x = 2\lambda + 1$, $y = \lambda - 3$ and $z = \lambda + 2$; where λ is the parameter.

   (i) Find the Cartesian equation of the line.

   (ii) Determine the coordinates of the point where the line meets the plane $x - y + z = 4$. *(05 marks)*

6. Evaluate $(1 + i)^8$.

7. Given that $x = \ln(xy)$ show that $\dfrac{dy}{dx} = \left(\dfrac{x-1}{x^2}\right)e^x$. *(05 marks)*

8. The gradient function of the curve is $2x + \dfrac{54}{x^2}$. If the y - coordinate of the stationary point of the curve is 7, find the equation of the curve.

   (05 marks)

SECTION B: (60 MARKS)

*Answer only **five** questions from this section. All questions carry equal marks*

9. (a) Find the principal argument of the number $\left(\frac{-\sqrt{3}+i}{1-i}\right)^3$ *(05 marks)*

(b) Find and sketch the locus $Arg(z+2-i)=\pm\frac{\pi}{4}$ *(07 marks)*

10. Given the curve $y=\frac{2x^2-72}{x^2-9}$,

(i) Find the equations of the asymptotes

(ii) State the equation of the line of symmetry of the curve, hence or otherwise determine the coordinates of the stationary point.

(iii) Sketch the curve; and deduce the range of values of y within which the curve does not lie. *(12 marks)*

11. (a) Given that $y=\tan x$; show that $\frac{d^2y}{dx^2}=2\tan x\frac{dy}{dx}$; hence find the first three terms of the maclaurin's expansion of $\tan x$. Hence evaluate $\tan 1.8^0$ to 4 decimal places.

(b) Express $11+8x-2x^2$ in the form $a+b(x+c)^2$, where a,b and c are constants, hence deduce the minimum positive value of $\frac{1}{12+8x-2x^2}$. *(12 marks)*

12. (a) Solve the equation: $2\sin^3\theta=\sin 3\theta$ for $0\le\theta\le 2\pi$. *(06 marks)*

(b) Prove that: $\frac{\sin\theta}{1+\cos\theta}=\tan\frac{1}{2}\theta$, hence solve the equation: $\sin\theta-\cos\theta=1$ for $-360^0\le\theta\le 90^0$. *(06 marks)*

13. (a) Evaluate : $\int_2^3\frac{dx}{x(1+x^2)}$ *(06 marks)*

(b) Find : $\int \frac{dx}{x^2\sqrt{9-x^2}}$ *(06 marks)*

14.(a) Calculate the distance of the point P(1,2,3) from the line $\frac{x}{3} = \frac{y+3}{4} = z$

(06 marks)

(b) Find the scalar product equation of the plane containing the point P (1, 2, 3) and the line $\frac{x}{3} = \frac{y+3}{4} = z$ in (a) above. *(06 marks)*

15.The parametric equations of a curve are given as $y = \frac{3}{2}t^2 - 6t + 1$ and $x = t^2 + t - 1$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t. *(05 marks)*

(b) Find and determine the nature of the stationary point of the curve.

(04 marks)

(c) Obtain the equation of the tangent to the curve at the point corresponding to $t = 1$. *(03 marks)*

16.The temperature of a body falls at a rate proportional to the amount by which the temperature of the body exceeds that of the surroundings. A body at 100^{0}C is placed in a room maintained at 20^{0}C. After 5 minutes the temperature of the body is 60^{0}C. Find the;

(a) temperature of the body after the next 5 minutes

(b) temperature of the body and the time taken to reach the point at which the rate of cooling is $-(\ln 1024)^0 C$ per minute. *(12 marks)*

END

P425/1
PURE MATHEMATICS
PAPER 1
Time: 3 hrs

Instructions.
Attempt ***all*** questions in section A.
Attempt any ***five*** questions from section B

SECTION A (40MARKS)

1. Solve the equation $(\log_3 x^2)(\log_{9x} 3) = 1$ (5marks)
2. Find the equations of the tangents to the curve $y = x^2 - x + 1$ from the point $(0, 0)$. (5 marks)
3. Solve the equation: $2 + \cos 2\theta = Sin^2\theta$ for $0^0 \leq \theta \leq 360^0$ (5marks)
4. $A(1, -3), B(5,2) and\ C(3,4)$ are vertices of a triangle. Find the equation of the median of the triangle passing through the vertex A. Hence or otherwise deduce the coordinates of the centroid of the triangle. (5marks)
5. Given that the vectors $\boldsymbol{a} = 3i - j + \mu k$ and $\boldsymbol{b} = \lambda i + 2j + 3k$ are parallel to each other. Find the i) values of λ and μ
   ii) ratio $|\boldsymbol{a}|:|\boldsymbol{b}|$. (5marks)
6. Find $\int \frac{d\theta}{\sec\theta - 1}$ (5marks)
7. Taking x=0.01, use the binomial expansion of $\sqrt{(1-2x)}$ to evaluate $\sqrt{2}$ to 4 decimal places. (5marks)
8. Using the mathematics of small changes, show that
   $sin^2(30.5^0) \approx \frac{180+\pi\sqrt{3}}{720}$ (5marks)

SECTION B (60MARKS)

9. a) Given that α and β are the roots of the quadratic equation $x^2 + \sqrt{3}x + 3 = 0$. find the quadratic equation whose roots are
   $\frac{1}{\alpha^2-1}$ and $\frac{1}{\beta^2-1}$ (6marks)
   b) Find the range of values of x for which the quadratic equation
   $xy^2 - 2y + 5 - 4x = 0$ has no real roots. (6marks)

10. a) Solve the equation: $\tan\theta = \cot(\theta - 60^0)$ for $0^0 \leq \theta \leq 270^0$ (6marks)
   b) Use the cosine rule on triangle ABC to prove that $\cos\frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$, where
   $s = \frac{1}{2}(a + b + c)$.
   (6marks)

11. a) Find the values of t for which the tangent to the curve $y^2 = 4x$ at the point $(t^2, 2t)$ passes through the point (-6,1). (6marks)

b) $P(ct, c/t)$ is a point on the curve $xy = c^2$, and the normal to the curve at the point $P(ct, c/t)$ meets the curve again at the point $Q(cT, c/T)$

Show that i) the equation of the normal PQ is $y = t^2 + \frac{c}{t} - ct^3$.

ii) $t^3T = -1$. (6marks)

12. a) Differentiate:

i) $5x^{5x}$

ii) $2x^2 \ln\sqrt{\cos 2x}$. (6marks)

b) Given that $y = cosecx + \cot x$, prove that $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$.

(6marks)

13. Given the lines $\boldsymbol{r} = \begin{pmatrix}4\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\frac{x+3}{3} = \frac{1-y}{1} = \frac{z+6}{2}$. find the

i) coordinates of their point of intersection.

ii) acute angle between the lines.

iii) Cartesian equation of the plane containing the lines. (12marks)

14. a) Sketch the equation $Z^3 + 8i = 0$. (6marks)

b) Sketch the locus $\arg(Z + 2 - i) = \frac{-\pi}{4}$ for the point representing $Z = x + iy$. Hence determine the minimum value of $|Z|$. (6marks)

15. a) Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec^2x tan^3x\, dx$. (5marks)

b) Find $\int \frac{(1+x)^2}{x^2(1+x^2)} dx$. (7 marks)

16. a) Solve the equation: $\sin x\frac{dy}{dx} + y = \sin x$ (5marks)

b) A space craft sets off from the earth for the moon. The speed of the spacecraft is directly proportional to the product of the distance travelled and the time taken in flight. After 1 hour, the distance travelled is 1000km, and after 2 hours the distance travelled is 8000km. Calculate how long it takes to reach the moon which is 400,000km from the earth. (7marks)

END

ITENDERO SECONDARY SCHOOL

S.6 ENTRANCE EXAMINATION

PRINCIPAL MATHEMATICS

PAPER ONE

INSTRUCTIONS:

ATTEMPT ALL QUESTIONS IN SECTION A AND ONLY FIVE FROM SECTION B

TIME: 3 HOURS

SECTION A

1. The 5th term of an arithmetic progression (A.P) is 12 and the sum of the first 5 terms is 80. Determine the first term and the common difference (5mks)
2. Determine the possible values of x in the equation $\log_2 x + \log_x 64 = 5$ (5mks)
3. If $x^2 + 3xy - y^2 = 3. find\ \frac{dy}{dx}\ and\ \frac{d^2y}{dx^2}\quad at\ (1,1)$ (5mks)
4. The roots of the equation $2x^2 - 4x + 1 = 0\ are\ \alpha\ and\ \beta$. Find the equations with the integral coefficients whose roots are
   i) $\frac{1}{\alpha} + \frac{1}{\beta}$
   ii) $\frac{\alpha}{\beta}\ ,\frac{\beta}{\alpha}$ (5mks)
5. Solve the equation
   $\cos 6x + \cos 4x + \cos 2x = 0\ for\ values\ of\ x\ from\ 0^0\ to\ 180^0\ inclusive$ (5mks)
6. The area of the segment cut off by t $y = 5\ from\ the\ curve\ y = x^2 + 1\ is\ rotated\ about\ y = 5$. Find the volume generated by the curve (5mks)
7. Expand $(1 - x)^{\frac{1}{3}}$ as far as the term x^3 . Use your expansion to deduce $\sqrt[3]{24}$. Correct to 3 significant figures (5mks)

8. A right circular cone of radius r has a maximum volume, the sum of its vertical height h , and the circumference is 15cm . if the radius varies , show that the maximum volume of the cone is $\frac{125}{3\pi} cm^3$ (5mks)

SECTION B

9a) Find the binomial expansion of

$(1-\frac{x}{2})^5$, *use your expansion to estimate* $(0.875)^5$ *to* 4 decimal places (7mks)

b)A financial credit society gives 2% compound interest per annum to its members .

If John deposits shs. 100,000 at the beginning of every year starting with 2004 , how much would he collect at the end of 2008 if there are no withdraws within this period (5mks)

10.Show that the tangent to the curve $y = 4 - 2x - 2x^2$ *at point* $(-1,,4)$ and $(\frac{1}{2}\ , 2\frac{1}{2})$ respectively passes through the point $(\frac{-1}{4}\ , 5\frac{1}{2})$. Calculate the area enclosed by the curve and the x- axis (12mks)

11. If $z = \frac{(2-i)(5+12i)}{(1+2i)^2}$;

a) find the

i) Modulus of z

ii) Argument of z (8mks)

b) Represent z on a complex plane (2mks)

c) Write z in polar form (2mks)

12. Find all solutions of the equation
$5\cos x - 4\sin x = 6 \ in\ the\ range\ -180^0 \le x \le 180^0$ (6mks)

b) If A ,B and C are angles of a triangle , prove that

$\cos A + \cos B + \cos C - 1 = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ (6mks)

13 a) Prove that the circles $x^2 + y^2 + 4x - 2y - 11 = 0\ and\ x^2 + y^2 - 4x - 8y + 11 = 0$ are orthogonal (5mks)

b) Find the equation of a circle which passes through points (5,7) (1, 3) and (2,2) (7mks)

14 a) Evaluate $80_{P_5} \div 80_{C_6}$ (4mks)

b) Solve for n $n_{C_4} = n_{C_2}$ (4mks)

c) A committee of five students ' school council is to be selected from eight male students and five female students . find how many possible committees can be obtained (4mks)

15. Find the integral of i) $\int_0^1 x\sqrt{4-3x^2}\ dx$ (6mks)

ii) $\int_0^{\frac{\pi}{3}} 2\sin 3x \cos x\ dx$ (6mks)

ALL THE BEST IS DRAWN ON TO YOU

P425/1
PURE MATHEMATICS
PAPER 1
JUNE, 2018
3 HOURS.

EDEN INTERNATIONAL SCHOOL

UGANDA ADVANCED CERTIFICATE OF EDUCATION
B.O.T II EXAMINATIONS ,2018
S.6

PURE MATHEMATICS

PAPER 1

3 HOURS

INSTRUCTIONS.

- *Attempt **ALL** the **EIGHT** questions in section **A** and **only** **FIVE** from section **B**.*
- *Begin each answer on a fresh sheet of paper.*
- *No paper should be given for rough work.*
- *Mathematical tables and squared paper are provided.*
- *Silent, non-programmable calculators may be used.*
- *State the degree of accuracy at the end of each answer attempted a calculator or table; and indicate **CAL** for calculator or **TAB** for mathematical tables.*

SECTION A (40MARKS)

Answer **all** *questions from this section.*

1. Solve; $\cos\theta + \sqrt{3}\sin\theta = 2$, for $0^0 \le \theta \le 360^0$ *(05 marks)*
2. The angle between r_1 and r_2 is $\cos^{-1}\left(\frac{4}{21}\right)$. If $r_1 = 6i + 3j - 2k$ and $r_2 = -2i + \lambda j - 4k$, find the possible values of λ. *(05 marks)*
3. Determine the coordinates of the points of intersection of the curve $x = 2t^2 - 1,\ y = 3(t+1)$ and the straight line $3x - 4y = 3$. *(05 marks)*
4. Given $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$. *(05 marks)*
5. The first, second, third and n^{th} terms of a series are 4, -3, -16 and $(an^2 + bn + c)$ respectively. Find the values a, b and c *(05 marks)*
6. Show that $\int_a^{3a} \frac{x+a}{x^2+2ax}dx = \frac{1}{2}\ln 5$. *(05 marks)*
7. The sum of the height and radius of the base of a right circular cone is 9 cm. Show that the maximum volume of the cone will be $36\pi\ cm^3$. *(05 marks)*
8. If the line 3x – 4y – 12 = 0 is the tangent to the circle with a centre at (1,1). Find the equation of that circle *(05 marks)*

SECTION B (60MARKS)

Answer the **five** *questions from this section. All questions carry equal marks*

9. a) Prove that in a triangle ABC, $\cot\frac{(A-C)}{2} = \frac{a+c}{a-c}\tan\frac{B}{2}$ *(07 marks)*

   b) Given that $\tan x = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$, where x and θ are acute, prove that $\sin x = \frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)$. *(05 marks)*

10. a) Given that $f''(x) = 2 - \frac{2}{\sqrt{x^3}}$, $f'(1) = 0$, $f(1) = 8$, find $f(x)$. *(04 marks)*

    b) Find and classify the nature of the stationary point on the curve $x = 4 - t^3$ and $y = t^2 - 2t$. *(08 marks)*

11. (a) Given that $x^3 + 5x^2 + ax + b$ is divisible by $x^2 + x - 2$,

find (i) the values of a and b. *(05 marks)*

(ii) the linear factor of the polynomial. *(02 marks)*

(b) Find the first three terms of the binomial expansion of $\sqrt{1-2x}$. Using $x = 0.1$, estimate $\sqrt{5}$ to 2 decimal places. *(05 marks)*

12. a) Find $\int x \sin x \cos x \, dx$ *(04 marks)*

b) Express $f(x) = \frac{1}{(x+2)(1+x)^2}$ into partial fractions and hence find the definite integral of $\int_0^1 f(x)dx$. *(08 marks)*

13. a) Given that L_1 is a line $r = \begin{pmatrix} 2 \\ 9 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and L_2 is a line $r = \begin{pmatrix} -3 \\ 7 \\ p \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$, if L_1 and L_2 intersect, find the value of p and the point of intersection. *(06 marks)*

b) The position vector of points P and Q are $2i - 3j + 4k$ and $3i - 7j + 12k$ respectively.

Determine;

i) the size of PQ. *(03 marks)*

ii) The Cartesian equation of PQ. *(03 marks)*

14. a) Given that $y = e^{(x+e^x)}$, prove that $y\left(1 + \frac{d^2y}{dx^2}\right) = \left(y + \frac{dy}{dx}\right)\frac{dy}{dx}$ *(06 marks)*

b) Find the area bounded by the curve $y = x + \sin x$, the $x-axis$ and the line $x = \pi$.

15. a) Prove by induction that $2^{4n} - 1$ is a multiple of 15 if n is a natural number.
(06 marks)

b) Find the number of arrangements of all the letters of the word **BOBBIT** in a row,

i) without restriction. *(01 marks)*

ii) In which the B's are together *(03 marks)*

iii) If the vowels are separated *(03 marks)*

16. a) Given that $z + 2i$ is a factor of $z^4 + 2z^3 + 7z^2 + 8z + 12$, solve the equation $z^4 + 2z^3 + 7z^2 + 8z + 12 = 0$.

b) The complex number z satisfies $\frac{z}{z+2} = 2 - i$. Find the real and imaginary parts of z and the modulus and argument of z.

(12 marks)

END

P425/1
Pure Mathematics
Paper 1
July/Aug. 2018
3 hours

ACEITEKA JOINT MOCK EXAMINATIONS 2018

MOCK EXAMINATIIONS 2018

Uganda Advanced Certificate of Education

Pure Mathematics Paper 1

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* ***all*** *the* ***eight*** *questions in Section* ***A*** *and only* ***five*** *questions in Section* ***B****.*
- *Indicate the five questions attempted in section B in the table aside.*
- *Additional question(s) answered will* ***not*** *be marked.*
- ***All*** *working* ***must*** *be shown clearly.*
- *Graph paper is provided.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Qn 1: An arithmetic progression contains $\boldsymbol{n}$ terms. The first term is 2 and its common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find $\boldsymbol{n}$. [5marks]

Qn 2: Find the equation of a circle which touches the line $3x + 4y = 9$ has a centre $(4, -7)$. [5marks]

Qn 3: Differentiate $\mathbf{cos}\, \boldsymbol{x}$ from first principles. [5marks]

Qn 4: Four letters of the word "**HYPERBOLA**" are to be arranged in a row. In how many of these arrangements are the vowels separate? [5marks]

Qn 5: Solve for x, $2\sin^2\left(\frac{x}{2}\right) - \cos x + 1 = 0$, where $0 \le x \le 2\pi$. [5marks]

Qn 6: Prove that the integral of cosec $\left(\frac{x}{2}\right)$ for x between π and $\frac{4\pi}{3}$ is $\ln 3$. [5marks]

Qn 7: Find the shortest distance of a point $A(1, 6, 3)$ from the line

$$\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + \beta\left(-\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right).$$ [5marks]

Qn 8: The surface area of a sphere is decreasing at a rate of $0.9\ \text{m}^2/\text{s}$ when the radius is 0.6 m. Find the rate of change of the volume of the sphere at this instant. [5marks]

SECTION B (60 MARKS)

Question 9:

(a). If the roots of the equation $x^2 + (x+1)^2 = k$ are α and β;

(i). Prove that $\alpha^3 + \beta^3 = \frac{1}{2}(1 - 3k)$.

(ii). Find a quadratic equation whose roots are α^3 and β^3.

(b). (i). Given that $|x| < \frac{1}{2}$, expand $\frac{1+5x}{\sqrt{1+2x}}$ upto the term in x^3.

(ii). By substituting $x = 0.04$ in (b)(i) above, deduce the approximation of $\frac{1}{\sqrt{3}}$ correct to 4 decimal places. [12marks]

Question 10:

Given that $y = \frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$

(i). Prove that $y + \tan^2\left(\frac{x}{2}\right) = 0$, and hence express the exact value of $\tan^2 15°$ in the form $p + q\sqrt{r}$ where p, q and r are integers.

(ii). Hence find the value of x between $0°$ and $360°$ for which $2y + \sec^2\left(\frac{x}{2}\right) = 0$. [12marks]

Question 11:

Given the curve $f(x) = \frac{2x^3 - x^2 - 25x - 12}{x^3 - x^2 - 5x + 5}$;

(a). Find the:

(i). value of x for which $f(x) = 0$.

(ii). assymptotes for $f(x)$.

(iii). x and $f(x)$ intercepts for the curve.

(b). Sketch the curve. [12marks]

Question 12:

A point representing the complex number Z moves such that $\left|\frac{Z-2}{Z-4}\right| > \frac{1}{2}$

(i). Prove that the locus of Z is a circle.

(ii). Find the centre and radius of this circle.

(iii). Represent Z on the argand diagram.

(iv). State the least and greatest values of $|Z|$. [12marks]

Question 13:

(a). Given two vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 12\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{k}$; find:

(i). the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$,

(ii). a vector that makes a right angle with $\underset{\sim}{a}$ and with $\underset{\sim}{b}$.

(b). Find the equation of the plane passing through the points $A(1,1,0)$, $B(3,-1,1)$, $C(-1,0,3)$ and find the shrotest distance of the point $(3,2,1)$ to the plane. [12marks]

Question 14:

(a). Using calculus of small increments, or otherwise, find $\sqrt{98}$ correct to one decimal place. [4marks]

(b). Use Maclaurin's theorem to expand $\ln(1 + ax)$, where a is a constant. Hence or otherwise expand $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right)$ up to the term in x^3. For what value of x is the expansion valid? [8marks]

Question 15:

A tangent to the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at a point, $P(6\cos\theta, 4\sin\theta)$ meets the minor axis at ***A***. If the normal at ***P*** meets the major axis at ***B***, find the:

(i). Coordinates of ***A***,

(ii). Coordinates of ***B***,

(iii). Locus of the midpoint of ***AB***. [12marks]

Question 16:

(a). Find the general solution of

$$(x^2 + 1)\frac{dy}{dx} + 2x - 2xy = 0$$

(b). A moth ball evaporates at a rate proportional to its volume, losing half of its volume every 4 weeks. If the volume of the moth ball is initially 15 cm^3 and becomes ineffective when its volume reaches 1 cm^3, how long is the moth ball effective? [12marks]

*****END*****

P425/1
Pure Mathematics
July/August 2018
3 hours

BUGANDA EXAMINATIONS COUNCIL MOCKS 2018

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3HOURS

INSTRUCTIONS TO CANDIDATES

- *Answer eight questions in Section A and ONLY 5 from Section B.*
- *All working MUST be clearly shown.*
- *Mathematical tables with a list of formulars and graph paper will be provided.*
- *Use a silent non programmable calculator.*
- *State the level of accuracy for answers got and indicate (tab) for Maths tables and (cal) for calculator used.*
- *Begin each answer on a fresh paper and use g = 9.8 m/s for numerical work*

SECTION A (40 MARKS)

1. Find the values of X in the range $o \leq x \leq \frac{\pi}{2}$ which satisfy the equation $8\text{Sec } x - 4 \tan x = 7$. *(05 marks)*

2. Solve the simultaneous equations by reducing to echelon form

$$x + 3y + z = 10$$
$$4x - y + 2z = 8$$
$$6x + y - 5z = {}^{-}7$$

(05 marks)

3. Evaluate: $\int_{1}^{3} \frac{3}{(16+25x^2)} dx$ *(05 marks)*

4. Express the following into partial fractions. *(05 marks)*

$$\frac{x+1}{x^2(x-2)}$$

5. Solve for y when $2\sqrt{(y-1)} - \sqrt{y+4} = 1$ *(05 marks)*

6. Given that $\propto$ and β are roots of the equation $x^2 + px + q = 0$
Express $\propto^2 - \beta^2$ and $\propto^3 + \beta^3$ in terms of p and q. *(05 marks)*

7. Show that $\text{Cos}3A = 4\cos^3 A - 3\cos A$ *(05 marks)*

8. Without using calculators or tables show that

$$\frac{\log\sqrt{27} + \log\sqrt{8} - \log\sqrt{125}}{\log 6 - \log 5} = \frac{3}{2}$$

(05 marks)

SECTION B (60 MARKS)

9. a) Solve for θ in the range 0 to 180° when $\cos\theta - \cos 7\theta = \sin 4\theta$ *(06 marks)*

b) Prove that for any triangle ABC.

$$\frac{a^2 - b^2}{c^2} = \frac{\text{Sin}(A-B)}{\text{Sin}(A+B)}$$

(06 marks)

10. a) Use de-moivre's theorem to show that:

$$\frac{1}{(1-i\sqrt{3})^5} = \frac{-1}{8}$$

(04 marks)

b) Given that z = 1+i is a root of $z^4 + 3z^2 - 6z + 10 = 0$.
Determine the remaining three roots of the polynomial. *(04 marks)*

c) Simplify to the form a+b*i* if $P = \frac{\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)^8}{\left(\cos\frac{3\pi}{5} - i\sin\frac{3\pi}{5}\right)^3}$ *(04 marks)*

11. i) Show that the curve $y = \frac{2x-3}{x^2+2x-3}$ does not exist in the range $\frac{1}{4} \le y \le 1$ *(04 marks)*

ii) Sketch the above curve by stating the turning points and asymptotes as well. *(08 marks)*

12. a) Determine the acute angle between the line $^x/_y = \frac{y-1}{-1} = \frac{2+3}{-5}$ and the plane $x - 2y + 4z + 3 = 0$. Find the point of intersection of the line and plane above.

b) Find the shortest distance from plane $2x - 4y + z = 3$ to point P(2, 1, 3) hence determine the line perpendicular to the above plane but passing through point P. *(06 marks)*

13. a) Find the particular solution for the differential equation. $\frac{d\theta}{dz} + t\cot\theta \quad 2\cos\theta$ for t = 3 when $\theta = \frac{\pi}{2}$. *(05 marks)*

b) The liquid is being heated in an oven maintained at a constant temperature of 180°C. It is assumed that the rate of increase in temperature of a liquid is proportional to (180- θ), where θ is the temperature of the liquid at any time t (minutes). If the temperature of the liquid raises from 0°C to 120°C in five minutes, find:

i) the temperature of the liquid in further 5 minutes.
ii) time taken for temperature to raise to 90°C. *(07 marks)*

14. a) The first term of an arithmetic progression is 73 and the ninth is 25. Find:

i) common difference

ii) the number of terms that must be added to give the sum of 96. *(6mks)*

b) A geometrical progression has first term as 15 and sum to infinity as 225.

Find the: i) the common ratio

ii) sum of the first ten terms. *(06 marks)*

15. a) Differentiate with respect to x.

i) $3x^x$ (ii) $\cos^2 3x$

b) Find the equation of the normal and tangent to the curve.

$xy^3 - 2x^2y^2 + x^4 - 1 = 0$ at a point P(1, 2).

16. a) A conic section is given by $x = 4\cos\theta$ and $y = 3\sin\theta$ show the conic section is an ellipse and determine its eccentricity and length of its Latus rectum.

b) If the line $y = mx + c$ is a tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2+b^2$ hence find the equations of tangents at p(-3, 3) to ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

END

P425/1
PURE
MATHEMATICS
PAPER 1
JUNE, 2018
3hrs

GAYAZA HIGH SCHOOL

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3hours

Instructions

Attempt all the eight questions in Section A and Not more than five from Section B.

Any additional question(s) will not be marked

All working must be shown clearly

Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40marks)

1. Solve the equation; $tan\theta tan2\theta = 1$for $0^0 \leq \theta \leq 360^0$. (5marks)

2. Differentiate; $Cos(x^2e^x)$ with respect to x. (5marks)

3. A committee of 4 people is to be chosen from 5 men and 7 women. Find the number of ways the committee can be chosen so that it contains people of both sexes and there are at least many women as men. (5marks)

4. A straight line PQ passes through the points $P(5,1,6)$and $Q(3,4,1)$. Calculate the angle between the line PQ and the line whose equation is $3x - 1 = 6y + 2 = 1 - z$. (5marks)

5. Solve the differential equation; $\frac{dy}{dx} + ytanx = secx$ (5marks)

6. Find the equation of the tangent to the curve $y^2 = 20x$ which makes an angle of 45^0 with the x-axis. (5marks)

7. Evaluate; $\int_0^1 \frac{x}{\sqrt{1-x^4}} dx$. (5marks)

8. Expand,$(1 - 2x)^{\frac{1}{2}}$ is ascending powers upto the term in x^3. Taking $x = \frac{1}{9}$, find the approximate value of $\sqrt{7}$ to 4 significant figures. (8marks)

SECTION B

9. a) Find the complex numbers which satisfy the equation;

$z^2 = -8 - 6i$ (5marks)

b) i) Express $z_1 = \frac{7+4i}{3-2i}$ in the form $p + qi$ where p and q are real.

ii) Sketch on the argand diagram the locus of the point which moves such that $|z - z_1| = \sqrt{5}$. Hence find the greatest value of $|z|$. (7marks)

10. a) Solve the equation $sin3xsinx = 2cos2x + 1$ for $0^0 \leq x \leq 360^0$.

(6marks)

b) Express; $\sqrt{5}cosx + 2sinx$ in the form $Rcos(x - \alpha)$ where $R > 0$ and $0 < \alpha < 90^0$. Hence solve the equation $\sqrt{5}cosx + 2sinx = 1.2$ for $0^0 \leq x \leq 360^0$. (6marks)

11. The points A, B and C have position vectors $\overrightarrow{OA} = \begin{pmatrix}1\\2\\0\end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix}3\\0\\1\end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ relative to the origin. A plane, P, is perpendicular to AB and contains point C. Determine the;

i) Cartesian equation of the line through A and B.

ii) Cartesian equation of the plane, P

iii) Point of intersection of the line AB and plane, P. (12marks)

12. Find;

a) $\int x\, cot^{-1}x\, dx$ (6marks)

b) $\int \frac{1}{4 - 9x^2}\, dx$ (6marks)

13. a) The eighteenth and twenty first terms of an arithmetic series are 25 and 32.5 respectively. Given that the sum of all terms of the series is 2750. Find the;

i) common difference and first term

ii) number of terms of the series. (7marks)

b) Prove by induction that $2^{n+2} + 3^{2n+1}$ is divisible by 7 for all positive integers. (5marks)

14. a) If $y = \sqrt{\frac{1\_cosx}{1-cosx}}$, find $\frac{dy}{dx}$. (5marks)

b) An open square based cuboidal tank has a volume of $32m^3$.

i) Determine the expression for the total internal surface area, S, of the tank in terms of side of the base.

ii) Find the dimensions of the tank for which its surface area, S is minimum. (7marks)

15. a) A circle, C, with centre lying on the line $y = 2x$ has radius $\sqrt{5}$ and passes through point $P(2,4)$. Find the possible equations of the circle. (6marks)

b) The parabola is given by the equation $y^2 - 4x - 4y + 8 = 0$. Find the;

i) coordinates of the vertex

ii) coordinates of the focus

iii) equation of the directrix (6marks)

16. In a certain chemical reaction, a substance is being formed and t minutes after the start of the reaction, there are m grams of the substance present. In the reaction, the rate of increase of mass, m is proportiona to $(50 - m)^2$. Given that when $t = 0, m = 0$ and $\frac{dm}{dt} = 5gs^{-1}$.

a) Show that; $\frac{dm}{dt} = 0.002\,(50 - m)^2$.

b) Determine the;

i) Mass of the substance after 10minutes

ii) Time taken for mass to become 45g. (12marks)

****END *****

P425/1
PURE
MATHEMATICS
Paper 1
AUGUST 2018
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS AUGUST 2018

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in section A and any **five** from section B.*
- *Any additional question (s) answered will not be marked*
- *All necessary working **must** be shown clearly*
- *Begin each answer on a fresh sheet of paper*
- *Graph paper is provided*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Find the coefficient of the term involving x^{-6} in the expansion of $\left(3x - \frac{2}{x^2}\right)^9$.
(05marks)

2. Use the identities $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find the value of $\sin 18^o$. Leave your answer in surd form. (05marks)

3. If the function $f(x) = x^3 + ax^2 + bx + c$ has stationary values at $x = \alpha$ and $x = \beta$. Show that $\frac{f(\alpha) - f(\beta)}{\alpha - \beta} = \frac{2}{9}(3b - a^2)$ (05marks)

4. Given that the angle between the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \lambda\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ is $\cos^{-1}\left(\frac{1}{2}\right)$, find the value of λ. (05marks)

5. Evaluate $\int_0^1 e^{\sqrt{x}}dx$. (05marks)

6. Prove by mathematical induction that $3^{2n+1} + 5^{2n-1}$ is divisible by 16, for $n \geq 1$. (05marks)

7. Find the equations of the lines that bisect the angles between the pairs of lines $3x = 4y + 2$, $12y = 5x + 2$. (05marks)

8. Solve the differential equation:
$(x + 1)\frac{dy}{dx} - 3y = (x + 1)^4$, given that $y(1) = 6$. (05marks)

SECTION B: (60MARKS)

*Answer any **five** questions from this section. All questions carry equal marks*

9. (a) Use Demoivre's Theorem to show that; $\tan 3\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta + \tan^4\theta}$. (05marks)

(b) Find the roots z_1 and z_2 of the equation, $(Z+i)^2 = 3-4i$. Hence represent the roots on the Argand diagram. (07marks)

10. (a) Solve the equation $2(\sin x + \sin 2x) - 1 = \cos 2x + \cos x$, for $0^o \le x \le 360^o$. (04marks)

(b) Express $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ in the form $q + r\cos(2x-\alpha)$, where q, r are constants and α is an acute angle. Hence find the maximum and minimum values of

$$\frac{9}{5\sin^2 x - 3\sin x\cos x + \cos^2 x}.$$ (08marks)

11. Given that $\dfrac{x^2-8x+5}{(2x+1)(x^2+9)} = \dfrac{A}{2x+1} + \dfrac{Bx+c}{x^2+9}$, find the values of A, B and C.

Hence show that $\displaystyle\int_0^3 \frac{x^2-8x+5}{(2x+1)(x^2+9)}dx = \frac{1}{2}In7 - \frac{\Pi}{3}$. (12marks)

12. The polynomial $f(x) = x^4 + 4x^3 + lx^2 + mx + n$ is a perfect square of second degree.

(a) Show that $m + 8 = 2l$ and $16n = m^2$. (06marks)

(b) If the polynomial $f(x)$ leaves a remainder 4 when divided $x+1$. Determine the possible values of l, m and n. (06marks)

13. The parametric equations of two planes Π_1 and Π_2 are;

$$\mathbf{r} = \begin{pmatrix}2\\4\\3\end{pmatrix} + \alpha\begin{pmatrix}-1\\0\\2\end{pmatrix} + \beta\begin{pmatrix}1\\2\\-8\end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix}-2\\-3\\0\end{pmatrix} + \mu\begin{pmatrix}-1\\-1\\-1\end{pmatrix} + \lambda\begin{pmatrix}-3\\-4\\-2\end{pmatrix}$$ respectively.

(a) Find the cartesian equation of each plane. (06marks)

(b) If l is the line of intersection of the planes, find the;

(i) equation of the line, l ,in vector form. (03marks)

(ii) coordinates of the foot of the perpendicular from the point $P(1,-5,-10)$ to the line, l . (03marks)

(a) Given that $y = \sqrt{1-\cos^2 x^2}$. Find the value of $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$. (06marks)

(b) If $y = e^{4x}\cos 3x$, prove that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$. (06marks)

14. (a) Find the equations of the circles passing through the points of intersection of the circles $x^2+y^2-18x-2y+8=0$ and $x^2+y^2-26x+6y=24$ and touching the line $y=10$. (05marks)

(b) P and Q are points $(ap^2,2ap)$ and $(aq^2,2aq)$ respectively on the parabola $y^2 = 4ax$. If the tangents to the parabola at the given points intersect at R and are inclined at angle of 45^o . Show that the locus of R is the curve, $y^2 - x^2 - 6ax - a^2 = 0$. (07marks)

15. Army worms wipe out a community at a rate proportional to the population present at any time. If the initial population is 4million and it reduces from 2.5million to one-fifth of a million in 5months. How long does it take the initial population to reduce to a million? (12marks)

P425/1

PURE

MATHEMATICS

Paper 1

Jul/Aug 2018

3 Hours

MUKONO EXAMINATIONS COUNCIL

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES

*Answer **all** the eight questions in section **A** and any **five** from section **B***

*Any addition question(s) answered will **not** be marked*

*All necessary working **must** be clearly shown*

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

*Answer **all** questions in this section*

1. Solve for x in the equation

   $log_9(21x - 5) = log_3(2x)$. *(5 marks)*

2. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{2x}\right)^9$ *(5 marks)*

3. Given that $r - 2cos\theta + 6sin\theta = 6$ is a circle, find;
   (i.) Cartesian equation of the circle.
   (ii.) Coordinates of its Centre and the radius. *(5 marks)*

4. Points A and B are (-1, -2, 3) and (2, 1, -3) respectively. If point P divides the line AB externally in the ratio 1: 4. Find the Cartesian equation of the plane containing P and perpendicular to the line AB. *(5 marks)*

5. Solve $4sin2\theta = 3sin^2\theta$ $for - 180^0 \leq \theta \leq 180^0$ *(5 marks)*

6. Differentiate; $3x^{2x}sin^3 2x$. With respect to x *(5 marks)*

7. Find $\int \frac{4x^2+x+1}{x^3-1} dx$. *(5 marks)*

8. Find the area bounded by the curve $x = y^2 - 4$ and the y – axis. *(5 marks)*

SECTION B (60 marks)

*Answer **any five** in this section*

9. (a.) Express the complex numbers $z_1 = 1 - 4i$ *and* $z_2 = 2 + i$ in the polar form Hence find $z_1(z_2)^2$. ***(6 marks)***

   (b.) Find the values of x and y if, $\frac{x}{3-i} - \frac{y}{5+2i} = \frac{4+17i}{17+i}$. *(6 marks)*

10. (a) The polynomial $f(x)$ leaves a remainder of 3 when divided by $x + 3$ and a remainder of 18 when divided by $x - 2$. Find the remainder when $f(x)$ is divided by $x^2 + x - 6$. *(6 marks)*

    (b) The roots of the equation $25x^2 + x + 1 = 0$ are α^2 and β^2 . Find the equation with integral coefficients whose roots are $\frac{1}{\alpha}$ *and* $\frac{1}{\beta}$. *(6 marks)*

11. (a) A line $r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$ is parallel to the plane $3x + 6y - 2z = 15$. Find; (i) the value of a .

(ii) the shortest distance between the line and the plane. *(6 marks)*

(b) Find the acute angle between the line $r = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ and the plane $4x - 7y - 4z = 20$. *(6 marks)*

12. (a) Find the locus of a point P which moves so that its distance from $(2,2)$ is half its distance from the line $x + y + 4 = 0$. *(6 marks)*

(b) Find the length of the tangents to the circle $x^2 + y^2 - 2x + 4y - 3 = 0$ from the centre of the circle $x^2 + y^2 + 6x + 8y - 1 = 0$ *(6 marks)*

13. (a) Solve the equation $cos5x + 1 = 2sin^2x$ for $0^0 \leq x \leq 180^0$. *(5 marks)*

(b) Given that $f(\theta) = 4cos\theta + 5sin\theta$.

(i) Express $f(\theta)$ *in the form* $R\sin(\theta + \beta)$, where R is a constant and β an acute angle.

(ii) Determine the maximum value of $2 - f(\theta)$ and the value of θ for which it occurs.

(iii) Solve the equation $f(\theta) = 2.2$ *for* $0^0 \leq x \leq 360^0$. *(7 marks)*

14. (a) Evaluate $\int_0^{\sqrt{3}} \frac{x^3}{(1+x^2)^{1/2}} dx$. *(6 marks)*

(b.) Use the substitution $t = tan\, {}^{x}/_{2}$ to evaluate $\int_{\frac{\pi}{6}}^{\pi} \frac{1}{1-2sinx+cosx} dx$. *(6 marks)*

15. A curve is given parametrically as $x = 2t$ and $y = \frac{1}{t} - t$.

(a.) Find the Cartesian equation of the curve. *(4 marks)*

(b.) Sketch the curve.

(c.) Find the area enclosed by the curve, x-axis and line x = 1. *(8 marks)*

16. (a) Solve the differential equation

$$tanx\frac{dy}{dx} - y = sin^2x \ , if\ y = 4\ when\ x = \frac{\pi}{6}.$$ *(5 marks)*

(b) A student walks to school at a speed proportional to the square root of the distance he still has to cover. If the student covered 900m in 100 minutes and the school is 2500m from home, find how long he takes to get to school. *(7 marks)*

END

P425/1
PURE MATHEMATICS
Paper 1
August, 2018
3hours

UNNASE MOCK EXAMINATIONS 2018

Uganda Advanced Certificate of Education

PURE MATHEMATICS
paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and any **five** from section **B**.

Any additional question(s) answered will not be marked.

All necessary working **must** be clearly shown.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A. (40 MARKS)

Answer **all** questions in this section.

1. Find the first four terms of the expansion $\sqrt[3]{1+3x}$ .Hence evaluate $\sqrt[3]{1.03}$ to 4s.f. **(5marks)**

2. Find the equation of the tangent to the curve

   $x^3y-3x^2y^2+x^3-2x=0$ at the point $P(2,1)$ **(5marks)**

3. Prove that $\frac{\sin 2A}{1-\cos 2A}=\cot A$ **(5marks)**

4. If $Z_1=6\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$, $Z_2=3\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$.Express Z_1Z_2 in the form $a+bi$ hence find $|Z_1Z_2|$ **(5marks)**

5. A right circular cone is constructed so as to have a total surface area A and radius r, prove that its volume is $V=\frac{1}{3}r\sqrt{A^2-2\pi Ar}$ **(5marks)**

6. Evaluate $\int_0^1 \frac{x^3}{1+x^2}dx$ to 4 significant figures. **(5marks)**

7. Prove that $A(1,5)$ and $B(7,-4)$ are on different sides of the line $2x-3y=0$. **(5marks)**

8. Find the coordinates of the point of intersection of the line $\frac{x-3}{-1}=\frac{y-1}{2}=\frac{z+3}{4}$ with the plane $3x-y+2z=8$. **(5marks)**

SECTION B(60 MARKS)

Answer any **five** questions from this section. **All** questions carry equal marks.

9. Express $\frac{2x^2+1}{(x-1)(x+5)(x^2+1)}$ in partial fractions hence

$$\int \frac{2x^2+1}{(x-1)(x+5)(x^2+1)}dx$$ **(12 marks)**

10. The lines l_1 and l_2 have equations;

$$l_1; r = \begin{pmatrix}1\\0\\-4\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\-3\end{pmatrix}, l_2; r = \begin{pmatrix}4\\6\\2\end{pmatrix} + \mu\begin{pmatrix}a\\4\\9\end{pmatrix}$$ where a is a constant.

a) Find the acute angle between l_1 and the $x-axis$. **(04marks)**

b) The point A has position vector $2i-2j+bk$ and the line l_2 passes through the point A find the;

i) value of **a** and **b**.

ii) perpendicular distance of A from the line l_1 **(08 marks)**

11a) Given that $y = \frac{4lnx-3}{4lnx+3}$ and show that $\frac{dy}{dx} = \frac{24}{x(4lnx+3)^2}$ **(03 marks)**

b) The gradient of the curve at (x, y) is $\left(x-\frac{1}{x}\right)$ and the curve passes through he point $(1,2)$. Show that the area enclosed by the curve, the $x-axis$ and $x=1, x=2$ is given by $\frac{11}{3}-2\ln 2$ **(09 marks)**

12a) The 9^{th} term of an $A.P$ is -1 and the sum of the first nine terms is 45. Find the common difference and the sum of the first twenty terms.

(06 marks)

b) Find the values of k for which the quadratic equations

$x^2 + kx - 6k = 0$ and $x^2 - 2x - k = 0$ have common root. **(06 marks)**

13a) Solve the equation $6\tan^2 x - 4\sin^2 x = 1$ for $0 \le x \le 2\pi$ **(06 marks)**

b) A triangle ABC has area 20cm², given that $\overline{AC} = 10cm$, $\overline{BC} = 6cm$ and that Angle ACB is obtuse, find the;

i) Angle ACB **(03 marks)**

ii) Length AB **(03 marks)**

14a) i) Find the equation of the tangent to the curve $y^2 = \frac{4}{x^2}$ at the point $P\left(p^2, \frac{2}{p}\right)$.

ii) If this tangent passes through the point $(1,2)$ determine the possible values of p. **(8marks)**

b) Determine the equation of a parabola given the focus $S(4,0)$ and the directrix $x = 4$. **(04 marks)**

15a) The sum of the squares of the roots of the equation $x^2 + px + q = 0$ is 56 and the sum of the reciprocal of the roots is 2, If α and β are the roots, find the values of **p** and **q**. **(06marks)**

b) When $P(x) = x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$, the remainder is $2x + 11$. Given that $x + 1$ is a factor of $P(x)$, find the values of **a**, **b** and **c**.

(06marks)

16a) Solve the differential equation $ye^{y^2}\frac{dy}{dx} = e^{2x}$ and that $y\left(\frac{1}{2}\right)$ 1

(05marks)

b) During a fermentation process, the rate of decomposition of a substance at any time t, is directly proportional to the product of the active ferment x. If the constant of proportionality k = 0.5 and the value of x at any time t is $\frac{4}{(1+t)^2}$

i) If initially y = 10, find y as a function of t.

ii) Deduce the amount of substance remaining as t becomes very large.

(07 marks)

END

P425/1
PURE MATHEMATICS
PAPER 1
July/August 2018
3HRS

WESTERN JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

MATHEMATICS

PAPER 1

3 Hours

INSTRUCTIONS TO CANDIDATES.

- *All necessary working must be shown*
- *Section **A** is compulsory*
- *Answer only **five** in section **B***

  ***N.B:** Extra question(s) will not be marked*
- *Un necessary use of calculators/tables will lead to loss of marks.*

SECTION A: (40 MARKS)

Answer *all* questions in this section

1. By using row reduction to echelon form, solve simultaneous equations
   $x + y - z = 1$
   $3x + 4y - 2z = 3$
   $-x + y + 4z = 2$ (05 marks)

2. The line $y = mx$ meets the curve $y^2 = 4x$ at the origin O and at a point A. Find the equation of the locus of the mid-point of OA as m varies. (05 marks)

3. If A, B and C are angles of a triangle, prove that
   $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$ (05 marks)

4. Differentiate $tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to x (05 marks)

5. Find the perpendicular distance of the point $(3, 0, 1)$ from the line whose

   Cartesian equation is $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}$ (05 marks)

6. Solve the inequality $\left|\frac{1}{1+2x}\right| < 1$ (05 marks)

7. Find $\int_0^{\pi/2} x\cos^2 3x\ dx$ (05 marks)

8. A cylinder has radius ***r*** and height $8r$. The radius increases from 4cm to 4.1cm; Find the approximate increase in the volume (use π = 3.14) (05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section.
All questions carry equal marks

9. (a) If Z_1 and Z_2 are complex numbers, solve the simultaneous equations
   $4z_1 + 3z_2 = 23$, $z_1 + iZ_2 = 6 + 8i$ giving both answers in the form $x + iy$
   (06 marks)

   (b) If $(a + bi)^2 = -5 + 12i$, Find **a** and **b** given that they are both real.
   Give the two square roots of $-5 + 12i$ (06 marks)

10. (a) Find the equation of the circle which touches the line $3x - 4y = 3$ at the point $(5, 3)$ and passes through the point $(-2, 4)$. (05 marks)

    (b) A curve has the parametric equations $x = 3t$, $y = \frac{3}{t}$ Find the equation of the tangent to the curve at the point $\left(3t, \frac{3}{t}\right)$. The point P has coordinates (-5, 8) and the tangents from P to the curve touch the curve at A and B and the length of chord AB (07 marks)

11. (a) If $y = e^{-x}\ \ln x$, show that $x\frac{d^2y}{dx^2} + (2x+1)\frac{dy}{dx} + (x+1)y = 0$ (05 marks)

(b) Express the function $f(x) = \frac{x+2}{(x^2+1)(2x-1)}$ as a sum of partial fractions.

Hence find $\int_2^3 f(x)\,dx$, correct to 4 decimal places (07 marks)

12. Two lines have vector equations

$$\vec{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$ and

$$\vec{r} \quad \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$ Find the position vector

of the point of intersection of the two lines and the cartesian equation of the plane containing the two lines.

(b) Find the acute angle between the line $\frac{x-6}{5} = \frac{y-1}{-1} = \frac{z+1}{1}$ and the plane $7x - y + 5z = -5$, giving your answer to the nearest degree. (05 marks)

13. Find the coordinates of the points of intersection of the curves.

$$y = \frac{x}{x+3} \text{ and } y = \frac{x}{x^2+1}$$

Sketch the cures on the same diagram, showing any asymptotes or turning points.

Show that the area of the finite region in the first quadrant enclosed by the two curves is $\frac{7}{2}\ Ln\,5 - 3Ln\ 3 - 2$ (12 marks)

14. (a) In the expansion of $(1 + ax)^n$, the first three terms are $1 - \frac{5x}{2} + \frac{75x^2}{8}$

Find ***n*** and ***a*** and state the range of values of x for which the expansion is valid (06 marks)

(b) Expand $(1 + x)^{½}$ in ascending powers of x as far as the term in x^2

and hence find an approximation for $\sqrt{1.08}$. Deduce that $\sqrt{12} \approx 3.464$

(06 marks)

15. (a) Solve the equation for $-180^0 \leq \theta \leq 180^0$, $3 + 2\sin 2\theta = 2\sin\theta + 3\cos^2\theta$ (06 marks)

(b) Given that $3\sin x - \cos x = R\sin(x - \alpha)$ *where* $R > 0$ *and* $0^0 < \alpha < 90^0$, Find the values of R and α correct to 1 decimal place.

Hence find one value of x between 0^0 *and* 360^0 for which the curve

$y = 3\sin x - \cos x$ has a turning point (06 marks)

16. (a) Find y in terms of x, given that $x\frac{dy}{dx} = \cos^2 y$, $x > 0$ and that

$y = \frac{\pi}{3}$ *when* $x = 1$ (06 marks)

(b) The rate at which a body loses temperature at any instant is proportional to the amount by which the temperature of the body at that instant exceeds the temperature of its surroundings. A container of hot liquid is placed in a room of temperature. 18^0C and in 6 minutes the liquid cools from 82^0C to 50^0C

How long does it take for the liquid to cool from 26^0C to 20^0C? (06 marks)

END

P425/1
PURE MATHEMATICS
PAPER 1
July/August 2019
3 Hours

WESTERN JOINT MOCK EXAMINATIONS 2019

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3 Hours

INSTRUCTIONS TO CANDIDATES:

- *Answer all the eight questions in section A and any five from section B.*
- *Any additional question(s) answered will not be marked*
- *All necessary working must be shown clearly*
- *Begin each answer on a fresh page.*
- *Graph paper is provided.*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Solve the equation $log_x^8 - log_{x^2}^{16} = 1$ *(5marks)*

2. The curve $y = x^3 + ax^2 + b$ *has aminimum point at* $(4, -11)$. *Find* the co-ordinates of the maximum point on the curve. *(5marks)*

3. Solve, $6tan^2x - 4sin^2x = 1$ for values of x between 0 and 2π *(5marks)*

4. The points A, B and C have position vectors $2i + 5j - k$, $2k - 4j + 3j$ and $-i + 2j + k$ respectively. Find a vector perpendicular to both AB and AC *(5marks)*

5. Evaluate $\int_0^a x\sqrt{(a^2 - x^2}dx$ *(5marks)*

6. Find the circum centre of a triangle with vertices $A(-3,0)$, $B(7,0)$ and $C(9, -6)$ *(5marks)*

7. Prove by induction that $10^n + 3(4^{n+2}) + 5$ is divisible by 9 for all positive integral values of n. *(5marks)*

8. If $y^2 - 2y\sqrt{(1 + x^2)} + x^2 = 0$, show that $\frac{dy}{dx} = \frac{x}{\sqrt{(1+x^2)}}$ *(5marks)*

SECTION B (60 MARKS)

Answer five questions from this section.

9. If *x is so small that* x^3 *and higher powers of x may be neglected*, find the values of a and b such that $(1 - 8x)^{\frac{1}{4}} = \frac{1+ax}{1+bx}$

   By substituting $x = 0.05$, *find an approximation for* $(0.6)^{\frac{1}{4}}$ in the form $\frac{p}{q}$, *where p and q are integers.* *(12marks)*

10. (a) Solve for *x in the equation* $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ *(6marks)*

    (b) Prove that in any triangle ABC $\sin(\frac{B-C}{2}) = \frac{b-c}{a} \cos\frac{A}{2}$ *(6marks)*

11. (a) Find the Cartesian equation of the plane which contains the point *a*(0,2,5) *and is perpendicular to the line* $\frac{x-3}{2} = \frac{2-y}{2} = \frac{Z-z}{-1}$

    Show that the point B (-1, 3, 1) lies on the plane. *(6marks)*

(b) Find the position vector of the point C of intersection of the line and the plane in (a) above. (6marks)

12. (a) Differentiate $\tan^{-1}(\frac{1-x}{1+x})$ with respect to x. Simplify your answer. (5marks)

(b) Evaluate $\int_3^4 \frac{x^3}{x^2-x-2}\,dx$

Correct to three decimal places. (7marks)

13. Determine the nature of the turning points of the curve

sketch the curve stating clearly any asymptotes $y = \frac{x^2-6x+5}{2x-1}$ (12marks)

14. (a) If $Z = x + iy$ *and* $\bar{Z}$ *is the complex conjugate of* Z, *find the valves of* x and y such that $\frac{1}{z} + \frac{2}{\bar{z}} = 1 + i$ (6marks)

(b) Use De-moivre's theorem to show that

$$\frac{\cos 5x}{\cos x} = 1 - 12\sin^2 x + 16\sin^4 x$$ (6marks)

15. Given the equation $y = 5x - 2x^2$

(a) Show that the equation represents a parabola and state the length of its latus rectum. (4marks)

(b) Find the co-ordinates of the focus and the equation of the directrix (5marks)

(c) Sketch the parabola (3marks)

16. The rate of decomposition of a radioactive substance is proportional to the mass of the substance remaining. If x is the mass remaining at time t years and x *is the original mass*:

(a) Write down a differential equation connecting x, t and k where k is the constant of proportionality.

(b) Solve the differential equation if one third of the mass is left after 12 years.

(c) How much of the radioactive substance left after 3 years if the original mass is 15g (12marks)

END

5/1
RE MATHEMATICS
PER I
RS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PURE MATHEMATICS

PAPER I

3 HOURS

tructions to candidates

- Answer all the eight questions in section A and any five from section B.
- Any additional question(s) answered will not be marked
- All necessary working must be shown clearly
- Graph paper is provided
- Silent non -- programmable scientific calculators and mathematical tables w list of formulae may be used.

CTION A

If $f(x) = ax^2 + bx + c$ leaves remainders of 1, 25, 1 on division by $x - 1$, $x + 1$ and $x - 2$ respectively. Show that f(x) is a perfect square.

In how many different ways can the letters of the word Mathematics be arranged? In how many of these arrangements will two A's be adjacent.

Differentiate tanx from first principles

Find the acute angle between the two lines below 3x – 2y = 5 and 4x + 5y = 1.

A plane goes through the points with position vectors $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and parallel to the line $\underset{\sim}{r} = \underset{\sim}{i} - \underset{\sim}{j} + \lambda(3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})$. find the distance of this plane from the origin.

$\int_0^{\ln 2} \frac{dx}{1+e^x}$

Given that $y = t - \cos t,\ x = \sin t$, show that $\frac{d^2y}{dx^2} = \frac{1+\sin t}{\cos^3 t}$

Prove that $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$. Give A, B, C are angles of a triangle.

CTION B

a) Show that $i - 1$ is a root to the equation $Z^4 - 2Z^3 - Z^2 + 2Z + 10 = 0$, hence find the other roots.

b) Find the locus defined by Arg $\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{4}$

c) Simplify $\frac{(\cos\frac{2\pi}{7} - i\sin\frac{2\pi}{7})^3}{(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7})^4}$

1. a) Solve the equation $8\cos^4 x - 10\cos^2 x + 3 = 0$ for x in the range $0 \le x \le 1$

b) Prove that cos4A – cos4B – cos4C = -1 – 4sin2Bsin2Ccos2A. Given that A, I and C are angles of a triangle.

a) $\int \frac{1+\sin x}{1+\cos x}\, dx$

b) $\int \frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}}\, dx$

c) $\int \frac{dx}{\text{⊦}x^2 - 4x + 3}$

a) F nd the square root of $5 + 2\sqrt{6}$

b) In the equation $ax^2 + bx + c = 0$ one root is the square of the other. Prov that $(a - b)^3 = a(c - b)^3$

c) Given that A and B are roots of the equation $2x^2 + 5x - 12 = 0$. Find t value of A – B.

a) Find the term independent of x in the expansion $(2x^2 - \frac{1}{2x})^6$

b) Prove by induction that $p + pq + pq^2 + \ldots..pq^{n-1} = P\frac{(1-q^n}{1-q})$

a) A normal chord AB is drawn at the point $A(4,1)$ on the rectangular hyperl $xy = 4$. Find its length.

b) Find the equation of the tangent to the ellipse $x^2/_{a^2} + y^2/_{b^2} = 1$ at the p

$(a\cos\theta, b\sin\theta)$. Hence show that if the line $y = mx + c$ is a tangent to th above ellipse then $c^2 = a^2m^2 + b^2$. Find the equation of the tangents fror point (¯3, 3) to the ellipse $x^2/_{16} + y^2/_9 = 1$.

Differentiate the function $\sqrt{\frac{(1+x^2)^3}{2+x^2}}$ and simplify

b) Given $y = e^{-t}\cos(t + \beta)$. Show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 +$

a) Given that $OA = a$ and $OB = b$. point R is on OB such that $OR : RB =$ 4: 1. Point P is on AB such that $BP : PA = 2 : 3$ and when RP and OA are b produced they meet at Q. find

(i) OR and OP in terms of a and b.

(ii) OQ in terms of a.

b) Find the vector equation of a line of the intersection of the two planes $2x + 3y - x = 4$ and $x - y + 2Z = 5$.

P425/1
PURE MATHEMATICS
PAPER I
3 HRS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PURE MATHEMATICS

PAPER I

3 HOURS

Instructions to candidates

- Answer all the eight questions in section A and any five from section B.
- Any additional question(s) answered will not be marked
- All necessary working must be shown clearly
- Graph paper is provided
- Silent non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A

1. If $f(x) = ax^2 + bx + c$ leaves remainders of 1, 25, 1 on division by $x - 1$, $x + 1$ and $x - 2$ respectively. Show that f(x) is a perfect square.

2. In how many different ways can the letters of the word Mathematics be arranged? In how many of these arrangements will two A's be adjacent.

3. Differentiate tanx from first principles

4. Find the acute angle between the two lines below 3x – 2y = 5 and 4x + 5y = 1.

5. A plane goes through the points with position vectors $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and parallel to the line $\underset{\sim}{r} = \underset{\sim}{i} - \underset{\sim}{j} + \lambda(3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})$. find the distance of this plane from the origin.

6. $\int_0^{ln2} \frac{dx}{1+e^x}$

7. Given that $y = t - \cos t$, $x = \sin t$, show that $\frac{d^2y}{dx^2} = \frac{1+\sin t}{\cos^3 t}$

8. Prove that $sin^2A + sin^2B + sin^2C = 2 + 2cosAcosBcosC$. Give A, B, C are angles of a triangle.

SECTION B

9. a) Show that $i - 1$ is a root to the equation $Z^4 - 2Z^3 - Z^2 + 2Z + 10 = 0$, hence find the other roots.

   b) Find the locus defined by Arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$

   c) Simplify $\frac{(\cos\frac{2\pi}{7} - i\sin\frac{2\pi}{7})^3}{(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7})^4}$

10. a) Solve the equation $8\cos^4 x - 10cos^2x + 3 = 0$ for x in the range $0 \leq x \leq \pi$

    b) Prove that cos4A – cos4B – cos4C = -1 – 4sin2Bsin2Ccos2A. Given that A, B and C are angles of a triangle.

11. a) $\int \frac{1+sinx}{1+cosx}\, dx$

    b) $\int \frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}}\, dx$

c) $\int \frac{dx}{4x^2 - 4x+3}$

12. a) Find the square root of $5 + 2\sqrt{6}$
b) In the equation $ax^2 + bx + c = 0$ one root is the square of the other. Prove that $c(a - b)^3 = a(c - b)^3$
c) Given that A and B are roots of the equation $2x^2 + 5x - 12 = 0$. Find the value of A – B.

13. a) Find the term independent of x in the expansion $(2x^2 - \frac{1}{2x})^6$
b) Prove by induction that $p + pq + pq^2 + \ldots..pq^{n-1} = p\frac{(1-q^n}{1-q})$

14. a) A normal chord AB is drawn at the point $A(4,1)$ on the rectangular hyperbola $xy = 4$. Find its length.
b) Find the equation of the tangent to the ellipse $x^2/_{a^2} + y^2/_{b^2} = 1$ at the point $(acos\theta, bsin\theta)$. Hence show that if the line $y = mx + c$ is a tangent to the above ellipse then $c^2 = a^2m^2 + b^2$. Find the equation of the tangents from the point (3, 3) to the ellipse $x^2/_{16} + y^2/_9 = 1$.

15. Differentiate the function $\sqrt{\frac{(1+x^2)^3}{2+x^2}}$ and simplify
b) Given $y = e^{-t}cos(t + \beta)$. Show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ +

16. a) Given that $OA = a$ and $OB = b$. point R is on OB such that $OR: RB = 4:1$. Point P is on AB such that $BP: PA = 2:3$ and when RP and OA are both produced they meet at Q. find
(i) OR and OP in terms of a and b.
(ii) OQ in terms of a.
b) Find the vector equation of a line of the intersection of the two planes $2x + 3y - x = 4$ and $x - y + 2Z = 5$.

END

P425/1
PURE
MATHEMATICS
PAPER 1
Aug, 2016
3hrs

UNNASE MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3hours

Instructions

Attempt all the eight questions in Section A and Not more than five from Section B.

Any additional question(s) will not be marked

All working must be shown clearly

Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.

SECTION A (40MARKS)

Answer **all** questions in this section

1. Solve the simultaneous equations.

$P - 2q - 2r = 0$

$2p + 3q + r = 1$

$3p - q - 3r = 3$ (5marks)

2. Solve the equation, $5\sin 2x - 10sin^2x + 4 = 0$ for $-180^0 \leq x \leq 180^0$.

(5marks)

3. The position vectors of the points A and B are $3i - j + 2k$ and $2i + 2j + 3k$ respectively. Find the acute angle between the line AB and the line

$1 - x = \frac{y+3}{2} = \frac{4-z}{4}$. (5marks)

4. Find $\int \frac{cosx}{4+sin^2x} dx$ (5marks)

5. If $x^2 + y^2 = 2y$, show that $(1-y)^3 \frac{d^2y}{dx^2} = 1$ (5marks)

6. Find the equation of the tangent to the circle $(x-1)^2 + (y+2)^2 = 8$ at the point $(3, -4)$. (5marks)

7. Prove by induction that, $6^n - 1$ is divisible by 5 for all positive integral values of n. (5marks)

8. If $= \frac{1}{x^2}$, find $\frac{dy}{dx}$ from the first principles. (5marks)

SECTION B (60MARKS)

Answer any **five** questions from this Section.

9. a) i) Solve the simultaneous equations $Z_1 + 3Z_2 = 8$, $4Z_1 - 3iZ_2 = 17 + 9i$.

(5marks)

ii) Given that $Z = \frac{7-i}{-4-3i}$ find the modulus and argument of Z, Hence express Z in the polar form. (4marks)

b)

10. a) Solve the equation $2cos2\theta = 7sin\theta \ for \ 0^0 \leq \theta \leq 360^0$. (6marks)

b) Prove that; $\frac{1+cos\theta+sin\theta}{1-cos\theta+sin\theta} = cot\frac{\theta}{2}$ (6marks)

11. a) Find the Cartesian equation of a curve whose polar equation is given by $r = atan\theta$. (3marks)

b) Obtain the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(acos\theta, asin\theta)$. If the tangent cuts the x and y axes at points Q and R respectively, determine the locus of the midpoint of QR. (9marks)

12. a) Find, $\int \frac{x^2}{(x-1)^2(x^2+4)} dx$ (9marks)

b) A function is defined by $y = I_0 sin^2(\omega t + \alpha)$ for $0 \leq t \leq \frac{2\pi}{\omega}$ where α, ω and I_0 are constants. Determine the mean value of the function.

(3marks)

13. a) Find the angles between the vectors $a = 2i + 3j - k$ and

$b = 5i + 2j + k$. (4marks)

b) A plane has the points $A(2, \ 1, \ 3)$, $B(0, \ -6, \ 2)$ *and* $C(3, \ 2, \ 1)$ on it. Determine the Cartesian equation of the plane. (4marks)

c) The normal to the plane in (b) above is a directional vector to the line passing through (1, 1, 5). Find in Cartesian form, the equation of the line. (4marks)

14. a) Evaluate; $\int_0^{\frac{\pi}{2}} \frac{4d\theta}{3+cos\theta}$ (7marks)

b) Obtain the integral $\int xe^{-2x}dx$ (5marks)

15. a) The ninth term of an AP is -1 and the sum of the first nine terms is 45. Find the common difference and the sum of the first twenty terms.

(6marks)

b) In a geometric progression the first term is 7 and the n^{th} term is 448. The sum of the first n terms is 889, find the common ratio. (6marks)

16. a) Solve the differential equation, $x\frac{dy}{dx} = y + kx^2cosx$ given that $y = 2\pi$ when $x = \pi$. (5marks)

b) A certain chemical reaction is such that the rate of transformation of the reacting substance is proportional to its concentration. If initially the concentration of the reagent was 9.5gm per litre and if after 5 minutes the concentration was 3.5gm per litre, find what the concentration was after 2 minutes. (7marks)

END

MOCK EXAMINATIONS, 2018
Uganda Advanced Certificate of Education
PURE MATHEMATICS P425/1

PAPER 1

TIME: 2 HOURS

INSTRUCTIONS:
Answer all questions.

SECTION A

1. Show that the parametric equations $x = 5 + \frac{\sqrt{3}}{2}\cos\theta,\ y = -3 + \frac{\sqrt{3}}{2}\sin\theta$ represent a circle. Find the radius and centre of the circle. (05 marks)

2. Prove by induction that $3^{2n+2} - 8n - 9$ is divisible by 64 for all positive integral values of n. (05 marks)

3. Solve $6\cos 2x + 7 = 7\sin 2x$ for $0^o \leq x \leq 360^o$ (05 marks)

4. Show that $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}} = \frac{1}{2}\ln 3$ (05 marks)

5. (i) Given that the vectors $\boldsymbol{i} - \lambda\boldsymbol{i} + 4\boldsymbol{k}$ and $2\boldsymbol{i} + 4\boldsymbol{j} + 6\boldsymbol{k}$ are perpendicular, determine the value of λ.

   (ii) The position vectors of points A and B are given by $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - 4\boldsymbol{j} + 6\boldsymbol{k}$.
   Determine the position vector of a point P which divides the line segment AB externally in the ratio 2:3. (05 marks)

6. Mr. Kayiwa's age and his three children are in a geometrical progression (G.P), the sum of their ages is 140 years, the sum of the ages of the last two children is 14 years. Find Mr. Kayiwa's age. (05 marks)

7. An examination has two sections, section A containing 5 questions and section B also containing 5 questions. A student must answer 7 questions. Given that the student must answer atleast 3 questions from section A. Find the number of ways in which the student may select the 7 questions. (05 marks)

8. Given that $e^{x+y} = x^2$. Find $\frac{dy}{dx}$. (05 marks)

SECTION B (60 Marks)

9. (i) Prove that if x is 50 small that the cube and higher powers can be neglected then.
   $\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{1}{2}x^2$
   By taking $x = \frac{1}{9}$, prove that $\sqrt{5}$ a approximately $\frac{181}{81}$

   (ii) Using the substitution $Z = x^2 - x$.
   Solve the equation $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$. (12 marks)

10. (a) Show that the vectors $a = 2i + 3j + 4k$, $b = i + 2j - 3k$ and $c = i + 5j + k$ form a triangle. Determine the area of this triangle.

(b) Find the equation of the plane containing the line whose equation is $r = (t-1)i + (t+2)j + (2t-4)k$, which is parallel to the direction vector $2i + 3j + 1c$. Hence state the distance from the origin to this plane. (12 marks)

11. (a) Find the focus, directrix and length of the latus rectum for the parabola $y^2 = 4x - 8$. (04 marks)

(b) The tangents at the points $P(ap^2, 2aP)$ and $Q(aq^2, 2aq)$ to the parabola $y^2 = 4ax$ meet at a point R. Find the co-ordinates of R. If R lies on the line $2x + a = 0$, find the equation of the locus of the mid-point of PQ. (08 marks)

12. (a) Solve the equation $\sqrt{(x-3)} + \sqrt{(2x+1)} = \sqrt{3x+4)}$ (05 marks)

(b) By expressing the equation $x + 2y + Z = 8$, $2x - y + 3Z = 9$ and $3x + 4y - Z = 8$. In raw echelon form, sole the equations. (07 marks)

13. (a) Differentiate $\frac{2x}{x+1}$ from the first principles with respect to x. (05 marks)

(b) Evaluate $\int_{-3}^{2} \frac{dx}{x^2 + 6x + 34}$ (07 marks)

14. (a) Prove the following identities:

(i) $tan^{-1}x + Cot^{-1}x = \frac{\pi}{2}$

(ii) $Sin\theta = \frac{4\, Cot\theta Cot^2\theta - 1)}{(1 + Cot^2)^2}$ (08 marks)

(b) Solve for x in;
$4Sin2x - 3Cos2x = 3$ for $0^o \leq x \leq 360^o$

15. (a) Differentiate with respect to x.

(i) $\frac{e^{Sin^2x}}{e^{-Cos^2x}}$ (04 marks)

(ii) x^{Sinx} (03 marks)

(b) An open cylindrical container is made from a $12ccm^2$ metal sheet. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}\ cm^3$. (05 marks)

16. (a) Find $\int \frac{4u^2 + 3u - 2u}{(u+1)\ (2u+3)}\, du$ (07 marks)

(b) Evaluate $\int_{1}^{2} \frac{x}{\sqrt{7 - x^2}}\, dx$ (05 marks)

END

P425/1
PURE MATHEMATICS
PAPER 1
July/August 2017
3hrs

RESOURCEFUL MOCK EXAMINATIONS, 2017

Uganda Advanced Certificate of Education

PURE MATHEMATICS

(P425/1)

TIME: 3HOURS

INSTRUCTIONS TO CANDIDATES

- ✓ *Attempt all the questions in section A and five from section B.*
- ✓ *Working must be shown clearly*
- ✓ *Silent non programmable calculator may be used.*
- ✓ *Any additional question(s) answered will not be marked.*

SECTION A

1. Prove that

$$\tan^{-1} x + cot^{-1}x = \frac{\pi}{2}$$

2. The first term of an AP and G.P are each $\frac{2}{3}$ their common difference and common ratio are x and the sum of their first 3 terms is equal. Find the possible values of x.

3. $\int x\sqrt{3x-1}\, dx.$

4. Solve$3sin(2x + {}^{\pi}/_{6}) - \cos(2x + {}^{\pi}/_{6}) = 2$

5. Find the equation of the normal to the curve $\frac{y}{x+siny} = 3$at the point where $y =$

6. Show that when the quadratic expression.

$x^2 + bx + c = 0 \; and \; x^2 + px + q = 0$have a common root then

$$(c - q)2 = (b - p)(pc - bq)$$

7. Given that

$\mathrm{P} = \log_2 3 \; and \; q = \log_4 5$, show that $\log_{45} 2 = \frac{1}{2(p+q)}$

8. Use the substitution.

$$y = x + \frac{1}{x} \text{ to solve the equation } 2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$$

SECTION B

9. Describe the locus of the complex number z which moves in the argand diagram

$Arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{2}$

b) Find the fourth roots of -16i

10. If A, B and C are angles of a triangle prove that

$sin^2 A + sin^2 B + sin^2 C = 2 + 2cosAcosBcosC$

b) By expressing $6cos^2\theta + 8sin\theta cos\theta$ in the form $Rcos(2\theta - 2)$. Find the maximum and minimum value of $6cos^2\theta + 8sin\theta cos\theta = 4$

11. The curve with the equation $y = \frac{ax+b}{x(x+2)}$ where a and b are constants has a turning point at (1, -2). Find the values of a and b.

Find the equation of all the asymptotes.

Sketch the curve.

12. Differentiate

$y = 2x^{cosx}$

$y = \frac{e^{sinx}}{tanx}$

b) Prove that $\int_1^3 \left(\frac{3-x}{x-1}\right)^{\frac{1}{2}} dx = \pi$. Use the substitution $x = 3sin^2\theta + cos^2\theta$.

c) The displacement of a particle at time t is x measured from a fixed point and $\frac{dx}{dt} = \frac{C(e^{2act}-1)}{e^{2act}+1}$, prove that $x = \frac{C(e^{2act}-1)}{e^{2act}+1}$, if x = 3 when t = 1 and x = $\frac{75}{17}$, prove that c = 5

13. Show that the lines

$r = 2i - 3j + 4k + \lambda(3i - 2j + k)$ and $r = i + 3j + k + \mu(-i - 2j + k)$ intersect. Find the point of intersection.

b) OAB is a triangle with OA = $\underset{\sim}{a}$, $OB = \underset{\sim}{b}$, c is amidpoint of OB, D is the midpoint of AB and E is amidpoint of OA. OD and AC interest at F. if AF = hAC and OF = KOD. Find the values of h & k. show that B, F & E are collinear.

14. a) Solve $\frac{dy}{dx} + 2y\tan x = \cos^2 x$

$y(0) = 2$

b) A radioactive substance disintegrates at a rate proportional to it's mass one half of the given mass of a substance distergrates 136 days, calculate the time required for $\frac{5}{8}$ of a substance to disintergrate. If the original mass of a substance was 100gm. Calculate the mass after 34 days.

15. Find the equation of the tangents to the curve at $y = x^3$ at (t, t^3) prove that this tangent meets the curve again at $Q(-2t - 8t^3)$. Find the locus of the midpoint of PQ.

b) Given that $y = mx + c$ is a tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$. Show that

$(1 + m^2)r^2 = (c-b+am)^2$.

16. a) $\int_1^2 \frac{8x+6}{(2x-1)^2 (x+2)^2}$

b) $\int_0^{\pi/2} \frac{1}{2+\cos^2 x}\, dx$

END