P425/1
Pure Mathematics
Paper 1
July - August, 2024
3 hours

UMTA

UGANDA MUSLIM TEACHERS' ASSOCIATION

UMTA JOINT MOCK EXAMINATIONS - 2024

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

- *Attempt **all** the **eight** questions in section **A** and **five** questions from section **B**.*

- *Any additional question(s) answered will **not** be marked.*

- ***All** working **must** be shown clearly. Begin each question on a fresh sheet of paper.*

- *Silent, nonprogrammable scientific calculators and mathematical tables with a list of*

formulae may be used.

SECTION A

1. $\int x^4 \ln x \, dx$ (05 marks)

2. Find the acute angle between the following lines, $2x + 3y = 7$, $x = 6y + 5$ (05 marks)

3. If $y = \sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}}$ show that $\dfrac{dy}{dx} = \sec^2 x$. (05 marks)

4. Show that the vectors $2i - j + k$, $i - 3j - 5k$ and $3i - 4j - 4k$ are coplanar. (05 marks)

5. Solve for **x** from 0^0 to 360^0 Given that $\tan x + \tan 2x + \tan x \tan 2x = 1$ (05 marks)

6. Solve for **x** given $9 \log_x 5 = \log_5 x$. (05 marks)

7. Find the area bounded by the curve $y = (1-x)(x+2)$ and the x-axis. (05 marks)

8. Solve for **x** Given $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$ (05 marks)

SECTION B

9. Given the curve $y = \dfrac{3x+3}{x(3-x)}$;

(a) Find the region where the curve does not lie, hence determine the turning points and their nature.

(b) State the asymptotes and find the intercepts.

(c) Sketch the curve. (12 marks)

10. (a) Solve the equation $\sqrt{3 - x} - \sqrt{7 + x} = \sqrt{16 + 2x}$. (06 marks)

(b) Solve for **x, y,** and **z** given $\dfrac{x+2y}{-3} = \dfrac{y+2z}{4} = \dfrac{2x+z}{5}$ and $x + y + z = 2$. (06 marks)

11. (a) Show that $\dfrac{(\cos 4\theta + i\sin 4\theta)^3 (\cos 2\theta - i\sin 2\theta)^5}{(\cos 3\theta + i\sin 3\theta)^4 (\cos 5\theta - i\sin 5\theta)^6} = \cos 20\theta + i\sin 20\theta$. (06 marks)

(b) Shade the region on Argand diagram of $|z - 1 - i| < 3$. (06 marks)

12. Sketch the curves $y = 2x^2$ and $y = 10x - x^2$ on the same graph.

Find the volume generated when the area enclosed between the curves is rotated through 360^0

(12 marks)

13. (a) Prove that $\dfrac{sin5x - sin7x + sin8x - sin4x}{cos4x - cos5x - cos8x + cos7x} = \cot 6x.$

(06 marks)

(b) Find all the possible values of **x** from 0^0 to 360^0 of the equation

$4cosx - 6sinx = 5.$

(06 marks)

14. Partialise $f(x) = \dfrac{3x^3 + x + 1}{(x-2)(x+1)^3}$ Hence evaluate $\int_3^4 f(x)dx.$

(12 marks)

15. (a) The gradient of the tangent at any point *(x,y)* of the curve is $x - \dfrac{2y}{x}$

Given that the curve passes through (2,4). Find the equation of the curve. (06 marks)

(b) Use substitutions $y = vx$ to solve the differential equation

$$x^2 \frac{dy}{dx} = x^2 + y^2 + xy$$

(06 marks)

16. (a) Find the point of intersection between the line $r = i + j - 3k + t(2i + 2j + k$

and the plane $r.(6i - 3j + 2k) = 13$ and find the angle between the two. (06 marks)

(b) Show that the following vectors form a right angled triangle

$a = (3i - 2j + k), b = (i - 3j + 5k), c = (2i + j - 4k).$
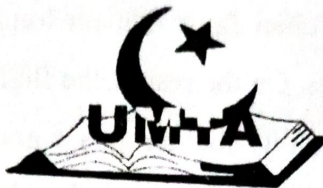
(06 marks)

END

P425/2
Applied Mathematics
Paper 2
July - August, 2024
3 hours

UMTA

UGANDA MUSLIM TEACHERS' ASSOCIATION

UMTA JOINT MOCK EXAMINATIONS 2024

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Applied Mathematics

Paper 2

3 hours

INSTRUCTIONS:

- *Answer **all** the **eight** questions in section A and any **five** from section **B**.*
- *Any additional question(s) answered will **not** be marked.*
- ***All** necessary working **must** be shown clearly.*
- *Begin each number on a fresh sheet of paper.*
- *Graph paper is provided.*
- *Silent, non-programmable scientific calculators and mathematical table with a list of formulae may be used.*
- *In numerical work, take acceleration due to gravity **g**, to be **9.8ms⁻²***

Include the allocation table on your answer sheet

Question	Marks
Section A	
9	
10	
11	
12	
13	
14	
15	
16	
Total	

SECTION A (40 MARKS)

*Answer **all** questions in this section.*

1. Over a period of time, Julian finds that on long-distance flights, he flies economy class on 82% of the flights. On the rest of the flights, he flies first class. When he flies economy class, the probability that he gets a good night's sleep is **x**. When he flies first class, the probability that he gets a good night's sleep is 0.9.

   (a) The probability that Julian gets a good night's sleep on a randomly chosen flight is 0.285. Find the value of **x**.   (02 marks)

   (b) Given that on a particular flight, Julian does not get a good night's sleep, find the probability that he is flying economy class.   (03 marks)

2. A particle is projected from a point **O** with initial velocity 30 ms⁻¹ at an angle $\tan^{-1}(2)$ to the horizontal. Find its height above **O** when its speed is 20 ms⁻¹.   (05 marks)

3. The cost of baking a cake is calculated from the cost of the items in 2022 and 2023.

Item	Price	
	2022 = 100 %	2023
Flour per kg.	6000	7800
Sugar per kg.	5000	4000
Eggs per egg.	500	700
Milk per litre	2000	2500

   Calculate the simple price index for the cost of making a cake. Comment on your result.   (05 marks)

4. Use trapezium rule with four sub intervals to estimate $\int_{0.2}^{1.0} \frac{x}{2 + \cos x} dx$ correct to **two** significant figures.   (05 marks)

5. Three forces of magnitudes **1N**, **8N** and $3\sqrt{2}$ **N** act along **CD**, **CB** and **AC** respectively as shown. **ABCD** is a square of side a units.
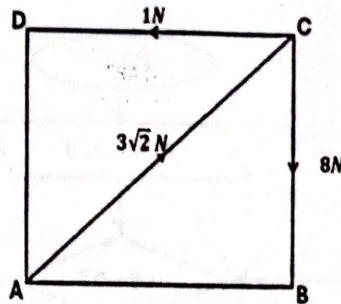
Consider **AB** and **AD** as x and y-axes respectively. Taking moments about **A**, show that the equation of line of action of the resultant is $2y + 5x - 7a = 0$. (05 marks)

6. The probability density function (pdf) of a Continuous random variable **X** is given by,

$$f(x) = \begin{cases} \dfrac{4}{9}x(1 - x^2) ; & -2 \le x \le 1 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Find the mode of **X**. (05 marks)

7. A body of weight **49N** lies on a rough plane which is inclined at $35°$ to the horizontal. The angle of friction between the plane and the body is $20°$. Find the magnitude of the least force that must be applied to the body in a direction parallel to and up the plane to hold the body in equilibrium. (05 marks)

8. Study the flow chart below.
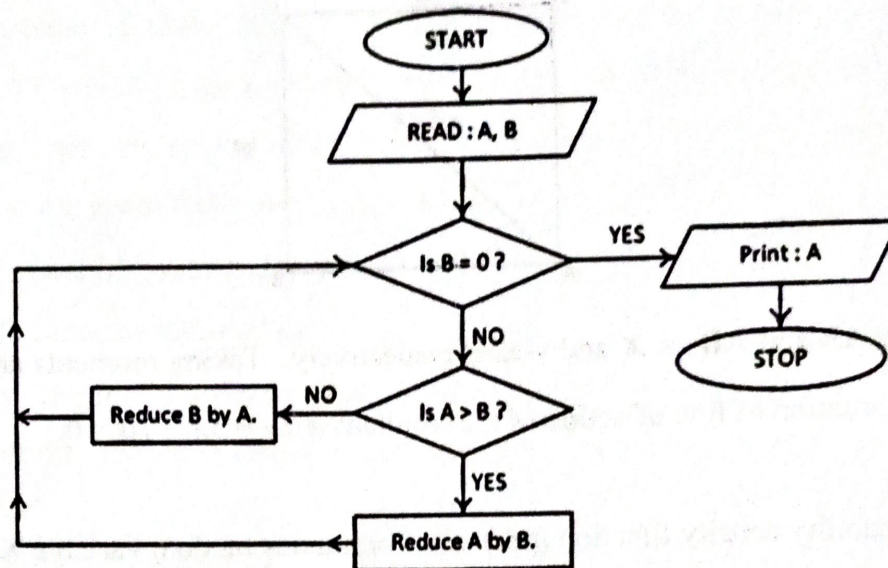
Perform a dry run for **A** = 8 and **B** = 10. (05 marks)

SECTION B (60 MARKS)

*Answer any* **five** *questions from this section.*

All questions carry equal marks.

9. The patients at a chest clinic were asked to keep a record of the number of cigarettes they smoked every day.

Number of cigarettes	Frequency
0 – 9	5
10 – 14	8
15 – 19	32
20 – 29	41
30 – 39	16
40 – 49	2

(a) Use the frequency distribution table to estimate the;

    (i)   Mean.

    (ii)   Number of patients who smoked 25 cigarettes and below.   (07 marks)

(b) Draw a histogram to represent the data. Use it to estimate the mode.   (05 marks)

10. The diagram below shows **three** particles **A, B** and **C** of mass 3kg, 2kg and 8kg respectively. One end of a light inextensible string is attached to a ceiling. The string passes under a moveable pulley **C** and then over a fixed smooth pulley. To the other end of the string is attached a light scale pan in which two weights **A** and **B** are placed with **A** on top of **B**.
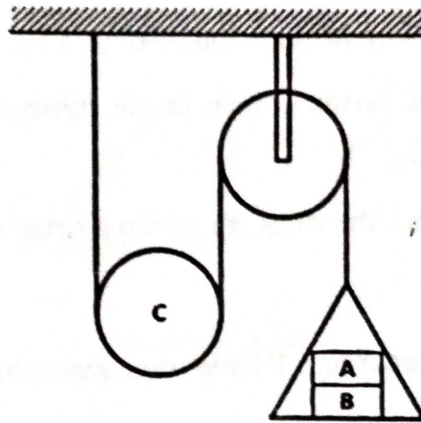
    The system is released from rest.

    (a) Calculate the;

        (i)   Acceleration of the particle, C.   (09 marks)

        (ii)   Tension in the string.   (03 marks)

    (b) Find the reaction between the weights **A** and **B**.

11. Show that **one** of the roots of the equation $x^2 - 3x + 1 = 0$ lies between 2 and 3. Hence use linear interpolation to find the root correct to **two** decimal places.

                                                           (12 marks)

12. The marks of 400 students in a certain test are normally distributed with a mean of 54 marks and a standard deviation of 9 marks.

(a) Given that the pass mark is **38**, estimate the number of students who passed the test. (05 marks)

(b) Find the probability that a student selected at random scored between 49 and 57 marks. (04 marks)

(c) If a sample of 10 students was taken at random, determine the probability that at least one scored between 49 and 57. (03 marks)

13. A ship **P** steaming at 20 kmh^{-1} in the direction N50°E is 120 km due West of a ship **Q** steaming at 12 kmh^{-1} in the direction N30°W. Find the;

(a) Shortest distance between them in the subsequent motion and the time taken to reach this position. (07 marks)

(b) The time for which the ships are within a range of 50 km from each other.

(05 marks)

14. Two decimal numbers **X** and **Y** have been approximated by **X** and **Y** with errors ΔX and ΔY respectively.

(a) Show that the maximum relative error made in approximating $x\sqrt{y}$ by $X\sqrt{Y}$ is given by $\left|\frac{\Delta X}{X}\right| + \frac{1}{2}\left|\frac{\Delta Y}{Y}\right|$. (08 marks)

(b) Given that **X** and **Y** have been measured as 1.824 and 3.9 respectively, determine the percentage error made in the expression $X\sqrt{Y}$. (05 marks)

15. A box contains 3 red, 2 green and 1 yellow balls. When a ball is drawn from the bag, it is returned together with a ball of the same colour. If two such random draws are made,

(a) Construct a probability distribution table for the number of red balls in the bag after the two draws. (06 marks)

(b) Determine the;

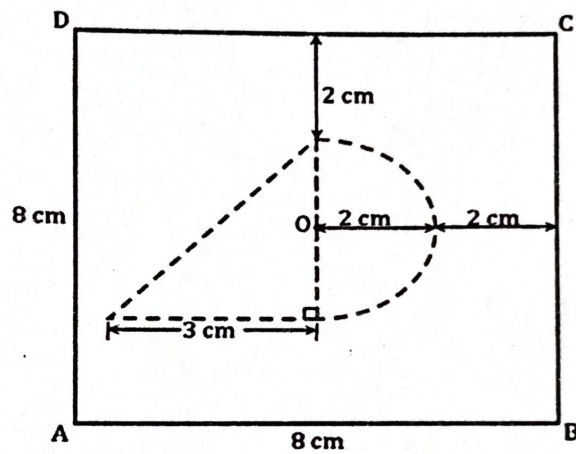   (i) Expected number of red balls.

    (ii) Median number of red balls.   (04 marks)

(c) Sketch the Cumulative probability distribution.   (02 marks)

16. **ABCD** is a uniform square lamina of side 8cm. A hole made up of a right-angled triangle and a Semi circle with centre **O** and radius 2cm was cut off from the square as shown in the diagram below.

(a) Find the position of the centre of gravity of the remaining sheet.   (09 marks)

(b) The remaining sheet is then suspended about **B**, determine the angle side **AB** makes with the horizontal.   (03 marks)

END