EQUILIBRIUM IN NATIONAL INCOME ANALYSIS

The equilibrium analysis can also be applied to other areas of economics. As a simple example we may cite the familiar Keynesian national income model,

$Y = C + I_0 + G_0$(Equilibrium condition)

$C = C_0 + bY_d$(Consumption function)

Where Y and C stand for the endogenous variables, national income and consumption expenditure, respectively. $C_0, and\ G_0$ Represent the exogenously determined investment and government expenditures.

Solving the two linear equations, we obtain the equilibrium national income and consumption expenditure.

$Y = C + bY + I_0 + G_0$

Collecting like terms;

$Y - bY = C_0 + I_0 + G_0$

$Y(1 - b) = C_0 + I_0 + G_0$

$\bar{Y} = \frac{C_0 + I_0 + G_0}{1 - b}$

Where the numerator stands for autonomous expenditure

$C = C_0 + bY$

But $Y = C + I_0 + G_0$

$C = C_0 + bY$

$C = C_0 + b(C + I_0 + G_0)$

$C = C_0 + bC + bI_0 + bG_0$

$C - bC = C_0 + b(I_0 + G_0)$

$C\frac{(1-b)}{1-b} = \frac{C_0 + b(I_0 + G_0)}{1-b}$

$\bar{C} = \frac{C_0 + b(I_0 + G_0)}{1-b}$

Example 2

$Y = C + I + G$

Derive the equilibrium income given;

$C = C_0 + bY_d$

$T = T_0 + tY$

$I = I_0, G = G_0, Tr = Tr_0$

$Y_d = Y - T + Tr$

$= Y - (T_0 + tY) + Tr_0$

$C = C_0 + b(Y - T_0 - tY + Tr_0)$

$Y = C_0 + b(Y - T_0 - tY + Tr_0) + I_0 + G_0$

$Y = C_0 + (bY - bT_0 - btY + bTr_0) + I_0 + G_0$

$Y - bY + btY = C_0 - bT_0 + bTr_0 + I_0 + G_0$

$Y(1 - b + bt) = C_0 - bT_0 + bTr_0 + I_0 + G_0$

$\boldsymbol{\bar{Y} = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0}{1 - b + bt}}$

Example 3

$Y = C + I_0$

$C = 200 + 0.6Y$

$I_0 = 50$

i. Determine the equilibrium income
ii. Determine the value of the multiplier
iii. What type of multiplier is implied in the model
iv. What happens to the multiplier if ; $C = 200 + 0.8Y$

<u>Solution</u>

i) $\bar{Y} = \frac{C_0 + I_0}{1 - b}$

$\bar{Y} = \frac{200 + 50}{1 - 0.6} = \frac{250}{0.4} = 625$

$$\underline{\underline{= \mathbf{625}}}$$

ii) $\bar{Y} = \frac{C_0+I_0}{1-b}$ ……………………………………………………………i

Let $\bar{Y}$ change by ΔY, and I_0 change by ΔI_0

$\bar{Y} + \Delta Y = \frac{C_0+I_0+\Delta I_0}{1-b}$ ………………………………………………………ii

Equation ii-i

$$Y + \Delta Y - Y = \frac{C_0+I_0+\Delta I_0}{1-b} - \frac{C_0+I_0}{1-b}$$

$$\Delta Y = \frac{C_0+I_0+\Delta I_0-C_0-I_0}{1-b}$$

$\Delta Y = \frac{\Delta I_0}{1-b}$, multiply through both sides of the equation by $\frac{1}{\Delta I_0}$

$$\Delta Y \times {}^{1}/_{\Delta I_0} = \frac{\Delta I_0}{1-b} \times {}^{1}/_{\Delta I_0}$$

$$\frac{\Delta Y}{\Delta I_0} = \frac{1}{1-0.6} = \mathbf{2.5}$$

iii) This is the investment multiplier

iv) $C = 200 + 0.8Y$

$$= \frac{1}{1-0.8} = \frac{1}{0.2} = \mathbf{5}$$

Here, the multiplier doubles

Example 4

Consider the following national income model

$$Y = C + I + G$$

$$C = 40 + 0.75Y_d$$

$$T = 20 + 0.2Y$$

$$Tr_0 = 10, I_0 = 30, G_0 = 20$$

i. What is the equilibrium level of national income
ii. What is the government budget position
iii. What is the value of the government budget position
iv. Calculate the net taxes
v. What is the average propensity to consume at equilibrium level of income

vi. What are the values of investment and government multiplier in the model

vii. What is the value of the balanced budget multiplier

Solution

i) $Y = C + I + G$

$C = C_0 + bY_d$

$T = T_0 + tY$

$I = I_0, G = G_0, Tr = Tr_0$

$Y_d = Y - T + Tr$

$= Y - (T_0 + tY) + Tr_0$

$C = C_0 + b(Y - T_0 - tY + Tr_0)$

$Y = C_0 + b(Y - T_0 - tY + Tr_0) + I_0 + G_0$

$Y = C_0 + (bY - bT_0 - btY + bTr_0) + I_0 + G_0$

$Y - bY + btY = C_0 - bT_0 + bTr_0 + I_0 + G_0$

$Y(1 - b + bt) = C_0 - bT_0 + bTr_0 + I_0 + G_0$

$\boldsymbol{\bar{Y} = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0}{1 - b + bt}}$

Given $\boldsymbol{C_0 = 40, b = 0.75, T_0 = 20, t = 0.2, Tr_0 = 10, I_0 = 30, G_0 = 20}$

$\bar{Y} = \frac{40 - (0.75 \times 20) + (0.75 \times 10) + 30 + 20}{1 - 0.75 + (0.75 \times 0.2)}$

$\bar{Y} = \frac{40 - 15 + 7.5 + 50}{0.25 + 0.15}$

$\bar{Y} = \frac{82.5}{0.4} = \boldsymbol{206.25\ units}$

ii) Budget position $(BP) = T - G_0 - Tr_0$

But $T = T_0 + tY$

$T = 20 + (0.2 \times 206.25)$

$T = 20 + 41.25$

$T = 61.25$

$BP = T - G_0 - Tr_0$

$= 61.25 - 20 - 10$

$= 31.25$

Since the difference between government revenue and expenditure is positive; the budget position is a surplus budget

iii) The value of the government budget position is 31.25 units

iv) The net taxes $= T - Tr_0$

But $T = 61.25, Tr_0 = 10$

Net taxes $= 61.25 - 10$

$\underline{\underline{= \mathbf{51.25}}}$

v) $APC = \frac{C}{Y}$

$C = C_0 + b(Y - T + Tr_0)$

$APC = \frac{40+0.75(206.25-51.25+10)}{206.25}$

$APC = \frac{40+154.69-38.44+7.5}{206.25}$

$APC = \frac{163.75}{206.25}$

$\underline{\underline{= \mathbf{0.79}}}$

Implying that 79% of the total income is consumed

Given; $\boldsymbol{\bar{Y} = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0}{1 - b + bt}}$ **1**

vi) Investment multiplier $= \frac{\Delta Y}{\Delta I_0}, \Delta Y = \Delta I_0$

$\bar{Y} + \Delta Y = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0 + \Delta I_0}{1 - b + bt}$ 2

Equation 2-1

$\bar{Y} + \Delta Y - \bar{Y} = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0 + \Delta I_0}{1 - b + bt} - \bar{Y}$

$\Delta Y = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0 + \Delta I_0}{1 - b + bt} - (\frac{C_0 - bT_0 + bTr_0 + I_0 + G_0}{1 - b + bt})$

$$\Delta\bar{Y} = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0 - C_0 + bT_0 - bTr_0 - I_0 - G_0}{1-b+bt}$$

$$\Delta Y = \frac{\Delta I_0}{1-b+bt}$$

$$\frac{\Delta Y}{\Delta I_0} = \frac{\Delta I_0}{1-b+bt} \times \frac{1}{\Delta I_0}$$

$$= \frac{1}{1-b+bt}$$

$$= \frac{1}{1-0.75+(0.75\times 0.2)} = \frac{1}{0.4}$$

$= 2.5$

Government multiplier $= \frac{\Delta Y}{\Delta G_0}, \Delta Y = \Delta G_0$

Let $Y = \frac{C_0 - bT_0 + bTr_0 + I_0 + G_0}{1-b+bt} = \frac{A}{1-b+bt}$ … … … … … … … … … … … … … 1

$Y + \Delta Y = \frac{A + \Delta G_0}{1-b+bt}$ … .2

$$Y + \Delta Y - Y = \frac{A + \Delta G_0}{1-b+bt} - \frac{A}{1-b+bt}$$

$$\Delta Y = \frac{A + \Delta G_0 - A}{1-b+bt}$$

$$\Delta Y = \frac{\Delta G_0}{1-b+bt}, \frac{\Delta Y}{\Delta G_0} = \frac{\Delta G_0}{1-b+bt} \times \frac{A}{\Delta G_0}$$

$$\frac{\Delta Y}{\Delta G_0} = \frac{1}{1-b+bt} = \frac{1}{1-0.75+0.1}$$

$$= \frac{1}{0.4}$$

$= 2.5$

vii) Let $Y = \frac{A}{1-b+bt}$ … … … … … … … … … … … … 1

$Y + \Delta Y = \frac{A - b\Delta T_0 + \Delta G_0}{1-b+bt}$ … … … … … … … … … .2

Equation 2-1

$$Y + \Delta Y - Y = \frac{A - b\Delta T_0 + \Delta G_0}{1-b+bt} - \frac{A}{1-b+bt}$$

$$\Delta Y = \frac{-b\Delta T_0 + \Delta G_0}{1-b+bt}, but\ \Delta T_0 = \Delta G_0$$

$$= \frac{-b\Delta G_0 + \Delta G_0}{1-b+bt}$$

$$= \frac{\Delta G_0(1-b)}{1-b+bt} \times \frac{1}{G_0}$$

$$\frac{\Delta Y}{\Delta G_0} = \frac{1-b}{1-b+bt}$$

$$\therefore \frac{\Delta Y}{\Delta G_0} = \frac{1-0.75}{1-0.75+(0.75\times0.2)}$$

$$= \frac{0.25}{0.4}$$

$$= 0.62$$

Exercise

Consider the following three sector model

$$Y = C + I + G$$

$$C = 20 + 0.5Y_d;\ Y_d = Y - T + Tr_0$$

$$T_0 = 10, t = 0.25, Tr_0 = 10, I_0 = 30, G_0 = 20$$

a) What instruments of fiscal policy are available in this model?
b) What is the equilibrium level of income?
c) What is the government budget position?
d) What is the value of the government budget position?
e) Calculate the net taxes
f) What is the APC at equilibrium level of income?
g) Calculate the level of autonomous expenditure at equilibrium level of income
h) What are the values of the tax and government multiplier?
i) What is the value of the balanced budget multiplier?