# NCDC
### NATIONAL CURRICULUM DEVELOPMENT CENTRE

# MATHEMATICS
# LEARNER'S BOOK
## SENIOR ONE

## LOWER SECONDARY
## CURRICULUM

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DEVELOPMENT CENTRE**

# MATHEMATICS
## LEARNER'S BOOK

## SENIOR ONE

**LOWER SECONDARY
CURRICULUM**

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# Contents

# Preface

This Learner's Textbook has been written in line with the revised Mathematics syllabus. The knowledge and skills which have been incorporated are what is partly required to produce a learner who has the competences that are required in the 21st century.

This has been done by providing a range of activities which will be conducted both within and outside the classroom setting. The learner is expected to be able to work as an individual, in pairs and groups according to the nature of the activities.

The teacher as a facilitator will prepare what the learners are to learn and this learner's book is one of the materials which are to be used to support the teaching and learning process.

**Associate Professor Betty Ezati**
Chairperson, NCDC Governing Council

# Acknowledgements

National Curriculum Development Centre (NCDC) would like to express its appreciation to all those who worked tirelessly towards the production of the Learner's Textbook.

Our gratitude goes to the various institutions which provided staff who worked as a panel, the Subject Specialist who initiated the work and the Production Unit at NCDC which ensured that the work produced meets the required standards. Our thanks go to **Enabel** which provided technical support in textbook development.

The Centre is indebted to the learners and teachers who worked with the NCDC Specialist and consultants from Cambridge Education and Curriculum Foundation.

Last but not least, NCDC would like to acknowledge all those behind the scenes who formed part of the team that worked hard to finalise the work on this Learner's Book.

NCDC is committed to uphold the ethics and values of publishing. In developing this material, several sources have been referred to which we might not fully acknowledge.

We welcome any suggestions for improvement to continue making our service delivery better. Please get to us through P. O. Box 7002 Kampala or email us through admin@ncdc.go.ug.

**Grace K. Baguma**
Director, National Curriculum Development Centre

# Topic 1

# NUMBER BASES

**Key Words**
base
binary
decimal

By the end of this topic, you should be able to:

   i)   identify numerals in base(s) up to base 16.
  ii)   identify place values of different bases using abacus.
 iii)   convert numbers from one base to another.
 iv)   manipulate numbers in different bases with respect to all four
       operations.

## Introduction

I Am an ordinary person, how many fingers do I have on:

i)    one hand?
ii)   two hands?

If you have heaps of oranges of ten, twelve and fifteen, how many groups of tens, fives and fours do you get in each? And how many are remaining in each heap?

**In order to answer the above questions, you can** use **your knowledge of decimal place value to develop your understanding of numbers written in other bases.**

## Sub-topic 1. 1: Identifying numbers of different bases on an abacus

In your primary education, you studied number bases such as bases five, two and ten (decimal base). Remember the numerals for all the various number bases you studied by doing the following activity:

### Activity 1. 1: Getting familiar with number bases

In your groups, identify situations in which you have ever used number bases in your life.

| Real life situation | Base | Reason for the base chosen |
|---|---|---|
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Question: Which possible base does each abacus below represent?

**a)**

ten
tens
(hundreds)  tens  ones

**b)**

five
fives  fives  ones

**c)**

six
sixes  sixes  ones

**d)**

nine
nines  nines  ones
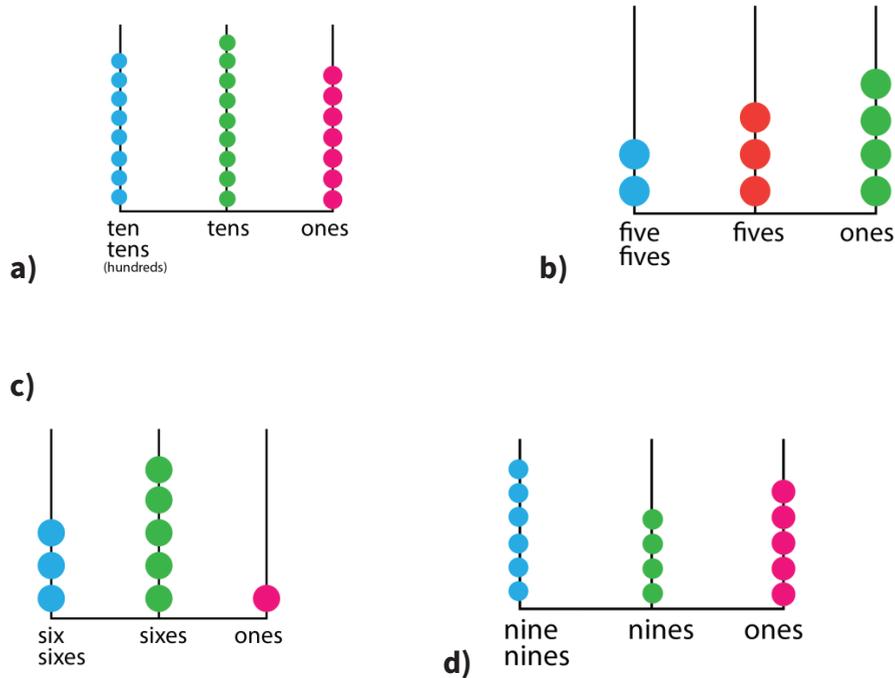
## Activity 1.2: List the numerals for the following bases

In your groups, list the numerals for the following bases:

   i)    Two (Binary) ii) Three. iii) five iv) seven v) eight. vi) nine vii) eleven viii) twelve ix) sixteen

Now study the table below and fill in the gaps.

| Base | Numerals |
|---|---|
| Two | 0, 1 |
| Three | 0, 1, 2 |
| Four | 0, 1, -, 3 |
| Five | 0, - , 2, - , 4 |
| Nine | 0, 1, 2, - , 4, - , 6, - , 8 |
| Twelve | 0, - , 2, - , 4, - , - , 7, - , 9, - , e |
| Sixteen | 0,1,2,3,4,5,-,-,-,9,t,e,-,-,-,- |

Compare your answers and note what happens to the base number when writing the numerals used in a particular base. Give reasons.

## Sub-topic 1. 2:  Place Values Using the Abacus

You have already learnt how to represent numbers on an abacus. The representation of numbers on an abacus helps you to identify the place value of digits in any base.

### Activity 1.3:  Making abaci

In groups work in pairs to make different abaci, in different bases.

Compare your work with  other members of the group .

### Activity 1.4: Reading and stating the value of digits in bases

In groups, represent the following numbers on an abacus:

a.      $123_{four}$
b.      $274_{ten}$
c.      $1312_{five}$

Read and state what each digit in the numbers above represents on an abacus using the stated bases.

### Exercise

State the place value of each numeral in the following numbers:

   a)   $321_{four}$   b) $354_{six}$   c) $247_{eight}$

State the value of each numeral in the following numbers:

   b)   $567_{nine}$ b) $381_{twelve}$   c) $11010_{two}$

Represent the following numbers on the abacus:

(a) $1101_{two}$ (b) $2102_{three}$ (c) $2021_{four}$ (d) $5645_{seven}$ (e) $8756_{nine}$

## 1.3  Converting Numbers

Numbers can be converted from one base to another, and when you do this, you get the same numbers written in different bases.

You learnt how to convert from base ten to any other base.

**Activity 1.5: Converting numbers from base ten to any other base**
In groups, convert the following numbers in base ten to bases indicated: 456, 1321, 5693, 56 and 436.

(a) Five (b) Nine (c) Eight

You can also convert numbers from any base to base ten (decimal).

**Example**: Convert (a) $101_{two}$ (b) $324_{five}$ (c) $756_{eight}$ to base ten.

**Solution:**

(a) $101_{two} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 1 \times 4 + 0 \times 1 + 1 \times 1 = \qquad 4+0+1 = 5$

(b) $324_{five} = (3 \times 5^2) + (2 \times 5^1) + (4 \times 5^0) = 3 \times 25 + 2 \times 5 + 4 \times 1 = 75+10+4 = 89$

(c) $756_{eight} = (7 \times 8^2) + (5 \times 8^1) + (6 \times 8^0) = 7 \times 64 + 5 \times 8 + 6 \times 1 = 448+40+6 = 494$

**Activity 1.6: Converting numbers in a given base to another base**
In pairs, discuss how to convert numbers in different bases to various bases in the exercise below.

**Exercise**

Convert the following numbers to the bases indicated: (a) $762_{eight}$ to base seven; (b) $234_{five}$ to base six; (c) $561_{seven}$ to base nine; (d) $654_{six}$ to base four; (e) $5432_{six}$ to twelve.
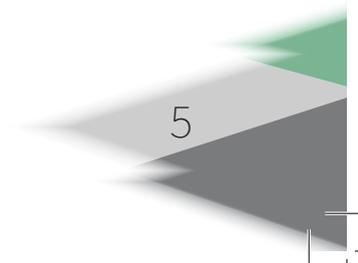
# 1.4: Operation on Numbers in Various Bases

James had two jackfruit trees in his compound. At one time one tree had 8 fruits ready and the other 7 fruits. He harvested them at the same time. He decided to put them in heaps of nine fruits. How many heaps of nine did he get and how many remained?

When you put the fruits in heaps of 9, you are adding in base 9.

**Addition**

The two jack fruit trees above had a total of 15 (that is 8 +7) ready fruits.

You can add numbers in various bases. For example, add the following numbers:

(a) $234_{five}$ to $23_{five}$ (b) $153_{seven}$ to $453_{seven}$

**Solution**

(a)
$$
\begin{array}{r}
235_{five} \\
+\ 23_{five} \\
\hline
312_{five}
\end{array}
$$

(b)
$$
\begin{array}{r}
153_{five} \\
+\ 453_{five} \\
\hline
636_{five}
\end{array}
$$

**Exercise:** Add the following numbers:

(a) $321_{four}$ to $122_{four}$. (b) $456_{seven}$ to $342_{seven}$

(c) $764_{eight}$ to $361_{eight}$. (d) $210_{three}$ to $211_{three}$

## Subtraction

Subtraction in other bases is done in the same way it is done in base ten.

**Examples**: Subtract (a) $342_{eight}$ from $567_{eight}$ (b) $432_{six}$ from $514_{six}$

**Solution:**

(a)
$$\begin{array}{r} 567_{eight} \\ - 342_{eight} \\ \hline 224_{eight} \end{array}$$

(b)
$$\begin{array}{r} 514_{six} \\ - 432_{six} \\ \hline 224_{six} \end{array}$$

## Exercise

Subtract the following numbers in the given bases:

(a) $351_{six}$ from $510_{six}$ (b) $672_{nine}$ from $854_{nine}$
(c) $845_{twelve}$ from $t23_{twelve}$ (d) $231_{five}$ from $421_{five}$

## Multiplication

Multiplication is done in the way it is done in base ten.

**Example:** Multiply $423_{five}$ by $12_{five}$

**Solution**
$$\begin{array}{r} 423 \\ \times 12 \\ \hline 1401 \\ + 4230 \\ \hline 11131 \end{array}$$

## Exercise:

Multiply the following:

(a) $241_{five}$ by $13_{five}$. (b) $345_{six}$ by $24_{six}$
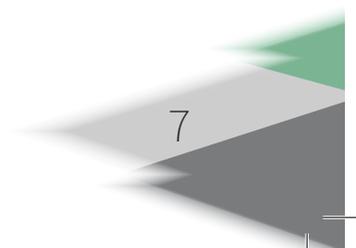
(c) $534_{seven}$ by $123_{seven}$.  (c) $156_{eleven}$ by $534_{eleven}$

## Division

The most common method of dividing numbers in different bases is by converting the numbers to base ten first and after division, you can convert the answer to the given base.

**Example**: Divide $1111_{two}$ by $101_{two}$

**Solution**: Convert $1111_{two}$ and $101_{two}$ to base ten

$1111_{two} = (1x2^3) + (1x2^2) + (1x2^1) + (1x2^0)$

$= 8 + 4 + 2 + 1$

$= 15.$

$101_{two} = (1x2^2) + (0x2^1) + (1x2^0)$

$= 4 + 1 = 5_{ten}$

Therefore, $1111_{two}$ divided $101_{two}$ is the same as 15 divided 5.

$15 \div 5 = 3$

$3_{ten} = 3 \div 2 = 1$ remainder $1 = 11_{two}$

Therefore, $1111_{two} \div 101_{two} = 11_{two}$

## Exercise:

1. Add: (a) $654_{seven}$ to $514_{seven}$ (b) $278_{nine}$ to $756_{nine}$
2. Subtract: (a) $412_{six}$ from $554_{six}$ (b) $435_{eight}$ from $764_{eight}$
3. Multiply: (a) $1121_{three}$ by $212_{three}$ (b) $312_{four}$ by $122_{four}$
4. Divide: (a) $100011_{two}$ by $111_{two}$ (b) $150_{nine}$ by $20_{nine}$

**Activity 1.6: Operations on numbers with mixed bases**

In your groups work in pairs discuss how you would carry out the four mathematical operations on numbers with mixed bases by getting your own examples. Compare your answers with other members of the group.

**Number Game**: You are given four boxes containing numbers in base ten. The boxes are labelled Box 1, Box 2, Box 3 and Box 4.

| 9  1  15  7 |
|---|

Box 1

| 6  14  2  7  15 |
|---|

Box 2

| 15  14  6  12  4  7 |
|---|

Box 3

| 15  14  9  12 |
|---|

Box 4

**Task:** Working in groups, select one number from any of the boxes given. Your mathematics teacher will ask you whether the number you selected appears in Box 1, Box 2, Box 3 and Box 4. From the responses you give, the teacher will tell you the number you selected. Discuss how the teacher was able to tell you the number you had selected.

**Situation of Integration**

A community is hit by famine and the government decides to give each member in the household a potato to solve their problem of hunger.

**Support:** Each package contains an equal number of potatoes of five.

There are 10 households in the community with 3, 5, 7, 4, 6, 5,8,12, 13 members respectively.

**Resources:** Knowledge of Bases, knowledge of mathematical operations

**Task:** Determine the number of packages of potatoes the government will take to that community. In case there are remaining potatoes, discuss what the government should do with them.

# Topic 2:
# WORKING WITH INTEGERS



**Key Words**

positive, negative, BODMAS, LCM, HCF

By the end of this topic, you should be able to:

   i)   identify, read and write natural numbers as numerals and words in million, billion and trillion.

  ii)  differentiate between natural numbers and whole numbers/integers.

 iii)  identify directed numbers.

 iv)  use directed numbers (limited to integers) in real life situations.

  v)  use the hierarchy of operations to carry out the four mathematical operations on integers.

 vi)  identify even, odd, prime and composite numbers.

 vii)  find the prime factorisation of any number.

viii)  relate common factors with HCF and multiples with LCM.

 ix)  work out and use divisibility tests of some numbers.

## Introduction

Sarah was sent to a shop up the hill to buy 1kg of sugar, a packet of salt and a packet of tealeaves. She was given UGX. 5,000 note but all items cost her UGX. 6,500. How much money did Sarah owe the shopkeeper?

In your day-to-day life, you use numbers to count items, to keep information, to transact business and many others. Since you use numbers in your day-to-today situations, knowledge of integers will be helpful to you.

## Subtopic 2.1: Natural Numbers

In lower primary, you learnt counting items using numbers one, two, three ---. In mathematics these numbers are called counting or natural numbers.

When zero is included in the set of natural numbers, they become whole numbers.

For example: N = $\{1,2,3,4,5,------\}$ This is a set of natural numbers.

$W = \{0,1,2,3,4,5.------\}$ This is a set of whole numbers.

### Activity 2.1: Natural numbers

There is a box and a board. In the box, there are number cards: some have numbers in figures and others in words. While the board has two sections: one section for natural and the other for non-natural numbers.

In groups, pick a card and place it in the appropriate section of the board.

Is it possible for a number to belong to two sections?

What can you say about the two categories of the numbers picked?

Where in real-life situations do we find such numbers?

**Activity: 2.2: Writing and reading numbers**
There are two boxes. In one box, number cards are written in figures and the others in words.

In groups, a member picks one card from one of the boxes. After all the cards have been picked, one member displays his/her card; then the others check their cards, and the matching card is displayed.

**Exercise**

**Write the following in words:**

1. 3,800
2. 8,008,008
3. 606,520,060
4. 9,000,909,800
5. 4,629,842,003
6. 1,629,284,729,000

**Write the following in figures:**

7. Six hundred two million eight thousand and eight
8. Two billion eighty-nine million four thousand seven
9. One trillion two hundred fifty billion eight hundred seventy-five million three hundred sixty thousand
10. State the value of digit four in the following numbers.
    i) 7,462,300,800
    ii) 24,629,293,005
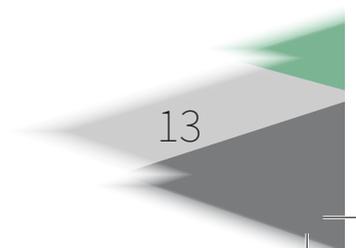
## Sub topic 2.2: Differentiating between natural numbers and whole numbers/integers

**Activity 2.3: Relating natural numbers and integers**

In groups, **read** the text below and answer the questions that follow:

Two learners—Mary and Joy—went to the school canteen to buy some snacks for their breakfast. Joy bought 3 pancakes at UGX.200 each and 1 ban at UGX. 300.

Mary checked her bag and found out that her money was stolen.  She borrowed some money from Joy.  She bought four 4 pancakes and 2 bans.

**Questions**

i)   Which of the two learners had more money?
ii)  How much money did Mary borrow from Joy?
iii) Peter said that Mary had negative UGX. 1400.  Was he correct? Give reasons for your answer.

## Sub-topic 2.3: Use Directed Numbers (Limited to Integers) in Real-life Situations

**Activity 2.4: Integers in real-life situations**

In groups, read the story below and answer the questions.

Once upon a time, there lived an old woman. She had hot and cold stones and a big pot of water. If she put one hot stone in the water, the temperature of the water would rise by 1 degree. If she took the hot stone out of the water again, the temperature would go down by 1 degree.

## Question 1

If the temperature of the water is 24 degrees and the old woman adds 5 hot stones, what is the new temperature of the water?

Now imagine that the temperature of the water is at 29 degrees. The old woman takes a spoon and takes out 3 of the hot stones from the pot.

## Question 2

What is the temperature of the water when the old woman removes 3 hot stones? Explain your answer.

The old woman also had cold stones. If she adds 1 cold stone to the water, the temperature goes down by 1 degree. The temperature of the water was 26 degrees. Then the old woman added 4 cold stones.

## Question 3

What is the temperature of the water after the old woman added 4 cold stones? Give a reason for your answer.

Just like the old woman could remove the hot stones and the temperature would decrease she could also remove the cold stones.

## Question 4

Imagine that the temperature of the water was 22 degrees and the old woman removes 3 cold stones. What happens to the temperature of the water?

What is the new temperature of the water? Explain your answer.

### Activity 2.5: Real-life situations
In groups, get a cup of hot water and a thermometer. Identify a timekeeper in your group. One of you reads the temperature on the
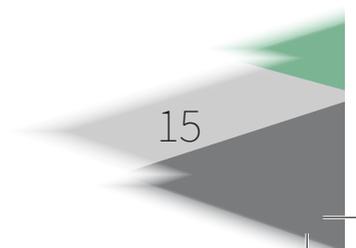
thermometer and the other members record in their notebooks. Put the thermometer back into the hot water and after 5 minutes take the reading on the thermometer. Repeat this at same interval of 5 minutes for duration of 25 minutes.

### Question 1

What is the change in temperature between the first reading and the second reading?

### Question 2

What is the change in temperature between the 2nd and 3rd reading?

### Question 3
What is the change in temperature between the 3rd and the 4th reading?

### Question 4
What is the difference in temperature between the 4th and the 5th reading?

## Sub-topic 2.4: Use the Hierarchy of Operations to Carry out the Four Mathematical Operations on Integers

**Activity 2.6: Operations on integers**

In groups, read the text below and answer the questions after.

Sarah moved 5 steps to the right from a fixed point. Then she moved 9 steps to the left.

**Question 1**

How far is Sarah from the fixed point?

**Question 2**

Peter gave his answer as 4 steps to the left of the fixed point and John as -4 (negative 4). Who is correct? Give reasons for your answer.

**Example** 1

a) 28 degrees    Remove 2 hot stones    26 degrees
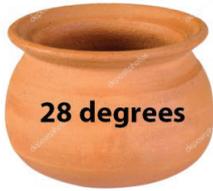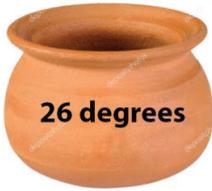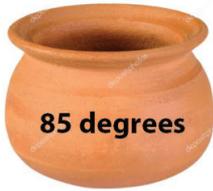
b) 85 degrees    Add 4 hot stones    89 degrees
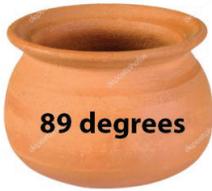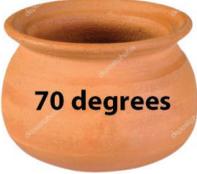
c)  70 degrees    Add 2 hot stones    68 degrees
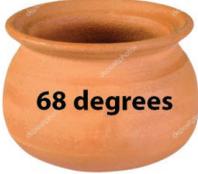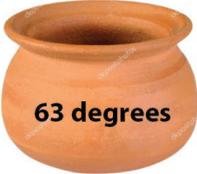
d)  63 degrees    Remove 3 hot stones    66 degrees
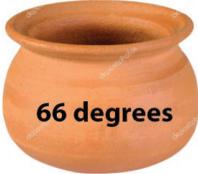
**Example 2**: A group of learners of Geography went for a tour to Kabale. They found out that the temperature at one time was $13^0$C. At around mid-night the temperature was $10^0$C. By how many degrees had the temperature dropped?

**Solution:**   $10^0$C - $13^0$C = - $3^0$C

**Example 2:** Using a number line work out:

a)      – 4 + +6
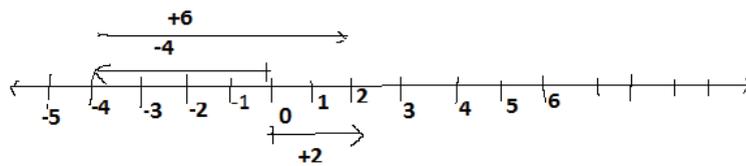


-4 + +6 = + 2

b)      +5 + - 9

$$+5 + -9 = -4$$

c)     **-6 - 4  = -6 + - 4**

**-6 - 4  = -6 + -4  = -10**



Note - x- = +, + x + = +

-x + = - , - ÷ - = +

- ÷ + = -
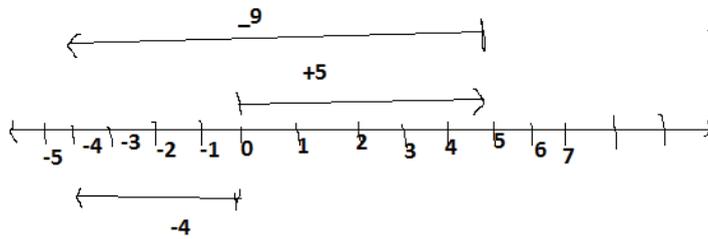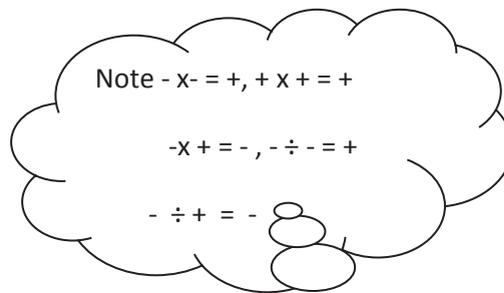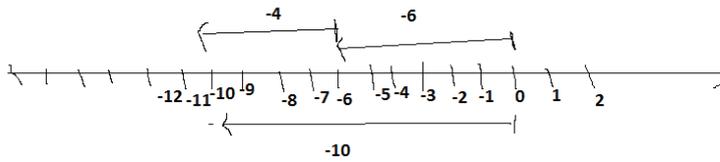
**Exercise**

1.  Work out the following in degrees:

**70**    Add 9 hot stones

**48**    Add 6 hot stones

**87**    Remove 5 hot stones

**6**    Remove 4 hot stones
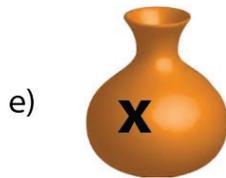
e)   **x**    Remove 5 cold stones    **25**
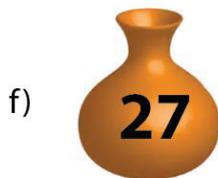
f)   **27**    Remove ............... **X**    **5**

2. Work out the following:

a)     8 + -6

b)     61 + +7

c)     49 - - 30

d)     77 - + 50

e)     -15 + + 20

f)     -3 - - 13

3. 2.Using a number line work out:

a) -2 ++ 3

b) +5 + - 6

c) – 8 - -5

4. A national park guide on one of the mountains in East Africa recorded the temperature as 15⁰C one day. At midnight the temperature was -7⁰C. By how many degrees had the temperature fallen?

5. Write down the next 3 terms in the sequence  - 9, -7, - 5, -3, - , - ; -

6. Look at the sequence of the numbers:

 -1, 3,-9, +27, -, - , -

Alex said the next three terms are +9, -36 and -729. Is Alex correct? Give reasons for your answer.

**Multiplication and Division**

Multiplication such as +4 × + 3 or -4 × + 3 are interpreted as repeated addition of positive or negative numbers.

+4 × + 3 = + 4 + +4 + +4 = +12

-4 × +3 = -4 + -4 + -4  =  -12



**Negative**

3 × 3 = 9

3 ×- 3 = -9

-3 × -3 =9

Justification of the above is as follows:

$3 \times 3 = 9$

$3 \times 2 = 6$

$3 \times 1 = 3$

$3 \times 0 = 0$

$3 \times -1 = -3$

$3 \times -2 = -6$

$3 \times -3 = -9$

Now reduce the first multiplier

$3 \times -3 = -9$

$2 \times -3 = -6$

$1 \times -3 = -3$

$0 \times -3 = 0$

$-1 \times -3 = 3$

$-2 \times -3 = 6$

$-3 \times -3 = 9$

The justification shows that any number multiplied by zero is zero; that a positive number multiplied by a positive number is a positive; a negative number multiplied by a positive number is a negative, and a negative number multiplied by a negative is a positive.

**Multiplication and division have the same rules:**

A negative number divided by a positive and a positive number divided by a negative number is a negative, Also a negative number divided by a negative is a positive.

 **Example**

+ 4  × -3 = -12

-12 ÷ - 3    = +4

- 12 ÷ +4   =  - 3

**Note:** Rules of integers

 a)  Positive number multiplied by a positive number is a positive.
 b)  Negative number multiplied by a positive number is a negative.
 c)  Negative number multiplied by a negative is a positive.
 d)  Negative number divided by a positive  is a negative
 e)  Positive number divided by a negative is a negative.
 f)  Negative number divided by a negative is a positive.

**Exercise**

Work out

1.  - 2 ×+ 4 × -3

2.   -4 ×+2 × - 3

3.  -3× -5 × +2

4.  -12 ×-5 ÷ +6

5  -15 ÷ 5 ×  -4

6.  -24 × + 4 ÷+2

7. In a certain test a correct answer scores 3marks and an incorrect answer, the child gets a penalty of two marks deducted. Joy guessed all the answers. She got 6 correct and 4 wrong. Work out her total marks.

8. Simplify   +6 - +7 ÷ +4 + + 6 × +7

9. Work out   7 of 13 – (18 ÷ 6 ₊3) ÷ (9 × 3 -25)

10. 56 - (38 - 35 ÷5 +2)

11. 69 ÷ (6 + (3 × 8 -7))

12. 4 of (5 + 2) - 2 (3 + 7) ÷ 5

## Sub-topic 2.5: Identify Even, Odd, Prime and Composite Numbers

Natural numbers can be classified into various groups of numbers. In your primary education, you learnt numbers such as even, odd, prime and composite.

### Activity 2.6: Identifying even, odd, prime and composite numbers

Each group is given a box containing number cards. In your groups pick the card and read the number. Identify which group of numbers it belongs to by filling the table below.

| No | Odd | Prime | Even | Composite |
|----|-----|-------|------|-----------|
|    |     |       |      |           |
|    |     |       |      |           |
|    |     |       |      |           |
|    |     |       |      |           |
|    |     |       |      |           |
|    |     |       |      |           |
|    |     |       |      |           |

**Question 1**

Are there numbers that belong to more than one group?

**Question 2**

How do you identify that a number is:

a) odd
b) even
c) prime
d) composite

## Sub-topic 2.6: Find the Prime Factors of any Number

In your primary education you studied multiples and factors of numbers. When two numbers are multiplied together, the product is called multiple. The two numbers multiplied together are called factors of the multiple.

**Note:** A multiple has two or more factors.

**For example:** The factors of 12 are $(1 \times 12), (2 \times 6)$, and $(3 \times 4)$; hence, the factors of 12 are $\{1,2,3,4,6,12\} = F_{12} = \{1,2,3,4,6,12\}$

The multiples of 3 are $\{3,6,9,12,15,18,21---\} = M_3 = \{3,6,9,12,15,18,21--\}$

**Exercise**

Find the factors of the following:

1. 42
2. 56
3. 36
4. 108

Find the multiples of the following:

5. 7
6. 12
7. 9
8. 5

**Note:** A factor of a number which is a prime number is called its prime factor. For example the factors of 36 are $\{1,2,3,4,6,9,12,36\}$

**Qn.** What are the prime factors of 36?

**Qn.** Write 36 as a product of its prime factors.

**Answer:**

| Prime Factor | Number |
|---|---|
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
|   | 1 |

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

This approach of determining prime factors is called prime factorisation.

This can be written in power notation.

**Exercise**

Find the prime factors of the following numbers. Give your answer in power form.

1. 108
2. 288
3. 180
4. 1232
5. 993
6. 2145

## Sub-topic 2.7: Relate Common Factors with HCF and Multiples with LCM

A number can have one or more common factors; for example, 2 and 4 are common factors of 8 and12. However, the highest common factor is 4. Therefore, the highest common factor (HCF) of 8 and 12 is 4.

**Activity 2.7:  Highest common factor**

In groups, find the HCF of the following:

i)    54, 48
ii)   42 ,63 ,105
iii)  132, 156,204,228

## Sub-topic 2.8: Work Out and Use Divisibility Tests of Some Numbers

**Activity 2.8: Identifying divisibility tests for some numbers**

1.  In your groups, pick a number card and determine which numbers on the chart divides it. Write a number under its divisor.
2.  What can you say about the numbers under each divisor? Give reasons for your answers.
3.  The relationship between the dividend and the divisor leads to divisibility t**ests.**

**Exercise**

Given the following numbers:

12, 132, 1212, 3243, 1112, 81, 18, 27, 279, 2580, 5750

Find out which of them are divisible by:

a) 3  b) 4  c) 6  d) 9  e) 10

**Exercise**

Find the HCF the following:

1. 2 × 2× 3 × 3× 3 ×3  × 5 × 5  ×5 × 11
2. 2 × 2 ×2  ×2 ×2 ×3  ×3  ×5 ×7  ×13
3. 2. $2^3$ ×$3^2$×$5^2$,   $2^5$ × $3^5$ ×$5^2$
4. 36, 60, 84

4. A rectangular field measures 616m by 456m. Fencing posts are placed along its sides at equal distances. What will be the distance between the posts if they are placed as far apart as possible? How many posts are required?

## Sub-topic 2.9: Least Common Multiple (LCM)

In the previous section of multiples and factors you learnt about multiples of numbers. For example, the multiples of 5 are 5, 10, 15,20,25,30, 35, 40, 45, 50, 55, 60, 65, 70, 75 ------. The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77 ------. From the above example, 35 and 70 are common multiples of 5 and at the same time of 7. However, 35 is smaller than 70, therefore, 35 is the least common multiple of 5 and 7.

There is another approach of getting LCM of numbers without listing the multiples of the numbers.

**Example**

Find the LCM of 8 and 12

| 2 | 8 | 12 |
|---|---|----|
| 2 | 4 | 6 |
| 2 | 2 | 3 |
| 3 | 1 | 3 |
|   | 1 | 1 |

2 × 2 × 2 × 3 = 24

The LCM of 8 and 12 is 24.

**Activity 2.9**: In your groups, find the LCM of the following:

a)   28, 42 ,98
b)   35,48 ,56, 70

**Exercise**

Find the LCM of the following numbers:

1.  14, 21
2.  18, 24, 96
3.  49, 84, 63
4.  60, 72, 84, 112
5.  Determine the smallest sum of money out of which a number of men, women and children may receive UGX. 75, Ush.90 and Ush.120 each.

# Topic 3:
# FRACTIONS, PERCENTAGES AND DECIMALS

**Key Words**

recurring, numerator, denominator, terminating, non-terminating, reciprocal, whole

By the end of this topic, you should be able to:

i) describe different types of fractions.
ii) convert improper fractions to mixed numbers and vice versa.
iii) work out problems from real-life situations.
iv) add, subtract, divide and multiply decimals.
v) convert fractions to decimals and vice versa.
vi) identify and classify decimals as terminating, non-terminating and recurring decimals.

vii)   convert recurring decimals into fractions.
viii)   convert fractions and decimals into percentages and vice versa.
ix)   calculate a percentage of a given quantity.
x)   work out real-life problems involving percentages.

## Introduction

In Chapter Two you studied place values in number bases. In this topic, you will use knowledge of place values to manipulate fractions, decimals and percentages. You will convert fractions to decimals, decimals to percentages and vice versa.

## Sub-topic 3.1: Describe Different Types of Fractions

**Activity 3.1**

Create a park of different cards and label them with different types of fractions, decimals and percentages.

From the park of the cards, you pick a card and place it in the most appropriate play area.

Observe the fractions in each play area by looking at the denominators and numerators.

In your groups explore and explain the common of the classification made in the different play areas.

**Exercise**

1.   Sarah shades 3/7 of a shape. What fraction of the shape is left unshaded?
2.   A cake is divided into 12 equal parts. John eats 3/12 of the cake and Kate eats another 1/12. What fraction of the cake is left?
3.   A car park contains 20 spaces. There are 17 cars parked in the car park.

a) What fraction of the car park is full?
b) What fraction of the car park is empty?
4.   Ali eats 3/10 of the sweets in a packet.
     Tariq eats another 4/10 of the sweets.

   a) What fraction of the sweets has been eaten?
   b) What fraction of the sweets is left?
5.
   a) Draw a square with its four lines of symmetry.
   b) Shade 3/8 of the shape.
   c) Shade another 2/8 of the shape.
   d) What is the total fraction now shaded?
   e) How much is left unshaded?

## Sub-topic 3.2:  Convert Improper Fractions to Mixed Numbers and Vice Versa

**Mixed Numbers and improper Fractions**

So far you have worked with fractions of the form a/b where a < b, e.g.
        ¾, 2/7, 5/6 …

You also need to work with what are sometimes called *improper* fractions, e.g. 5/4, 7/2, which are of the form a/b when **a** and **b** are whole numbers and **a > b.**

**Example**

Convert 13/4 into an improper fraction.

**Solution**

13 ÷ 4 = 3 remainder 1

This is written as 3 ¼.

**Exercise**

1.  Draw diagrams to show these improper fractions:
(a) 7/2 (b) 8/3 (c) 18/5

   Write each improper fraction as a mixed number.

2.  Convert these mixed numbers to improper fractions.
  (a) **1** 3/5 (b**) 7** 1/3 (c) **3** 4/5   (d) **6** 1/9

3.  Write these fractions in order of increasing size.
     6 ½ ,  18/5 ,   3 ¼ , 5 1/3 , 17/3

4.   In an office there are 2 ½ packets of paper. There are 500 sheets of
     paper in each full packet. How many sheets of paper are there in the
     office?
5.   A young child is 44 months old. Find the age of the baby in years as
     a mixed number in the simplest form.

## Sub-topic 3.3: Operations on Fractions

In the previous sub-topic, you studied how to find equivalent fractions.
In this sub-topic you are going to use the knowledge of equivalent
fractions to add and subtract fractions.

### 3.3.1: Work out problems from real-life situations

Now we start to use fractions in a practical way.

**Example**

(a) Find 1/5 of UGX. 10000

 (b) Find 4/5 of UGX. 100,000

You can, do this practically, but it is much easier to work out.

(a) 1/5 of 10000 = 1/5  x 10000 = 2000

 (b) 4/5 of 100000 = 4/5 x  100000 =  400000/5 = 80,000

**Exercise**

1. Find:

 (a) ½ of 12 (b) 1/8 of 40 (c) ¼ of 32

2. Find:

 (a) 2/9 of 18 (b) 7/9 of 45 (c) 7/8 of 56

3. In a test, there are 30 marks. Nasim gets 3/5 of the marks. How many marks does she get?

4. In a certain school there are 550 pupils. If 3/50 of the pupils are left-handed, how many left-handed pupils are there in the school?

**Activity 3.3: Addition of Fractions**

In your groups, use a sheet of paper to work out $\dfrac{1}{5}+\dfrac{3}{5}$. Fold the paper into five equal parts   shade off one part of the five equal parts

Shade the three parts of the five equal parts

How many parts have been shaded?

Represent the shaded parts in a fraction form. Show the working.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$
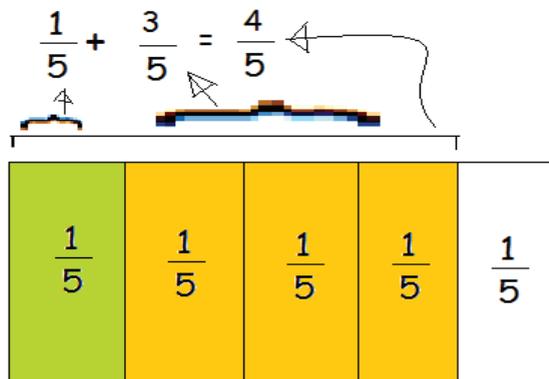
## Activity 3.4: Addition of Fractions with the Same Denominators

Slice a hexagon into 6 pieces:

Each piece ▲ is $\frac{1}{6}$ of the hexagon. Right?

And ▲▲▲▲ is $\frac{4}{6}$ of the hexagon.

So, what if we wanted to add

$$\frac{1}{6} + \frac{4}{6} \;?$$

Hmm... that would be

▲ + ▲▲▲▲

Count them up

$$= ▲\ ▲\ ▲\ ▲\ ▲ = \frac{5}{6}$$

1  2  3  4  5

So $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

In your groups, use the same method to work out the following:

a) $\dfrac{3}{7} + \dfrac{2}{7}$

b) $\dfrac{5}{9} + \dfrac{4}{9}$

### 3.3.2: Adding Fractions with the different Denominators

In the previous topic you studied about lowest common multiple. In this section, you will apply the knowledge of LCM.

$$\dfrac{1}{2} + \dfrac{1}{3}$$

Change the $\dfrac{1}{2}$ using the knowledge of equivalent fractions

$$\dfrac{1 \times 3}{2 \times 3} = \dfrac{3}{6}$$

Change the $\dfrac{1}{3}$ using the knowledge of equivalent fractions

$$\text{So } \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3}{6} + \dfrac{2}{6}$$

$$\dfrac{3+2}{6} = \dfrac{5}{6}$$

The main rule of this game is that we cannot add the fractions until the denominators are the same!

We need to find something called the least common denominator (LCD)..which is the LCM of our denominators, 2 and 3.

The LCM of 2 and 3 is 6. So, our LCD 6.

We need to make this our new denominator

### 3.3.3: Subtraction of Fractions with Same Denominators

Let's try $\dfrac{7}{8} - \dfrac{3}{8}$

Look at a Chapatti in a conical shape cut into 8 pieces. Each piece

 is $\dfrac{1}{8}$ of the Chapatti.

$\dfrac{7}{8}$
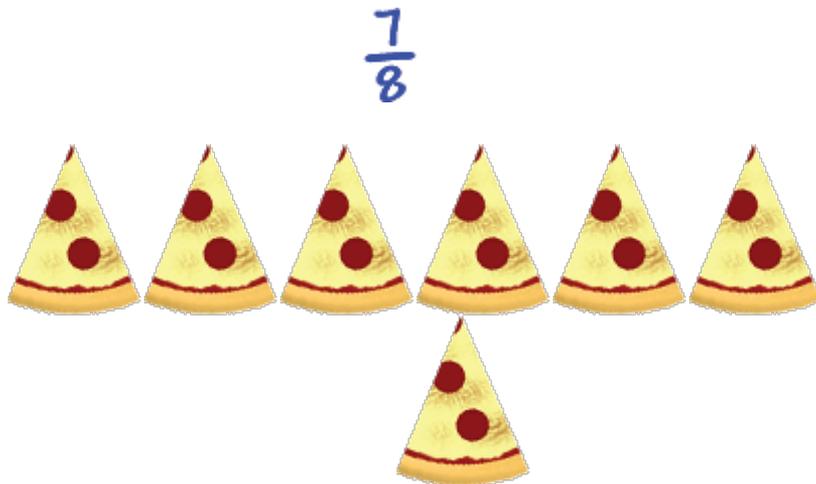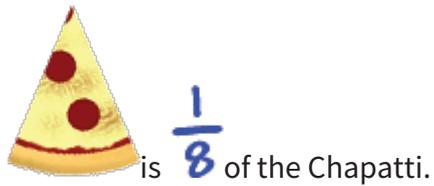


Take $\dfrac{3}{8}$ away (that's 3 pieces):
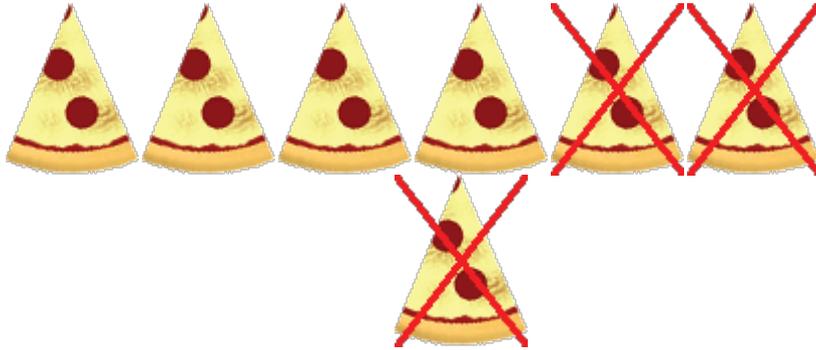
We're left with 4 pieces - that's.

So $\dfrac{7}{8} - \dfrac{3}{8} = \dfrac{4}{8}$

But, look what we really did!

We just subtracted the numerators!

$\dfrac{7}{8} - \dfrac{3}{8} = \dfrac{7-3}{8} = \dfrac{4}{8}$ which is $\dfrac{1}{2}$

### 3.3.4: Subtraction of Fractions with Different Denominators

Subtraction works the same way.

$$\dfrac{6}{11} - \dfrac{3}{22}$$

The LCM of 11 and 22 is 22… So, the LCD is 22.

We just need to change the $\dfrac{6}{11}$.

$$\frac{6 \times 2}{11 \times 2} = \frac{12}{22}$$

$$\text{So } \frac{6}{11} - \frac{3}{22} = \frac{12}{22} - \frac{3}{22} = \frac{12-3}{22} = \frac{9}{22}$$

Done!

### 3.3.5: Addition of Mixed Fractions

What if we need to add

$$3 + \frac{7}{8}?$$

Hey, remember, that's just $3\frac{7}{8}$.

Done!

That was easy, but, what about mixed numbers?

How about this?

$$3\frac{2}{5} + 1\frac{4}{7}$$

All we have to do is change these to improper fractions... Then we can add them!

$$3\frac{2}{5} + 1\frac{4}{7} = \frac{17}{5} + \frac{11}{7}$$

change to improper fractions

$$= \frac{17 \times 7}{5 \times 7} + \frac{11 \times 5}{7 \times 5} = \frac{119}{35} + \frac{55}{35}$$

change to the LCD of 35

$$= \frac{119 + 55}{35} = \frac{174}{35}$$

### 3.3.6: Subtraction of Mixed Fractions

$$5 - \frac{3}{8}?$$

Well, we can't just stick it together like we would if it was addition.

We need to get a common denominator... But, the 5 does not even have a denominator!

That's OK... Just think of a Chapatti cut into 8 pieces...
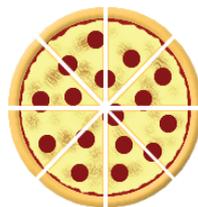
How many pieces would there be in 5 chapattis? Yep!

$$5 \times 8 = 40 \text{ pieces}$$

So $5 = \frac{40}{8}$

Check it: $\frac{40}{8}$ is the same as $40 \div 8$ which is $= 5$. Yep!

Back to the problem:

$$5 - \frac{3}{8} = \frac{40}{8} - \frac{3}{8} = \frac{40-3}{8} = \frac{37}{8} = 4\frac{5}{8}$$

What's $\frac{1}{3} \times 6$?

Well, that's $\frac{1}{3}$ of 6. Think about it:

You have 6 chapattis.



and you get to eat $\frac{1}{3}$ of them.

This is like splitting up the chapatti between 3 people:

| You get 2 chapattis | Your friend gets 2 chapattis | And your dog gets 2 chapattis |
| You get 2 chapattis | Your friend gets 2 chapattis | And your dog gets 2 chapattis |

So $\dfrac{1}{3}$ of 6 is 2.

But, how do we do this with just math? EASY!

We know how to multiply two fractions... Right?

So, just make both things be fractions. Check it out:

$$\dfrac{1}{3} \times 6$$

$\dfrac{1}{3}$ is already a fraction...

But, what about the 6?

Guess what? We can write 6 as $\dfrac{6}{1}$ .

Let's try

$$\dfrac{7}{8} - \dfrac{3}{8}$$

Look at a chapatti cut into 8 pieces. Each piece  is $\dfrac{1}{8}$ of the
Chapatti.

$$\dfrac{7}{8}$$
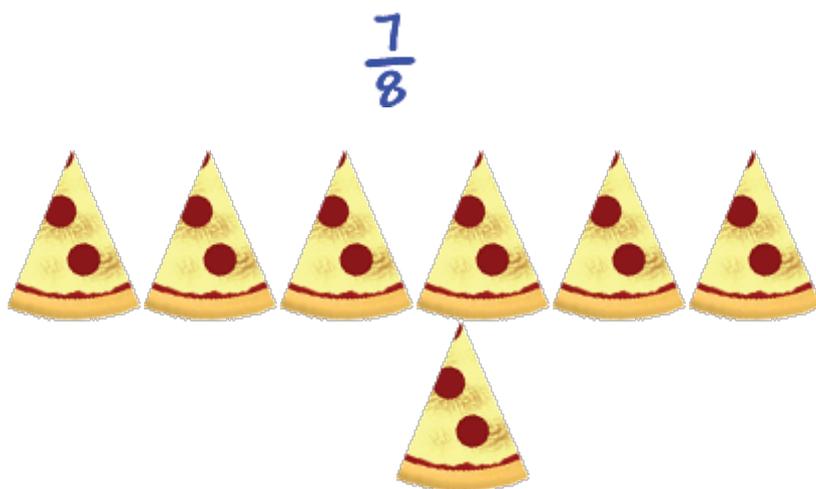
Take $\dfrac{3}{8}$ away (that's 3 pieces):
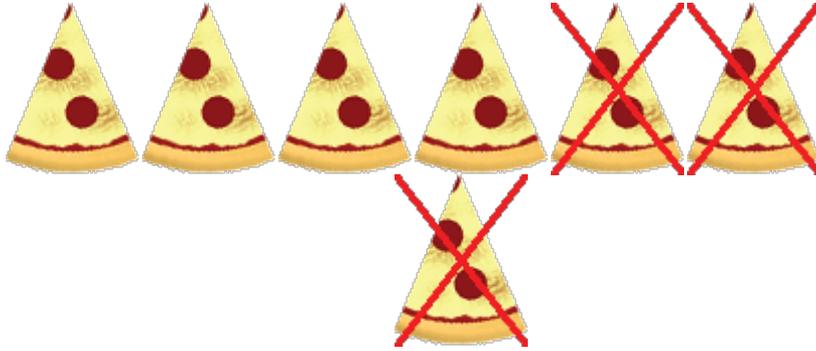


We're left with 4 pieces,  that's.

So $\dfrac{7}{8} - \dfrac{3}{8} = \dfrac{4}{8}$

But, look at what we really did!

We just subtracted the numerators!

$\dfrac{7}{8} - \dfrac{3}{8} = \dfrac{7-3}{8} = \dfrac{4}{8}$ which is $\dfrac{1}{2}$

### 3.3.7: Subtraction of Fractions with Different Denominators

Subtraction works the same way.

$\dfrac{6}{11} - \dfrac{3}{22}$

The LCM of 11 and 22 is 22... So the LCD is 22.

We just need to change the $\frac{6}{11}$.

$$\frac{6 \times 2}{11 \times 2} = \frac{12}{22}$$

So $\frac{6}{11} - \frac{3}{22} = \frac{12}{22} - \frac{3}{22} = \frac{12-3}{22} = \frac{9}{22}$

Done!

### 3.3.8: Multiplication of Fractions

What's $\frac{1}{3} \times 6$ ?

Well, that's $\frac{1}{3}$ of 6. Think about it...

You have 6 chapattis.



and you eat $\frac{1}{3}$ of them.

This is like splitting up the pizza between 3 people:

You get 2 chapattis | Your friend gets 2 chapattis | And your dog gets 2 chapattis

So $\frac{1}{3}$ of 6 is 2.

But, how do we do this with just math? EASY!

We know how to multiply two fractions... Right?

So, just make both things be fractions. Check it out:

$$\frac{1}{3} \times 6$$

$\frac{1}{3}$ is already a fraction...

But, what about the 6?

Guess what? We can write 6 as $\frac{6}{1}$ .

Think about it:

$\frac{6}{1}$ is the same as $6 \div 1$ ... which is 6!

(You can do this with any number!)

Back to the problem:

$$\frac{1}{3} \times 6 = \frac{1}{3} \times \frac{6}{1} = \frac{1 \times 6}{3 \times 1} = \frac{6}{3} = 2$$

Just what we figured!

### 3.3.9: Multiplying Mixed Fractions

What about this?

$$2\frac{3}{5} \times 3\frac{1}{7}$$

Yikes! I am sure I don't want to try to think about pizza for this one!

Let's stick to the math:

Again, let's change these into improper fractions and go for it!

$$2\frac{3}{5} \times 3\frac{1}{7} = \frac{13}{5} \times \frac{22}{7} = \frac{286}{35} = 8\frac{6}{35}$$

This is super easy!

Let's just do one:

$$\frac{1}{3} \times \frac{9}{10}$$

We just multiply straight across...

$$\frac{1}{3} \times \frac{9}{10} = \frac{1 \times 9}{3 \times 10} = \frac{9}{30} = \frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

*Then just reduce it*

Now, think about it...

$$\frac{1}{3} \times \frac{9}{10}\frac{1}{3} \text{ of } \frac{9}{10}$$

Cut a pizza into 10 pieces like
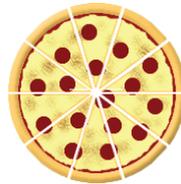
and look at 9 of the pieces:
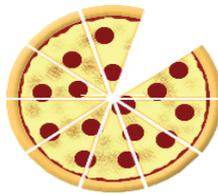
We want $\frac{1}{3}$ of these $\left(\frac{1}{3} \text{ of } \frac{9}{10}\right)$

That would be 3 pieces. Right?

That's $\frac{3}{10}$!

Doing math is cooooool!

Now that we understand what to do, we can just go for it.

### 3.3.10: Division of Mixed Fractions Flip And Multiply

**Check it out:**

$$\frac{1}{3} \div \frac{4}{5}$$

flip the second fraction...
and multiply!

$$\frac{1}{3} \times \frac{5}{4}$$

**That's it -- then GO FOR IT!**

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$$

**Done!**

**Look at another one:**

$$\frac{6}{11} \div \frac{1}{2}$$

$$\frac{6}{11} \div \frac{1}{2} = \frac{6}{11} \times \frac{2}{1} = \frac{6 \times 2}{11 \times 1} = \frac{12}{11} = 1\frac{1}{11}$$

Use the same trick you do when multiplying by changing everything to fractions and then go for it!

**Check it out:**

$$\frac{9}{17} \div 3$$

$$\frac{9}{17} \div 3 = \frac{9}{17} \div \frac{3}{1} = \frac{9}{17} \times \frac{1}{3} = \frac{9 \times 1}{17 \times 3}$$

$$= \frac{9}{51} = \frac{9 \div 3}{51 \div 3} = \frac{3}{17}$$

**How about another one?**

$$1\frac{2}{7} \div 5$$

$$1\frac{2}{7} \div 5 = \frac{9}{7} \div \frac{5}{1} = \frac{9}{7} \times \frac{1}{5}$$

$$= \frac{9 \times 1}{7 \times 5} = \frac{9}{35}$$

Use the same trick you do when multiplying by changing everything into fractions and then go for it!

## Sub-topic 3.4: Add, Subtract, Divide and Multiply Decimals

**Activity 3.5: Fractions and decimals**

In groups, copy and complete the table, by explaining how you have obtained the answer. The first three have been done for you

The column headings will help you

| Tens | Ones | Tenth $(\frac{1}{10})$ | Hundredth $(\frac{1}{100})$ | Thousandth $(\frac{1}{1000})$ | Fraction | Percentage |
|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |
|  |  | **5** |  |  | $\frac{1}{2}$ | **50** |
| 1 | 2 | 4 |  |  | $12\frac{2}{5}$ | 1240 |
|  |  | 2 | 5 |  | $\frac{1}{4}$ | 25 |
|  |  | 1 | 5 | 2 |  |  |
|  |  | 5 |  |  |  |  |
|  |  |  |  |  |  | 80 |
|  |  |  |  |  | $\frac{17}{20}$ |  |
|  |  |  |  |  |  | 64 |
|  |  | 0 | 0 | 4 |  |  |
|  |  |  |  |  | $\frac{3}{10}$ |  |
| 4 | 0 | 3 |  |  |  |  |

# Sub-topic 3.5: Convert Fractions to Decimals and Vice Versa

A fraction like ¾ means *three quarters*

*or three parts out of four*

*or three divided by four*

3 divided by 4 equals 0.75

So, the fraction ¾ is equal to 0.75 in decimal.

**Activity 3.6:** In pairs, convert the following fractions into decimals

a)  $\frac{2}{5}$
b)  (b) $\frac{1}{20}$  (c) $\frac{5}{8}$  (d) $\frac{2}{9}$  (e) $\frac{1}{11}$
c)  What do you notice about (d) and (e)?

# Sub-topic 3.6:  Identify and Classify Decimals as Terminating, Non-terminating and Recurring Decimals

Fractions like $\frac{3}{5}$, $\frac{1}{2}$, $\frac{3}{8}$ can be converted into decimals and they end or terminate: $\frac{3}{5} = 0.6$, ½ = 0.5 and $\frac{3}{8} = 0.375$.

Fractions like $\frac{2}{3}$, $\frac{2}{15}$, 1/11 do not end or terminate when converted into decimals, $\frac{2}{3} = 0.66666\ldots$, $\frac{2}{15} = 0.133333\ldots$ and
$\frac{1}{11} = 0.090909\ldots$
These decimals are referred to as **recurring decimals**

**Exercise**

1.  Write the following fractions as decimals:

(a) $^3/_8$ (b) $^7/_{10}$ (c) $^{17}/_{50}$ (d) $^{13}/_{25}$

2.  Write the following as fractions in their lowest terms:

(a) 0.25 (b) 0.08 (c) 0.35 (d) 0.125

3.  Write the following fractions as recurring decimals:

(a)a $^2/_{11}$ (b) $^1/_3$ (c) $^1/_6$ (d) $^7/_9$

## Sub-topic 3.7: Convert Recurring Decimals into Fractions

Recurring decimals can be converted into fractions.

**Example:** Convert this recurring decimal into a fraction: 0.333…
Note that the decimal repeats itself after one decimal place.
Let $r = 0.333…$ (1)
Multiply both sides of the equation by 10 i.e. $10 \times r = 10 \times 0.333$
$10r = 3.333$ (2)

Subtract equation (1) from equation (2):
That is, $10r = 3.333$
      $- (r = 0.333)$

      $9r = 3$

      $r = ^3/_9 = ^1/_3.$

**Exercise**

1. Convert the following recurring decimals into fractions
    a) 0.77---, b) 0.133--- , c)1.25656 ---, d) 0.2727 ---, e) 0.01313
2. Convert the following numbers into recurring decimals

a) $\frac{1}{3}$ , b) $\frac{1}{9}$ , c) $\frac{2}{6}$

## Sub-topic 3.8: Convert Fractions and Decimals into Percentages and Vice Versa

**Activity 3.7: Fraction percentage game**

| I am $\frac{7}{20}$ | Who is 67%? | I am $\frac{67}{100}$ | Who is 13%? | I am $\frac{13}{100}$ | Who is 22%? |
|---|---|---|---|---|---|
| I am 11 | Who is 5%? | I am 1 | Who is72%? | I am 18 | Who is 87%? |
| I am 87 | Who is 4%? | I am 1 | Who is 34%? | I am 8 | Who is 42%? |
| I am 21 | Who is 52%? | I am 13 | Who is 45%? | I am 9 | Who is 58%? |
| I am 29 | Who is 64%? | I am 16 | Who is32%? | I am 17 | Who is2%? |
| I am 1 | Who is 92%? | I am 23 | Who is 98%? | I am 49 | Who is 44%? |
| I am 11 | Who is 82%? | I am 41 | Who is 65%? | I am 13 | Who is 14%? |

From the fraction percentage game, identify the equivalent percentage for each fraction.

In your groups, use percentage to identify the smallest and largest fractions from the fraction percentage game.

## Sub –topic 3.9 Calculate a Percentage of a Given Quantity

The percentage of a quantity can always be calculated in terms of percentage increase or percentage decrease.

**Example 1**: Find the 10% of 50,000

**Solution:** $\frac{10}{100}$ x 50,000 = 5,000.

**Example 2:** Opio had 60 goats. Now he has 63 goats. What is the percentage increase?

**Solution:** The increase in the number of goats is 63 – 60 = 3.
Percentage increase is $\frac{3}{60}$ x 100 = 5%.

**Activity 3.8:** The table below shows students' marks in two mathematics tests. For each one, calculate the percentage difference. Say if it is an increase or a decrease.

|     | Student   | First Test | Second Test |
| --- | --------- | ---------- | ----------- |
| (a) | Marion    | 50         | 45          |
| (b) | James     | 40         | 52          |
| (c) | Christina | 20         | 35          |
| (d) | Sarah     | 60         | 50          |

## Sub-topic 3.10: Works out Real-life Problems Involving Percentages

**Exercise**

1. In a closing-down sale, a shop offers 50% cut of the original prices. What fraction is taken off the prices?
2. In a survey one in five people said they preferred a particular brand of Coca Cola. What is this figure as a percentage?
3. Peter pays tax at the rate of 25% of his income. What fraction of Peter's income is this?
4. When Carol was buying a house, she had to make a deposit of $\frac{1}{10}$ of the value of the house. What percentage was this?
5. I bought a coat in the January sales with $\frac{1}{3}$ price cut of the selling price. What percentage was taken off the price of the coat?
6. Adikinyi bought some fabric that was 1.75 metres long. How could this be written as a fraction?
7. A car park contains 20 spaces. There are 17 cars parked in the car park.
   a. What fraction of the car park is full?
   b. What fraction of the car park is empty?

## Sub-topic 3.11: Identifying and classifying decimal as terminating, non-terminating and recurring decimals

**Activity 3.6: Decimal as terminating, non-terminating and recurring decimals**

In groups list some terminating, none terminating and recurring decimals. In pairs prove them. Compare your answers with the members of the group.

**Situation of Integration**

A primary school has two sections, that is, lower primary (P1-P4) and upper primary (P5-P7). The head teacher of the primary school needs to draw a timetable for both sections. The sections should start and end their morning lessons at the same time before break time, start and end their break time at the same time. The after break lessons should start at the same time. The lunchtime for both sections should start at the same time.

**Support:** The time to start lessons for the two sections is 8.00am. The duration of the lesson for the lower section is 30 minutes and that of the upper section is 40minutes.

**Resources:** Knowledge of fractions, percentages, natural numbers, factors, multiples, lowest common multiples, and the subjects taught in all classes and of time.

**Task:** Help the head teacher by drawing the timetable up to lunchtime for the two sections. How many lessons does each section have up to lunchtime?

Express the total number of lessons for the lower primary as a fraction of the total number of lessons for the whole School. (Consider lessons up to lunch time.)

# Topic 4:

# RECTANGULAR CARTESIAN COORDINATES IN 2 DIMENSIONS



**Key words:** coordinates, axes, plot, scale

By the end of this topic, you should be able to:
   i)    identify the y-axis and x-axis.
   ii)   draw and label the Cartesian plane.
   iii)  read and plot points on the Cartesian plane.
   iv)   choose and use appropriate scale for a given data set.
   v)    identify places on a map using coordinates (apply coordinates in real-life situations).

## Introduction

This topic is key in building the concept of location. The knowledge achieved from this topic can be used in locating places. In order to locate places you need a starting point (reference point).

## Sub-topic 4.1: Identify the X-axis and Y-axis

**Activity 4.1: Plotting Points**

Now, plot the following points on a graph, (6,4), (5,9), (11,3), (5,6) and (3, 4).

The x number comes first then the y number: (X, Y). These numbers are called coordinates.

**Exercise**

1. Use a graph paper to:

   a) Join the points with coordinates (0, 3), (5,6), and (5,0) to draw a triangle.
   b) On the same diagram join the points with coordinates (2, 0), (2, 6) and (7, 3) to draw a second triangle.
   c) Describe the shape you have now drawn.

2. On the same graph paper join these points in order.

   a) (4, 6), (5, 7), (6, 6), (4, 6).
   b) (5, 8), (4, 8), (4, 7), (5, 8), (6, 8), (6,7), (5, 8).
   c) (4, 5), (5, 4), (6, 5), (5, 3), (4, 5).
   d) (5, 2), (3, 4), (3, 5), (2, 5), (2, 8), (3, 8), (3, 9), (7, 9), (7, 8), (8, 8), (8, 5), (7, 5), (7, 4), (5, 2).

We can also use negative numbers in coordinates. We can bring in coordinate axes with positive and negative numbers.

**Exercise**

1. (a) Draw a set of axes and mark the points with coordinates (4, 0), (-4, 0), (0, 4),

(0, -4), (1, 2), (1, -2), (3, 3), (3, -3), (2, 1), (2, -1), (-1, 2), (-1, -2), (-3, 3), (-3, -3),(-2, 1), (-2, -1)

   (b)  Join the points to form an 8 pointed star.

2. (a) On a graph paper, draw the rectangles with corners at the following points with coordinates:

   a)   (-6, 6), (-5, 6), (-5, 4), (-6, 4)
   b)   (-2, 1), (-3, 1), (-3, 3), (-2, 3)
   c)   (3, 1), (3,3), (4, 3), (4, 1).
   d)   (10, 1), (10, 3), (9, 3), (9, 1)
   e)   (12, 4), (13, 4), (13, 6), (12, 6)

(b)  Join the points with coordinates:

   (1, -5), (1, -1), (2, 0), (5, 0), (6, -1), (6, -5)

## Sub-topic 4.2: Plotting Polygons (shapes)

Here we look at polygons plotted on coordinate axes, but first, recall the names of polygons.

**Names of polygons**

| Number of sides | Name |
|---|---|
| 3 | Triangle |
| 4 | Quadrilateral |
| 5 | Pentagon |
| 6 | Hexagon |
| 7 | Heptagon |
| 8 | Octagon |
| 9 | Nonagon |
| 10 | Decagon |

**Note:**

In a regular polygon:

(a) all the sides are the same.

(b) all the angles are of the same size.

**Activity 4.2: The line AB is one side of a square**
**What are the possible coordinates of the corners of the square?**
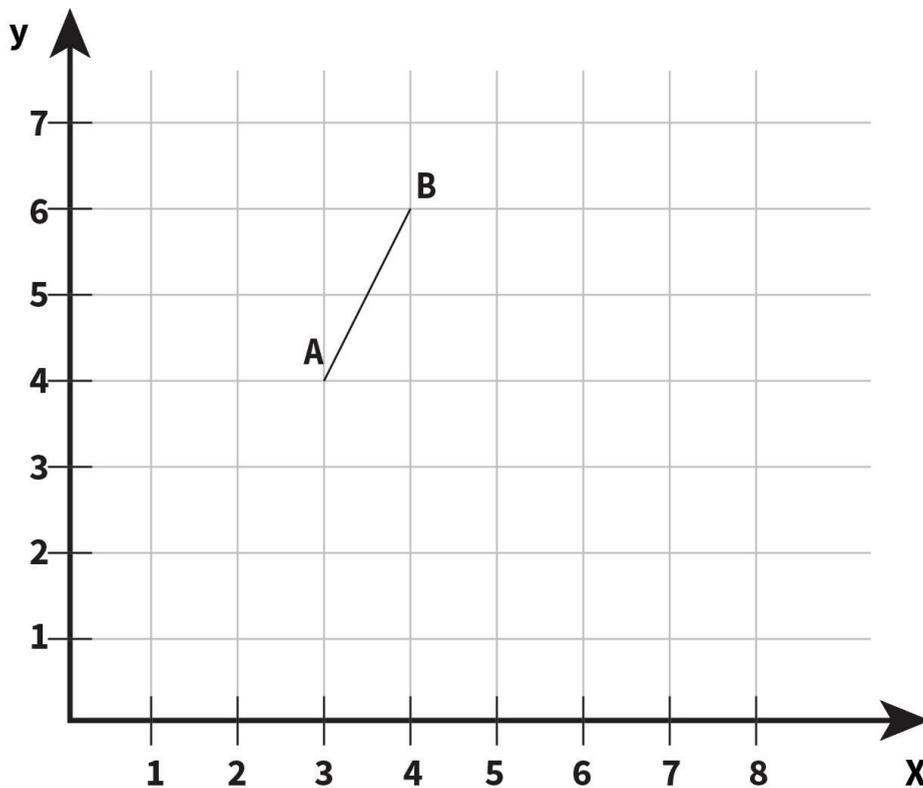


**Exercise**

1. In each case the coordinates of 3 corners of a square are given.
   Find the coordinates of the other corner.
   (a) (2, -2), (2, 3) and (-3, 3)

(b) (2, 3), (3, 4) and (1, 4)

(c ) (2, 2), (4, 4) and (4, 0)

(d) (-6, 2), (-5, -5) and (1, 3)

(e) (-5, -2), (-2, -1), and (-1, -4)

2.  The coordinates of 3 corners of a rectangle are given below. Find the coordinates of the other corner of each rectangle.
(a) (-4, 2), (-4, 1) and (6, 1)

(b) (0, 2), (-2, 0) and (4, -6)

(c ) (-4, 5), (-2, -1) and (1, 0)

(d) (-5, 1), (-2, 5) and (6, -1)

3.  (a) The coordinates of 2 corners of a square are (-4, 4) and (1, -1). Explain why it is possible to draw three different squares using these two points.
(b) Draw the three different squares.

(c ) If the coordinates of the corners had been (-5, 1) and (1, 3) would it still be possible to draw 3 squares? Draw the possible squares.

4.  Half of a heptagon with one line of symmetry can be drawn by joining the points with coordinates: (2, 4), (-2, 1), (-2, -1), (0, -3) and (2, -3). Join the coordinates. You have drawn one half of the heptagon. Complete the heptagon. Write down the coordinates.

## Sub-topic 4.3:  Use of Appropriate Scale for Given Data

**Activity4.3: Plot the following points on the axes: (5, 50), (10,100), (15,150), (20,200), (35, 350)**

Do you realise that on the horizontal axis there are 5 units for each space?

On the vertical axis there are 50 units for each space. So, what is the scale for the axes?

**Exercise**

1. For each part, draw a pair of axes with suitable scales and plot the points:
   (a) (1, 15); (4, 35); (8, 45)
   (b) (15, 100); (35, 500); (40, 700)
2.  Plot the points (2, 60); (4, 50); (0, 70); (7, 60)


**Situation of Integration**

 A Senior One learner has reported in her class and has settled at her desk.

**Support**: The classroom is arranged in rows and columns. It is big a big class with each learner having his/ her own desk.

**Resources**: Knowledge of horizontal and vertical lines i.e. rows and columns, coordinates

**Knowledge**: counting numbers


**Task**: The mathematics teacher has asked her to explain how she can access her seat, starting from the entrance of the class.  Discuss whether there are other ways of reaching her seat.

# Topic 5:

# GEOMETRIC CONSTRUCTION SKILLS



**Key Words:** perpendicular lines, parallel lines, circumcircle, arcs

By the end of this topic, you should be able to:

i) draw perpendicular and parallel lines.

ii) construct perpendiculars, angle bisectors, mediators and parallel lines.

iii) use compass and a ruler to construct special angles ($60^0$, $45^0$).

iv) describe a locus.

v) relate parallel lines, perpendicular bisector, angle bisector, straight line and a circle as loci.

vi) draw polygons.

vii) measure lines and angles.

viii) construct geometrical figures such as triangle, square, rectangle, rhombus, parallelogram.

## Introduction

In this topic you will learn how to construct lines, angles and geometric figures. Skills developed from this topic can be applied in day-to-day life.

## Sub-topic 5.1: Draw perpendicular and parallel lines

**Activity 5.1:  Drawing perpendicular and parallel lines**

(a)  In your groups, list objects in real-life situations that can be used to draw lines.

(b)  Use the objects in (a) above to draw perpendicular lines, parallel lines and intersecting lines.

**Activity 5.2: Identifying lines**
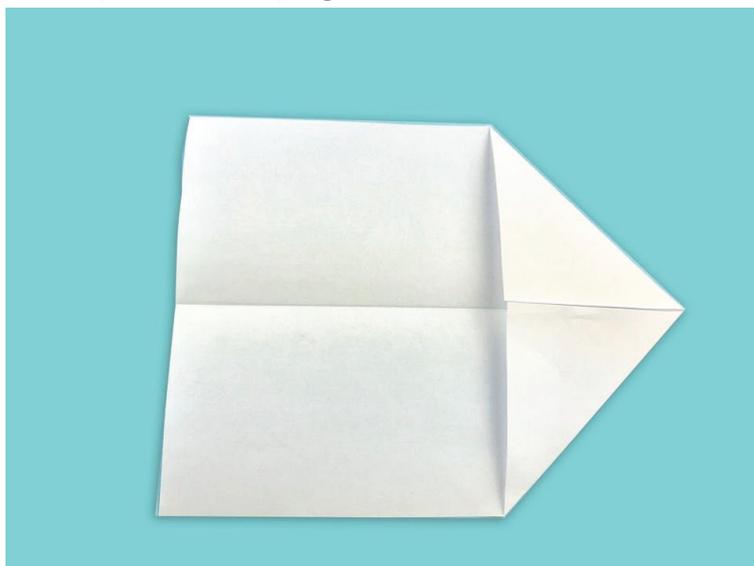
In your groups, take a sheet of paper; divide it into half, then into half again in the same way. Now fold your paper again. What kind of lines do you see?

Next, fold the same paper into half in the opposite direction. Unfold your paper now.
How is the new line you have created, related to the previous lines?
In real-life situations, where do we come across perpendicular lines and parallel lines?
Which letters in the alphabet have the above lines?

In this sub-topic, you will have more hands-on work on perpendicular and parallel lines

## Sub-topic 5.2: Construction of Perpendicular Lines

### Activity 5.3: Construction of perpendicular line from an external point to a given line

In your groups, work in pairs.
Given line segment AB and point C outside the line, construct a perpendicular line from point C to line AB.
Taking the centre as C and any radius, draw two arcs on line AB at x and y.
Now taking x as the centre and any radius, draw an arc below or above the line opposite point C without changing the radius. Taking y as the centre, draw an arc to intersect the previous arc. Join the intersection of the arcs to point C .Compare your answers and make notes.

### Activity 5.4: Construction of a Perpendicular line to a given point on a given line segment

In your groups, work in pairs.

Given line PQ and point Z on PQ.  Taking Z as the centre and any radius, draw two arcs on either side of Z name the arcs x and y . Now taking x as the centre and any radius draw an arc either above or below the line, without changing the radius now taking y  as the centre draw an arc to meet the previous arc  join the intersection of the arcs to point  Z . Compare your answers with other group members.

### Activity 5.5: Construction of a Perpendicular Bisector

In your groups work as an individual.
Given line segment AB.  Taking A as centre and AB as the radius, draw two arcs below and above the line, then now taking B as the centre and without changing the radius, draw arcs to meet the previous arcs. Join the intersection of the arc. What do you notice? Compare your work with your group members.

### Activity 5.6: Construction of parallel lines

In your groups, work in pairs.

Given line AB and point C outside the line. Take C as the centre, draw an arc at point A taking AB as radius and A as the centre, draw an arc at point B.  Now take radius AC and taking B as the centre, draw an arc above B, then taking radius AB and C as the centre, draw an arc to meet the previous arc at D. Join the intersection of the arcs (D) to point C. What do you notice. Name and describe shape ABCD. Compare your answer with members of the your group.

## Sub –topic 5.2: Using a Ruler, Pencil and Pair of Compasses, Construct Special Angles

**Activity5.7**:  **Construction of special angles**
In pairs, construct the following angles: 90º, 45º, 15º, 30º, 60º, 120º, 75º.
In your groups, compare your answers.
Using a protractor, measure your angles.

# Sub-topic 5.3: Describing Locus Question

What is the path traced out by the tip of the seconds-hand of a clock in the course of each minute?

**Activity 5.8**: **Discovering what Locus is**

In your groups, discuss what happens if a goat is tied to a rope of length 4 metres and around the place where the goat is, there are gardens at a distance of 5 metres.

In your groups, draw sketches of the area where the goat can feed from.

In real-life situations, where are such scenarios applied?

**Activity5.8: Sketching and Describing Loci**

In your groups, sketch and describe what happens about the following:
  a)  A mark on the floor as the door opens and closes.
  b)  The centre of a bicycle wheel as the bicycle travels along a straight line.
  c)  A man is walking and keeping the same distance from two trees P and Q.
  d)  A student is walking in the assembly hall keeping the same distance from two opposite walls.
  e)  Compare your answers with other groups.

## 5.3.1: Relating Lines and Angles to Loci

According to the activities above, Locus is a trace of a point under some conditions.

**Activity5.9**: **Demonstration of some simple Loci**
  a)  In your groups, demonstrate how one can walk the same distance from a given point.
  b)  How one can walk the same distance from two fixed points.
  c)  How one can walk the same distance from a line.

d)  How one can walk the same distance from two intersecting lines.

In your different groups compare your answers.

**Exercise**
1.  Construct the locus of a point equidistant from a fixed point.
2.  Construct a locus of a point equidistant from a given line.
3.  Construct the locus of a point equidistant from two intersecting lines.
4.  Construct a triangle ABC where AB = 12cm, AC=9cm and Angle BAC = 60º.  Find the point with the triangle where the distance from that point to all the vertices of the triangle is equal taking that point as the centre and the distance from the centre to the vertices as the radius draw a circle. (vi, vii are implied.)

## Sub-topic 5.4: Construction of Geometric Figure

Construction of geometric figures most of the time is application of locus.

### Activity5.10: **Construction of geometrical figures**
In pairs, construct a perpendicular bisector of any line segment. Measure the distance from the perpendicular line to any of the points on either side of the perpendicular bisector. What have you found out? In your groups, construct an equilateral triangle with length 6cm. Construct a circumcircle of the triangle. What type of locus is applied here?

**Exercise**
1.  Construct a triangle ABC in which AB = 8.5, BC = 6cm and angle B = 30º.
    Construct a circle through the vertices of the triangle. Work out the area of the circle.
2.  Construct triangle PQR with PQ = QR= 7cm angle Q = 45º.  Construct a circumcircle of the triangle.

3. Construct a parallelogram ABCD in which AB=5cm, BC=4cm and angle B is 120º.
4. Construct an equilateral triangle ABC of sides 7cm.Bisect AB and BC and let the bisectors intersect at X.  With X as the centre and radius XA, draw a circle.

**Situation of Integration**

In a village, there is an old man who wants to construct a rectangular small house of wattle and mud.

**Support:** A string, sticks, panga, tape measure and human resource.

**Resources:** Knowledge of horizontal and vertical lines i.e. rows and columns, knowledge of construction of geometric figures.

**Task:** The community asks you to accurately construct the foundation plan for this old man's house.

Explain to the class how you have accurately constructed the foundation plan.  Discuss whether there are other ways of constructing an accurate foundation plan.

# Topic 6:

# SEQUENCE AND PATTERNS
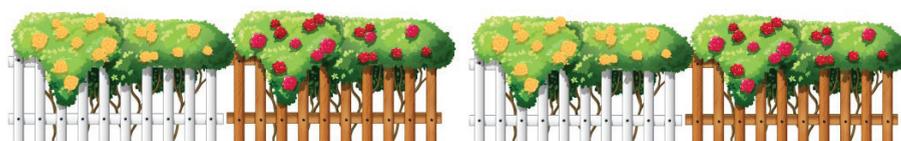
By the end of this topic, you should be able to:

i)   draw and identify the patterns.
ii)  describe a general rule of a given pattern.
iii) describe a sequence.
iv)  determine a term in a sequence.
v)   find the missing numbers in a given sequence.

## Introduction

In this topic you will learn how to identify and describe general rules for patterns. You will be able to determine a term in the sequence and find the missing numbers in the sequence.

## Sub-topic 6.1: Draw and Identify the Patterns

### Activity 6.1: Identifying number pattens

In groups, work in pairs.

Look at the following sequences, how can you get the next number? Compare your answers with other members.

i) 3, 6, 9, 12, 15, …
ii) 2, 4, 6, 8, 10, 12, …

In (i), in order to get the next number, you add 2 to the previous number. The numbers in this sequence are multiples of 3.

Sequence (ii), represents the multiples of 2.

**Exercise**
State the multiples of 3 found in this table:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

This square shows multiples of a number. What is this number?

Write down the numbers that should go in each of these boxes. The number square will help you with some of them.

a)  The fifth multiple of … is …

b)  The …th multiple of … is 36

c) The 12$^{th}$ multiple of … is …

d) The 20$^{th}$ multiple of … is …

e) The …th multiple of … is 96.

f)  The 100$^{th}$ multiple of … is …

**Solution**

  a)    the 5$^{th}$ multiple of 4 is 20
  b)    the 9$^{th}$ multiple of 4 is 36
  c)    the 12th multiple of 4 is 48
  d)    the 20th multiple of 4 is 80
  e)    the 24th multiple of 4 is 96
  f)    the 100$^{th}$ multiple of 4 is 400

**Exercise**

1.  On a number square like this one, shade all the multiples of 6. Then answer the questions after the table.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

a)  What is the 4th multiple of 6?
b)  What is the 10th multiple of 6?
c)  What is the 12th multiple of 6?
d)  What is the 100th multiple of 6?

2.  The multiples of a number have been shaded on this square. What is the number?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Copy each statement about these multiples and write down the numbers that go in the spaces.

a)  The 3rd multiple of … is …
b)  The 9th multiple of … is …
c)  The 200th multiple of … is …
d)  The …th multiple of … is 66
e)  The … th multiple of … is 330.

3.
a)  Write down the first 8 multiples of 8.
b)  Write down the first 8 multiples of 6.
c)  What is the smallest number that is a multiple of both 6 and 8?
d)  What are the next two numbers that are multiples of both 6 and 8?

4. a) Write down the first 6 multiples of 12.
   b) What is the 10th multiple of 12?

   c) What is the 100th multiple of 12?

   d) What is the 500th multiple of 12?

   e) If 48 is the nth multiple of 12, what is n?

   f) If 96 is the nth multiple of 12, what is n ?

5. a) What multiples have been shaded in this number square?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

   b) What is the first multiple not shown in the number square?

6. a) Explain why 12 is a multiple of 6 and 4.
   b) Is 12 a multiple of any other numbers?

7.   The number 24 is a multiple of 2 and a multiple of 3. What other numbers is it a multiple of?

8.   Two multiples of a number have been shaded on this number square. What is the number?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|----|----|----|----|----|----|----|----|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

9. Two multiples of a number have been shaded on this number square

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

a) What is the number?

b) What is the 19th multiple of this number?

10. Three multiples of a number are 34, 170 and 255. What is the number?

11. Three multiples of a number are 38, 95 and 133. What is the number?

12. Four multiples of a number are 49, 77, 133 and 203. What is the number?

## Sub-topic 6.2: Describing the General Rule

**Activity 6.2: Finding the Next Term**

In your, groups  work in pairs.

Can you use the given numbers of the sequence to deduce the pattern and hence find the next term?

**Example:** What are the next 3 numbers in the sequence:

  a) 12, 17, 22 …?

  b) 50, 47, 44, 41, 38, …?

Compare your answers with other group members

**Solution**

a) To find the pattern, it is usually helpful to first find the differences between each term i.e. the difference between 12 and 17 is 5; the difference between 17 and 22 is 5.

So the next term is found by adding 5 to the previous term. This gives you 27, 32, 37.

 b)  Again you find the difference between:

    i)   50 and 47 is -3.
   ii)   47 and 44 is -3.
  iii)   44 and 41 is -3.
   iv)   41 and 38 is -3.

So, the next term is found by taking away 3 from the previous term, giving you 35, 32, 29.

**Exercise**

1. Copy the following exercise and find the sequence in each case, giving the next three numbers.
   a) 18, 30, 42, 54, 66, …
   b) 4.1, 4.7, 5.3, 5.9, 6.5, …
   c) 8, 14, 20,…, 32, …
   d) 3, 11, …, 27, 35, …
   e) 3.42, 3.56, 3.70, 3.84, 3.98, …
   f) 10, 9.5, 9, 8.5, 8, 7.5, …

2. Copy each sequence and fill in the missing numbers.

   a) 2, 4, …, 16, 32, …
   b) 100, 81, 64, …, 36, …
   c) 6, 9, …, 21, 30, 30, …
   d) 0, 1.5, 4, …, 12, …
   e) 1, 7, 17, …, 49, …

## Sub-topic 6.3: Generating Number Sequence

### Activity 6.3: Generating a sequence

In your groups work in pairs.

You can use formulae to generate sequences. For example, the formula 5n, with n = 1, 2, 3, 4, … generates the sequence 5x1, 5x2, 5x3, 5x4, …

The sequence generated is 5, 10, 15, 20, …

**Example**: What sequence do you generate by using the following formula?

a) 5n – 1

b) 6n + 2

**Solution**

a) putting n = 1, 2, 3, 4, … gives  4, 9,  14,  19,  …
b) putting n = 1, 2, 3, 4, … gives 8,  14,  20,  26, …

You can find the formula for this sequence, 11, 21, 31, 41,  51,  61,  …

How you can find the sequence. The sequence begins with 11, and 11 = 10 + 1. Continue to add 10 each time the formula is 10n + 1.

Compare your answers with other members in the group.

**Exercise**

1.  What number comes out of each of these number machines?

(a)  7      (b)  8      (c)  14

( + 2 ) → ?      ( - 5 ) → ?      ( +9 ) → ?
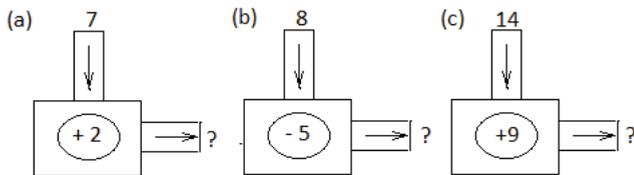
2.  The sequence 1, 2, 3, 4, 5, … is put into each number machine. What does each machine do?

(a)    1, 2, 3, 4, 5, ....

( ? ) ⟶ 11, 12, 13, 15, ....

(b)    1, 2, 3, 4, 5, ....

( ? ) ⟶ 10, 20, 30, 40, 50,...

(c)    1, 2, 3, 4, 5, ....

( ? ) ⟶ 7, 14, 21, 28, 35, ...
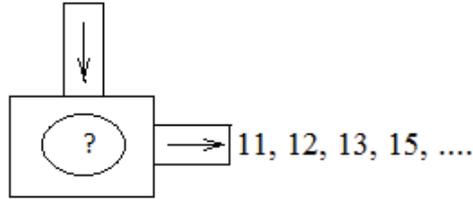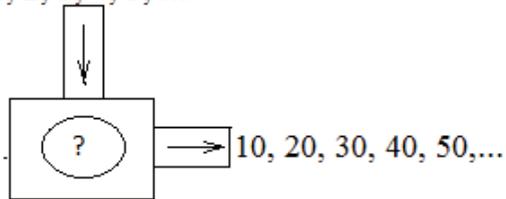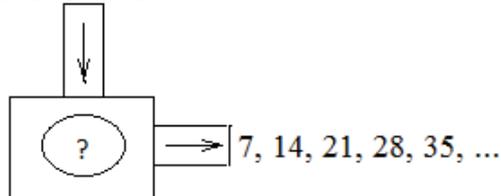
(d)    1, 2, 3, 4, 5, ....

( ? ) ⟶ 2.5, 3.5, 4.5, 5.5, 6.5, ...
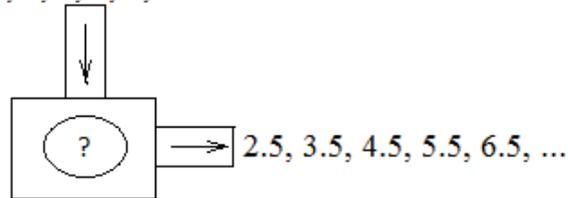
3.  Write down the first 5 terms of the sequence given by each of these formulae:

   a) 9n   b) 12n  c) 2n + 4    d)  3n – 1  e)  3n - 2

4. a) What is the 10th term of the sequence 2n + 1?

   b)  What is the 8th term of the sequence 3n + 6?

   c) What is the 5th term of the sequence 4n + 1?

   d) What is the 7th term of the sequence 5n – 1 ?

5. Draw double machines that could be used to get each of these sequences from 1, 2, 3, 4, 5 …

Also write down the formula for each sequence of the following:

   a)   5, 9, 13, 17, 21, …
   b)   2, 5, 8, 11, 14, …
   c)   6, 11, 16, 21, 26, …
   d)   4, 9, 14, 19, 24, …
   e)   102, 202, 302, 402, 502, …

## Sub-topic 6.4: Formulae for General Terms

### Activity 6.4 : Identifying the nth term

In your groups work in pairs.

**Note:** It is very helpful not only to be able to write down the next few terms in a sequence, but also to be able to write down, for example, the 100th or even the 1000th term.

**Example**: For the sequence 3, 7, 11, 15, …, …

   Find:

   a)  the next three terms.
   b)  the 100th term.
   c)  the 1000th term.

**Answer**

   a)  You can see that 4 is added each time to get the next term.
       So you obtain 19, 23, 27.
   b)  To find the 100th term, starting at 3, you add 3 to 4 times ninety nine times giving
$$3 + 4 \times 99 = 3 + 396 = 399$$
   c)  Similarly, the 1000th term is
$$3 + 4 \times 999 = 3 + 3996 = 3999$$

I can go one step further and write down the formula for a general term, i.e. the nth term.

This is $3 + 4 \times (n - 1) = 3 + 4n - 4$

$$= 4n - 1.$$

Compare your answers with other members of the group and the examples given.

### Exercise

1. For each sequence, write down the difference between each term and formula for the nth term.

        a)  3, 5, 7,  9,  11,  …
        b)  5, 11, 17,  23,  29,  …
        c)  4, 7, 10,  13,  16,  …
        d)  2, 5, 8,  11,  14,  …
        e)  6, 10, 14,  18,  22,  …

2.  a) Write down the first 6 multiples of 11.

    b)  What is the formula for the nth term of the sequence of the multiples of 11?

    c)  What is the formula for the nth term of this sequence?

3.  The formula for the nth term of this sequence is $n^2$.

      1, 4, 9,  16,  25,  …

What is the formula for the nth term of the following sequences?

        a)  0, 3, 8,  15,  24,  …
        b)  10, 13, 18,  25,  34,
        c)  2, 8, 18,  32,  50,  …
        d)  1, 8, 27,  64,  125,  …

**Situation of Integration**

There is a family in the neighbourhood of your school. The family has a rectangular compound on which they want to put up a hedge around. The hedge shall be made up of plants of different colours.

**Support**: Physical instruments like hoes, machetes, tape measure

**Resources**: Knowledge of construction of figures like rectangles, patterns, sequences

**Task**: The family requests you to plant the hedge around their rectangular compound so that it looks beautiful.

Explain how you will plant the hedge, making sure that the plants at the corners of the compound are the same in terms of colour.

Discuss whether there are other ways of planting the hedge.
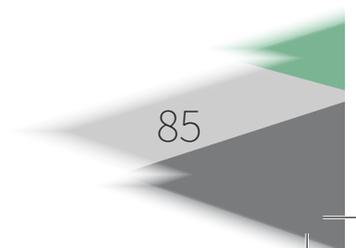
# Topic 7:
# BEARINGS

The diagram below shows the bearing of Kabale from where the lady is standing.
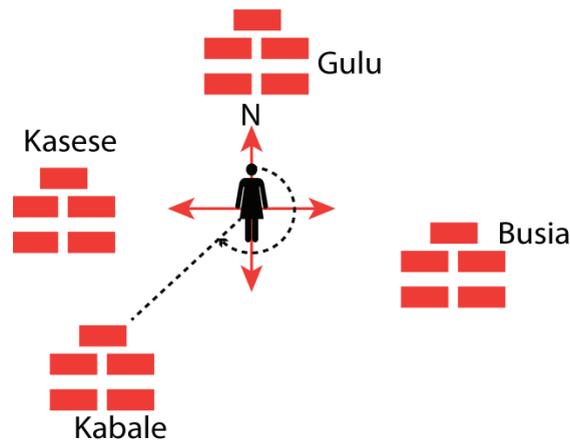


**Key words:** angle, direction, bearing, scale, line, turn

By the end of this topic, you should be able to:

i)   review the compass.
ii)  describe the direction of a place from a given point using cardinal points.
iii) describe the bearing of a place from a given point.
iv)  draw suitable sketches from the given information.
v)   choose and use appropriate scale to draw an accurate drawing.
vi)  differentiate between a sketch and a scale drawing.
vii) apply bearings in real life situations.

**Introduction**

In this topic you will learn how to tell the bearing of a point from a given point. You will determine accurately the distance between two points.

## Sub-topic 7.1: Angles and Turns

You will need to understand clearly, what the terms such as turn, half-turn, etc. mean in terms of angles. There are 360° in one complete turn, so the following are true.
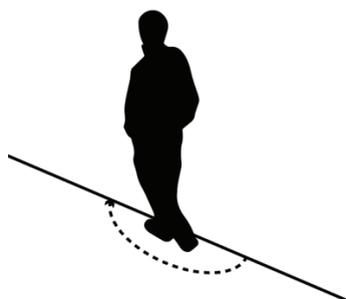
You also need to refer to compass points: north (N), south(S), east(E), west(W), northeast (NE), southeast (SE), southwest (SW) and northwest (NW)
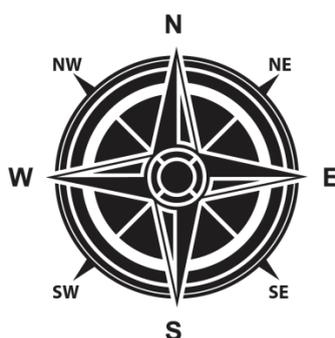
## Activity 7.1: Identifying the angles in relation to the compass direction

Work in pairs

Do the following turns and in each case state the size of the angle you have turned through.

   i)   Turn from N to S clockwise or anticlockwise
  ii)   Turn from NE to SE clockwise
 iii)   Turning clockwise from NE to E


### Example
What angle do you turn through if you turn:

(a)  from NE to NW anticlockwise?

(b) from E to N clockwise?

Compare your answers with the rest of the members in class.

### Solution
(a) 90$^o$ (or ¼ turn)

(b) 270$^o$ (¾ turn)


### Exercise
1. What angle do you turn through if you turn clockwise from:

 (a) N to E? (b) W to NW? (c) SE to NW?  (d) NE to N?  (e) W to NE?

 (f) S to SW? (g) S to SE?  (h)  SE to SW?  (i)  E to SW?

2. In what direction will you be facing if you turn:

 (a) 180$^o$ clockwise from NE?

 (b) 180$^o$ anticlockwise from SE?

(c ) 90° clockwise from SW?

(d) 45° clockwise from N?

(e) 225° clockwise from SW?

(f) 135° anticlockwise from N?

(g) 315° clockwise from SW?

3. The sails of a windmill complete one full turn every 40 seconds.

  (a) How long does it take the sails to turn through:

   (i) 180° (ii) 90° (iii) 45°?

  (b) What angle do the sails turn through in:

   (i) 30 seconds? (ii) 15seconds? (iii) 25 seconds?

## Sub-topic 7.2: Bearings

The bearing of a point is the number of degrees in the angle measured in a clockwise direction from North line to the line joining the centre of the compass with the point. A bearing is used to present the direction of one-point relative to another point.

### Activity7.2: Estimating bearings of some places within the school compound

In groups, work in pairs and outside the classroom.

From your school flag post, estimate the bearings of each building found in the School.

**Note**: Three figures are used to give bearings.

  All bearings are measured in a horizontal plane.

Compare your answers with the other members of the group.

**Exercise**

1. Find the bearing of each of the following directions:

(a) S   (b) NE     (c) N    (d) NW

2.  Find the bearing of each of the following directions:

(a) N60$^0$E   (b) N35$^0$E   (c) N90$^0$W (d) S400E

3. Draw a scale diagram to show the position of a ship which is 270 km away from a port on a bearing of 110$^{o.}$

**Situation of Integration**

Ajok is in Kampala City and has been told to use a car to move to Lira town. She has never gone to Lira.  She has been given the map of Uganda showing routes through which she can access Lira town.

**Support**: Mathematical instruments, pencil, paper, pens, tracing paper and map of Uganda

**Resources**: Knowledge of construction of figures like triangles, lengths of sides of triangles, operations on numbers.

**Task**: Ajok wants to use the short distance from Kampala to Lira.

Explain how Ajok can determine the shortest distance. Using the map given to her is it possible for Ajok to use the shortest distance she has determined. Explain your answer.
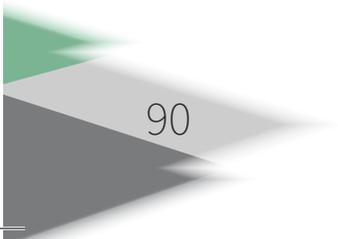
## Topic 8:

## GENERAL AND ANGLE PROPERTIES OF GEOMETRIC FIGURES
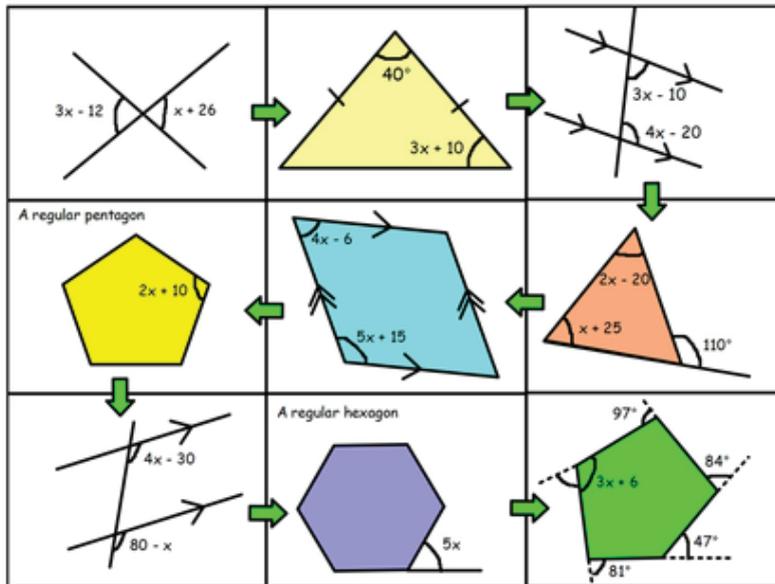


**Key words**: line segment**,** transversal, parallel

By the end of this topic, you should be able to:
    a.  identify different angles.
    b.  solve problems involving angles on a straight line, angles on transversal and parallel lines.
    c.  state and use angle properties of polygons in solving problems.

### Introduction

In bearings you studied angle turns, and in this topic you will study angles on the straight line, parallel lines and angle properties of polygons. Equipped with the knowledge from this topic, you will be able to solve problems related with angle properties.

You will need to understand clearly what the terms such as turn, half-turn, etc. mean in terms of angles. There are $360^o$ in one complete turn, so the following are true.
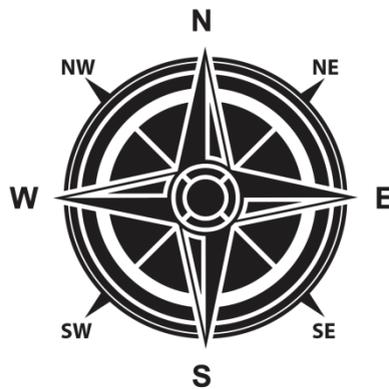


i)  Turning from N to S is $180^o$ clockwise or anticlockwise.
ii)  Turning from NE to SE is $90^o$ clockwise (or $270^o$ anticlockwise).
iii)  Turning clockwise from NE to E is $45^o$ (or $315^o$ anticlockwise).

### Example

What angle do you turn through if you turn:

a)   from NE to NW anticlockwise?
b)   from E to N clockwise?

### Solution

c)   $90^o$ (or ¼ turn)
d)   $270^o$ (¾ turn)

## Sub-topic 8.1:  Identify Different Angles

**Activity8.1: Identifying objects that form angles**

In  your groups, work in pairs.

Identify objects in you class, which make $90^0$, $180^0$, $360^0$

A protractor can be used to measure angles.

**Note:**

The angle around the circle is $360^o$.

The angle around a point on a line is $180^o$.

A right angle is $90^o$

**Compare your answers with other members of the group and classify them**

**Exercise**

1. For each of the following angles, first estimate the angles and then
    measure the angle to see how good your estimate was.

a)   b)

c)   d)
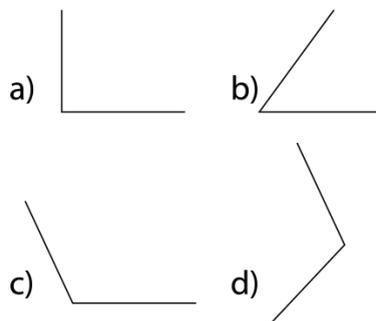
2. Draw the following angles

(a) 20° (b) 42° (c ) 80°  (d) 105°  (e) 170° (f) 200°  (g) 275°  (h) 305°

3. Immaculate finds out the favourite sports for members of her class. She works out the angles in the list shown below for a pie chart. Draw the pie chart.

| Sport | Angle |
|---|---|
| Football | $110^0$ |
| Swimming | 70° |
| Tennis | 80° |
| Rugby | 40° |
| Hockey | 30° |
| Badminton | 10° |
| Other | 20° |

**Exercise**
1. (a) Draw a triangle with one obtuse angle.
   (b) Draw a triangle with no obtuse angles.
2. Draw a four-sided shape with:
   a)   one reflex angle.
   b)   two obtuse angles.

## Sub- topic 8.2: Angles on a Line and Angles at a Point

Remember that:
   a)   angles on a line add up to 180°
And:
   b)   angles at a point add up to 360°

These are two important results, which help when finding the size of unknown angles.

**Activity 8.2: Identifying angles**

Work as individuals

Draw two intersecting lines. Use your mathematical instruments to measure the angles formed at the intersecting point.

 i) How many angles have been formed at the point of intersection?
 ii) What is the size of each angle formed?

 Compare your work with your friends and note your findings.

A polygon is a closed plane figure with straight sides.
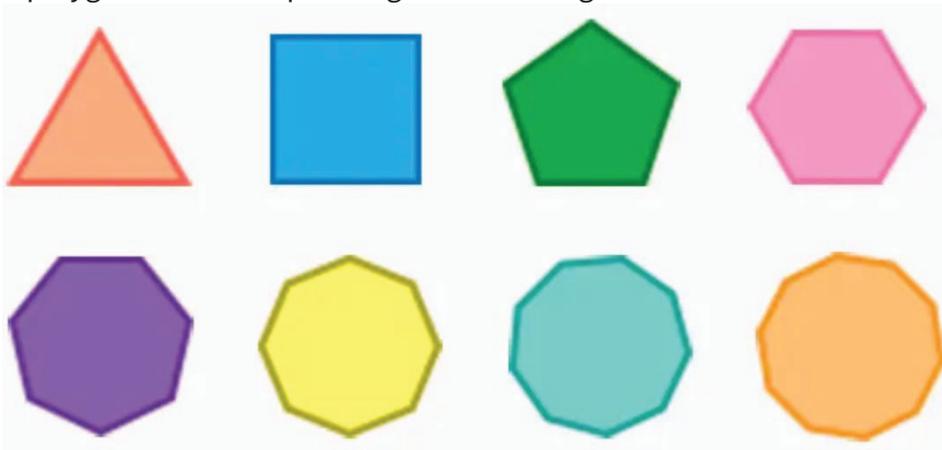


**Activity 8.3: Identifying the polygons**

In pairs:

Find the number of sides of different polygons and their corresponding names. Also determine the number and size of interior and exterior angles of the regular polygons.

Compare your answers with other members'.

**Exercise**

1. If the vertices of a regular hexagon are joined to the centre of the hexagon, what is the size of each of the six angles at the centre? Use your answer to construct a regular hexagon ABCDEF of side 3cm. Start with a circle of radius 3cm. Measure the length of the diagonal AC.
2. Find the sum of the interior angles of a polygon with 22 sides.
3. The interior angle of a regular polygon is $162^0$. How many sides has the polygon?

**Activity of Integration**

A diagram of a table showing coffee production in Uganda from year 2015 to year 2019

| Year | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|
| Production (tonnes) | 20 | 23 | 18 | 30 | 49 |

**Task:** The chairperson of Karo Farmers Association was asked to represent the information above on pie chart. As a senior one learner help him solve the challenge.

**Support:** Mathematical set

**Resource:** Knowledge of angles

# Topic 9:

# DATA COLLECTION AND PRESENTATION

**Key words:** data, chart, pie, quantitative, qualitative, discrete, continuous, hypothesis

By the end of this topic, you should be able to:

a) understand the differences between types of data.
b) collect and represent simple data from local environment using bar chart, pie chart and line graph.

### Introduction

In this topic, you will learn different types of data, data collection, presentation and analysis.

## Sub-topic 9.1: Types of Data

**Qualitative data** is data that is not given numerically; e.g. favourite colour, place of birth, favourite food, and type of car.

**Quantitative data** is numerical. There are two types of quantitative data: discrete and continuous data. Discrete data can only take specific numeric values e. g. shoe size, number of brothers, number of cars in a car park. Continuous data can take any numerical value e.g. height, mass, length.

### Activity 9.1: Identifying types of data

In your groups identify which of the following terms best describes each of the information listed (i) to (vii)?

Give reasons for your response.

- Qualitative data
- Continuous Quantitative Data
- Discrete Quantitative Data

| | | | |
|---|---|---|---|
| i) | Age | v) | Aces |
| ii) | Birth place | vi) | First serve School |
| iii) | Height | vii) | School life |
| iv) | World Ranking | | |

In your groups identify more examples.

## Exercise

1. Mr Okot starts to make a database for his lesson.

| Name | Age | Primary school | Transport to School | Height | Reading Glasses |
|---|---|---|---|---|---|
| Alice | 11 | St. Johns | Bus | 145cm | yes |
| Ben | 12 | St. Andrews | Walk | 160 cm | no |
| Carol | 12 | Hilltop | Car | 161 cm | no |
| David | 12 | Hilltop | | 152 cm | no |
| Eddie | 11 | St. Andrews | Walk | 158 cm | yes |
| Fredrick | | St. Andrews | Bike | 164 cm | no |
| Graham | 12 | St. Johns | Bus | 166 cm | yes |

a)  What is missing from Mr Okot's database?
b)  Which columns in the database contain quantitative data?
c)  Which columns in the database contain qualitative data?
d)  Write down what Mr Okot would put in his database if you joined his class.

2. Which of the following would give:

(a) qualitative data

(b) discrète quantitative data

(c ) continuous quantitative data

(i) Mass          (ii) Number of cars

(iii) Favourite football team      (iv) Colour of car

(v) Price of chocolate bars       (vi) Amount of pocket money

(vii) Distance from home to school    (viii) Number of pets

(ix) Number of sweets in a jar       (x) Mass of crisps in a packet.

3.  The table below shows a database that has no entries.

| Name | Age | Favourite food | Favourite TV show | Favourite pop group | Time spent watching TV yesterday |
|---|---|---|---|---|---|
|  |  |  |  |  |  |

a)   Collect data from 10 people to complete the data base.
b)   State whether each column contains:
   i)   qualitative data.
   ii)  continuous quantitative data.
   iii) or discrete quantitative data.
c)   Answer the following questions:
   i)    What is the most popular TV show?
   ii)   Who is the oldest?
   iii)  What is the favourite pop group for the youngest person?
d)   Write 3 more questions you could answer using your database and write the answers to them.

## Sub-topic 9.2: Collecting Data

In this section, you will see how data is collected, organized and interpreted, using a tally chart and then displayed using:

  i)  Pictograms
  ii) Bar charts
  iii) Pie charts

**Note:**

A hypothesis is an idea that you want to investigate to see if it is true or false. For example, you might think that most people in your school get

there by bus. You could investigate this using a survey. A tally chart can be used to record your data.

### Activity 9.2: Collecting data

In groups identify the means of transport each learner use to come to school. As a class identify how many of you use the same means of transport.

    i)  Which means of transport is used by the majority?
    ii) Which one is the least used means of transport?

### Example

The learners in a class were asked how they got to school.

| Method of Travel | Tally | Frequency |
|---|---|---|
| Walk | ///// /// | 9 |
| Bike | /// | 3 |
| Car | ///// / | 6 |
| Bus | ///// ///// // | 12 |
| | TOTAL | 30 |

Illustrate this data using:

    a)  a pictogram
    b)  a bar chart
    c)  a pie chart

What are the main conclusions that can be deduced from the data?

**Solution**

walk ☇☇☇☇☇

bike ☇☇

car ☇☇☇

bus ☇☇☇☇☇☇
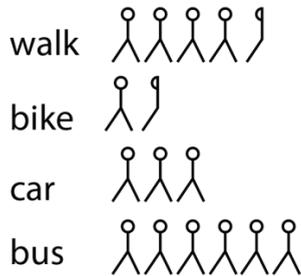
(a) If (stick man) is taken to represent 2 people, then the pictogram
    looks like:

    i)    Walk (4 and a half stick men)
    ii)   Bike (1 and a half stick men)
    iii)  Car (3 stick men)
    iv)  Bus (6 stick men)

(b) A bar chart for the data is illustrated below:
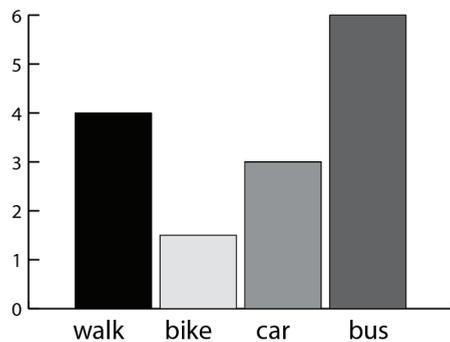


(c)  To illustrate the data with a pie chart, you need to find out what
    angle is equivalent to one pupil. Since there are 360° in a circle and
    30 pupils, then angle per pupil is $360 \div 30 = 12°$.

    To find the angle for walk, when there are 9 pupils, it is simply:

$$9 \times 12 = 108^0$$

The complete calculations are shown below:

| Method of travel | Frequency | Calculation | Angle |
|---|---|---|---|
| Walk | 9 | 9 x (360 ÷ 30) | 108° |
| Bike | 3 | 3 x(360 ÷ 30) | 36° |
| Car | 6 | 6 x (360 ÷ 30) | 72° |
| Bus | 12 | 12 x (360 ÷ 30) | 144° |
| | | TOTAL | 360° |

The corresponding pie chart is shown below:
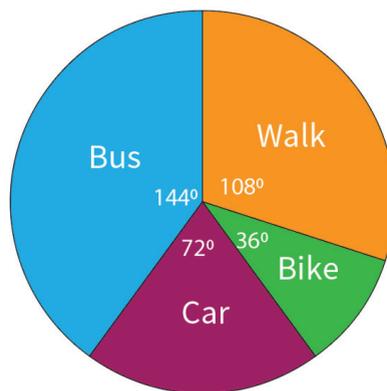


From the data we can see that:

- the most common way of getting to school is by bus. (This is called the modal class or the mode.)
- the least popular way of getting to school is by bike.

**Exercise**

1. The children in a class were asked to state their favourite crisps.

   The results are given in the tally chart below:

| Flavour | Tally | Frequency |
|---|---|---|
| Ready Salted | ///// | |
| Salt and Vinegar | ///// //// | |
| Cheese and onion | ///// // | |
| Prawn Cocktail | /// | |
| Smokey Bacon | ///// / | |
| | TOTAL | |

   (a) Copy and complete the table by filling the frequencies.

   (b) Represent the data on a bar chart.

   (c) Draw a pictogram for this data.

   (d) Copy and complete the following table and draw a pie chart.

| Flavour | Frequency | Calculation | Angle |
|---|---|---|---|
| Ready Salted | 5 | 5 x ( 360° ÷ 30) | 60° |
| | | | |

TOTAL

(e) What flavour is the mode?

2. (a) Do you think salt and vinegar crisps will be most popular crisps in your class?

   (b) Carry out a favourite crisps survey for your class. Present the results in a bar chart and state which flavour is the mode.

   (c) Was your hypothesis in (a) correct?

3. "Most *children in my class are 1.3m tall.*"

(a) Collect data to test this hypothesis.
(b) Present your data in a suitable diagram.
(c) Was the original hypothesis correct?

4. Is the music group that is most popular with the boys in your class the same as the music group that is most popular with girls?

(a) Write down a hypothesis that will enable you to answer this question.
(b) Collect suitable data from your class.
(c) Present your data using a suitable diagram.
(d) Was the hypothesis correct?

### Situation of Integration

The Games Master at your school wants to buy football boots for the three teams in the school. The three teams are the under 18 years, under 16 years and the under 14 years. The Games Master does not know the foot size for each of the players.

**Support**: pens, paper, tape measure, team members

**Resources**: Knowledge of tabulation, of tallying, of approximation, of central measures and of collection of suitable data.

**Task**: The total number of players for the three teams is 54. The Games Master wants to know the size of the boots for each player and the number of pairs for each size.

Explain how the Games Master will get the required data and how to determine the total cost for buying the football boots for the 54 players.

Is there another way of getting the required data other than what you have explained above?

# Topic 10:

# REFLECTION



By the end of this topic, you should be able to:

i) identify lines and planes of symmetry for different figures.
ii) state and use properties of reflection as a transformation.
iii) make geometrical deductions using reflection (distinguish between direct and opposite congruence).
iv) apply reflection in the Cartesian plane.

## Introduction

In this topic, you will learn how to identify the lines of symmetry, state the properties of reflection as a transformation, make geometrical deductions and apply reflection in Cartesian plane.

The image of a figure by reflection is its mirror image in the axis or plane of reflection. For example the mirror image of the letter p for

reflection with respect to a vertical axis would look like q. Its image by reflection in a horizontal axis would look like b.

## Sub-topic 10.1: Identify Lines of Symmetry for Different Figures

**Activity 10.1: Identifying lines of symmetry**

In pairs:

1. Fold a piece of paper in half
2. Open the paper and put in one drop of ink on the fold
3. Close the paper over the ink and press down hard on the paper.
4. When the ink has dried, open up your paper.
   (a) Look at both sides of the fold line. Are they the same size and shape?
   (b) Look at any two corresponding points on the ink blot, one on either sides of the fold.
      (i) What can you say about the distance from one point to the fold line and the distance from the corresponding point to the fold line?
      (ii) If a line joins two corresponding points, what is the angle between the line and the fold?

### Exercise 1

1. Draw a rectangle on a tracing paper. Fold it to find the lines of symmetry. How many lines of symmetry does a rectangle have?
2. Find the number of lines of symmetry of (a) a square  (b) an equilateral triangle  (c) a rhombus

# Sub-topic 10.2: Reflection in the Cartesian Plane

**Activity 10.2: Reflecting in a Cartesian plane**

In your groups, work as pairs.

Plot the points P (5, 4), Q (-1, 3) and R (0, -2) on squared paper.

 a) A mirror is placed on the x axis. Where would the images of the tree points be?
  What are the coordinates of the image points P', Q' and R'?

 b) Draw another pair of axes. Plot the same points again. Take the line y = 2 as the mirror line. Where would the images of the three points be? What are the coordinates of the new image points P', Q' and R'?

 c) Draw another pair of axes. Draw the line x = 4. Plot the points (1, -3). Using the line x = 4 as the mirror line, find the image of the point (1, -3).
  Compare your answers with other members in your group.

## Exercise 2

1. Find the image of the point (2, 5) under reflection in the y axis.
2. After a point has been reflected in the x axis, its image is at (3, 2). Find the coordinates of the object point.
3. The points A(4, 2) , B(1, 3)  and C(1,-2)are reflected in the line y = x. Find the coordinates of A' , B' and C', the images of A and B.

**Situation of Integration**

One of your relatives wants to make a barbershop /hairdresser. He approaches you for help.

As a senior one graduate draw a plan of how you can help your relative make his /her barber shop be up to date.

**Support:** Interior plan of the shop

**Task:** Advice the barber to make sure the customers can view themselves with their images not distorted.

**Resource:** knowledge of reflection

# Topic 11: Equation of Lines and Curves

**Key words:** variable, curve, substitution

By the end of this topic, you should be able to:

 i) form linear equations with given points.
 ii) draw the graph of a line given its equation.
 iii) differentiate between a line and a curve.

## Introduction

In this topic you will tell the difference between a line and a curve, how to form linear equations and draw graphs for the given linear equations.

## Sub-topic 11.1: Fundamental Algebraic Skills

In this section, you will look at some fundamental algebraic skills by examining codes and how to use formulae.

**Example**

If a = 4, b = 7 and c = 3, calculate:

(a)  6 + b (b) 2a + b (c) ab (d) a (b – c) (d) a (b – c)

**Solution**

(a)  6 + b =  6 + 7  = 13

(b)  2a + b =  2x4 + 7  = 8 + 7  = 15

(c)  ab = 4x7  = 28

(d)  a (b – c)  =  4 x (7 – 3) = 4 x 4 = 16

**Example**

Simplify where possible:

(a) 2x + 4x     (b) 5p + 7q – 3p + 2q

(c) y + 8y – 5y    (d) 3t + 4s

**Solution**

(a)   2x + 4x = 6x

(b)   5p + 7q – 3p + 2q = 5p -3p + 7q+2q = 2p – 9q

(c) y + 8y – 5y = 9y – 5y = 4y

(d) 3t + 4s = 3t + 4s

**Exercise**

1. If a = 2; b = 6; c = 10 and d = 3, calculate:

(a) a + b     (b) c – b     (c) d + 7

(d) 3a + d   (e) 4a       (f) ad

(g) 3b      (h) 2c     (i) 3c - b

(j) 6a + b    (k) 3a + 2b   (l) 4a – d

2. If a = 3; b = -1; c = 2 and d = -4, calculate:

(a) a – b    (b) a + d   (c) b + d

(d) b – d   (e) 3d     (f) 5(d – c)

(g) a (b + c)   (h) d(b + a)   (i) c(b – a)

(j) a (2b – c)   (k) d(2a – 3b)   (l) c(d – 2)

3. Simplify, where possible:

(a) 2a + 3a     (b)  5b + 8b

(c)  6c – 4c    (d)  5d + 4d + 7d

(e) 6e + 9e – 5e   (f) 8f + 6f – 13f

(g) 9g + 7g – 8g - 2g -6g (h) 5p + 2h

(i) 3a + 4b – 2a     (j) 6x + 3y – 2x –y

(k) 8t – 6t + 7s – 2s

(l)  11m +3n – 5p + 2q -2n +9q -8m + 14p

4. Sam asks his friend to think of a number, multiply it with 2 and then add 5. If the number his friend starts with is x, write down a formula for the number her friend gets.

## Subtopic 11.2: Function Machines

In this section you will look at how to find the input and output of function machines.

INPUT → FUNCTION MACHINE→ OUTPUT

**Activity 11.1: Function machine activity**

In pairs try out the numbers the first one is done for you.

Calculate the output of each of these function machines:

(a)  4 →x5 →?

(b)  5 →x2 →-1 →?

(c) -3 →+8→x7→?

(d) 8 →+6→x9→?

(e) -5→+3→x7→?

Compare your answers with members of the group.

**Solution**

(a) The input is simply multiplied by 5 to give 20:

   4 → x5 → 20

**Exercise**

1. What is the output of each of these function machines:

 (a) 4→+6 →?                          (d) 14→÷2→?

 (b) 3 →x10→?                         (e) 21→÷3→?

 (c) 10→-7→?                          (f) 100→×5→?

2. What is the output of each of these function machines:

 (a) 3→×4→-7→?                        (d) -2→×6→+20→?

 (b) 10→-8→×7→?                       (e)  7→+2→÷3→?

 (c) 8→-5→×5→?                        (f) -5→+8→×9→?

3. What is the input of each of these function machines:

(a)? →×5→30     (b)? → +8→ 12

 (c)? → -9→ 11    (d)? → +4→ 5

(e)? → +12→ 21   (f)? → ×7→ 42

4. A number is multiplied by 10, and then 6 is added to get 36. What is
   the number?

5. Karen asked her teacher, Maria, how old she was. The teacher
   replied that if she double her age, added 7 and then divided by 3, she
   would get 21. How old is Karen's teacher?

6. A bus has a maximum number of passengers when it leaves the bus station. At first stop, half of passengers alighted. At the next stop 7 people alighted and at the next stop 16 people alighted. There are now 17 people on the bus. How many passengers were on the bus when it left the bus station?

## Sub-topic 11.3: Linear Equations

An equation is a statement, such as 3x + 2 = 17, which contains an unknown number. In this case, it is x. The aim of this section is to show how to find the unknown number, x.

All equations contain an ''equals'' sign.

To solve the equation, you need to reorganize it so that the unknown value is by itself on one side of the equation. This is done by performing operations on the equation. When you do this, in order to keep the equality of the sides, you must remember that *__"Whatever you do to one side of an equation, you must also do the same to the other side__"*.

**Example**

Solve these equations:

(a) x + 2 = 8  (b) x‑ 4 = 3    (c ) 3x = 12

(d) 2x + 5 = 11 (e) 3 – 2x = 7

**Solution**

(a) To solve this equation, subtract 2 from each side of the equation:

$$X + 2 = 8$$

$$X + 2 - 2 = 8 - 2$$

$$X = 6$$

(b) To solve this equation, add 4 to both sides of the equation:

X – 4 = 3

X – 4 + 4 = 3 + 4

X = 7

(c) To solve this equation, divide both sides of the equation by3:

3x = 12

3x ÷ 3 = 12 ÷ 3

X = 4

(d) This equation must be solved in 2 stages.

First, subtract 5 from both sides:

2x + 5 = 11

2x + 5 -5 = 11 – 5

2x = 6

Then, divide both sides of the equation by 2:

2x ÷ 2 = 6 ÷ 2

X = 3.

(e) First, subtract 3 from both sides:

3 – 2x = 7

3 – 3 – 2x = 7 – 3

-2x = 4

Then divide both sides by (-2);

$-2x \div -2 = 4 \div -2$

$X = -2.$

**Example 3**

You ask a friend to think of a number. He then multiplies it by 5 and subtracts 7. He gets the answer 43

a) Use this information to write down an equation for x, the unknown number.

b) Solve your equation for x

**Solution**

a) As x = number your friend thought of, then

5x

X ⟶ [ x 5 ] ⟶ [ -7 ] ⟶
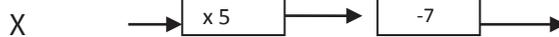
So 5x -7 = 43

b) First, add 7 to both sides of the equation to give

5x = 50

Then divide both sides by 5 to give

$X = 10$

And this is the number that your friend thought of.

**Exercises**

1. **Solve these equations:**

a) $x + 2 = 8$    b) $x + 5 = 11$    c) $x - 6 = 2$

d) $x - 4 = 3$    e) $2x = 18$    f) $3x = 24$

g) $\dfrac{x}{6} = 4$   h) $\dfrac{x}{5} = 9$   i)  6x = 54

j)  x + 12 = 10   k) x + 5 =3   l)  x -22 = -4

n) 10x = 0

m)   $\dfrac{x}{7} = -2$   o) $\dfrac{x}{2} = 5$

**2.   Solve these equations**

a)      2x + 4 =14      b)      3x + 7 =25      c)      4x +2 = 22

d)      6x – 4 = 26      e)      5x – 3 = 32      f)      11x – 4 =29

g)      3x + = 4 = 25      h)      5x – 8 = 37      i)      6x + 7 = 31

j)      3x + 11 = 5      k)      6x + 2 = -10      l)      7x + 44 +2

3.      Solve these equations, giving your answers as fractions or

        mixed numbers

a)      3x = 4      b)      5x = 7      c)      2x + 8 = 13

d)      8x + 2 = 5      e)      2x +6 =9      f)      4x = 7 = 10


4.      Solve these equations:

a)      x + 2 =2x -1      b)      8x – 1 = 4x + 11      c)      5x + 2 = 6x - 4

d)      11x – 4 = 2x = 23      e)      5x +1 = 6x -8      f)      3x + 2 +5x + x =44

g)      6x + 2 – 2x = x + 23      h)      2X – 3 = 6x + x -58      i)      3x + 2 = x -8

j)      4x – 2= 2x - 8      k)      3x + 82 = 10 x + 12      l)      6x – 10 = 2x - 14

# Topic 14: Time and Time Tables

**Key words**: quarter**,** timetable, half, departure, arrival

By the end of this topic, you should be able to:

    i)   identify and use units of time.
    ii)  use and interpret different representations of time.
    iii) apply the understanding of time in a range of relevant real life contexts.

### Introduction

In this topic, you will learn various units of time, such as minutes, seconds, hours, day, week, month, year. You will be able to understand and apply time in a range of relevant real-life contexts.

## Sub-topic 14.1: Telling the Time

In this section we look at different ways of writing times; for example, **7:45** is the same time as **quarter to eight.**

On a clock face, this can be represented as shown below.



Also remember that

> One hour  =  60 minutes

So that

> Half an hour = 30 minutes

Quarter of an hour = 15 minutes
Three quarters of an hour  =  45 minutes

**Example**
Write each time using digits and show the position of the hands on a clock face:
(a) twenty-five past eight.
(b) quarter to ten.

**Solution**
(a) Twenty-five past eight using digits is **8:25**



(b) Quarter to ten can be thought of as:
    15 minutes to 10 o'clock
Or
    45 minutes past 9 o'clock
So, using digits, quarter to ten is **9:45**

**Exercise**

1. Draw the time given below on clock faces:
 (a) ten past five    (b) ten minutes to nine    (c ) quarter to seven
(d) quarter past twelve (e) half past ten  (f)  twenty nine minutes to five
(g) ten minutes to two (h) twenty five minutes to six   (i)  twenty past
        four

2. Draw the following time on clock faces:
(a) 4:00 (b)  5:30  (c )  7:15  (d) 8:20  (e)  2:45  (f)  3:50
(g) 1:55 (h)  6:05   (i) 11:35

3. Write the following time in words:
(a) 9:30 (b)  4:00  (c ) 4:25  (d) 8:45  (e)  7:35  (f)  9:05

4. Write these times using digits:
(a)  eight o'clock   (b)  quarter to seven  (c )  ten past five
(d)  half past six  (e)  ten to three  (f)  five to four
(g)  twenty five to nine (h) twenty to three

## Sub-topic 14.2: 12-hour and 24-hour Clocks

The 24-hour clock system can be used to tell if time is in the morning or the afternoon. Alternatively, time can be given as **am** or **pm.**

**Activity 14.1: Converting from 12 hour to 24 hour and vice versa**
In pairs:
i) Write these times in 24-hour clock time:
(a) 3:06 am   (b) 8:14 pm (c) 9:45am  (d) 3:06pm
ii) Write these times in 12-hour clock time:
(a) 03:00 (b) 09:45  (c) 13:07  (d)  22:15

**Solution**
(a)  As this is **a.m.** the time remains the same except you add a zero in front of 3, so the time becomes **0306 in a 24-hour clock.**
(b) As this is **pm**, you add 12 to the hours to give you **2014 in a 24-hour clock.**

**Example**

Write these times using **am** or **pm in a 12-hour clock.**

(a) 14:28   (b) 07:42

**Solution**

(a) As the hours figure, 14, is greater than 12, subtract 12 and write as a pm time. The answer is**2:28pm.**

(b) As the hours figure, 07, is less than 12, simply remove the zero and then write the time as am. The answer is **7:42 am.**

**Exercise**

1. Convert the following time to the 24-hour clock:
   (a) 9:24am  (b) 11:28pm  (c ) 11:14a.m  (d) 7:13pm
2. Write the following time in the 24-hour clock:
   (a) quarter to eight o'clock in the morning
   (b) ten minutes to midnight
   (c) ten past nine o'clock in the morning
   (d)  half past two o'clock in the afternoon
3. Write the following24-hour clock in words
   (a) 14 :30 (b)  15:55 (c) 07:45
4. Sarah leaves home at 09:00 and returns 7 hours later. Write the time that Sarah gets home in the 24-hour clock and in the twelve-hour clock using **am** or **pm.**

## Sub-topic 14.3: Units of Time

In this section we explore the different units of time.

| | | |
|---|---|---|
| 1 minute | = | 60 seconds |
| 1 hour | = | 60 minutes |
| 1 day | = | 24 hours |
| 1 week | = | 7 days |
| 1 year | = | 365 0r 366 days |

**Example**

1. How many hours are there in May?

**Solution**

Number of hours in May = 31 x 24 = 744 hours

**Activity 14.2**

In pairs find out if 25 February is a Friday. What will be the date on the next Friday?

(a) If it is not a leap year.

(b) If it is a leap year?

Compare your answers with members of the group before you check the solution.

**Solution**

(a) You could write out the 7 days like this:

| | |
|---|---|
| Friday | 25 |
| Saturday | 26 |
| Sunday | 27 |
| Monday | 28 |
| Tuesday | 1 |
| Wednesday | 2 |
| Thursday | 3 |
| Friday | 4 |

**Or**

25 + 7 = 32

32 – 28 = 4

So the next Friday will be 4th March.

(b) Using the addition method:

25 + 7 = 32

32 – 29 = 3

So, in a leap year, the next Friday will be 3rd March.

**Exercise**

1. How many hours are there in a week?
2. How many hours are there in:

(a) September?

(c) February?
(d) one year?
3.  3. Rupert goes on holiday on Monday 20th June. He returns 14 days later. On what date does he return from his holiday?
4.  4. If 3rd October is a Monday:
    (a) What day of the week will 1st November be?
    (b) What will be the date of the first Monday in November?
5.  Immaculate goes to the bank every Tuesday. The last time she went was on Tuesday 20th October.
    (a) What will be the dates of her next 2 visits to the bank?
    (b) On the second Tuesday in November she is ill and goes to the bank on Wednesday instead. What is the date of that Wednesday?

## Sub-topic 14.4: Timetables

In this section we consider how to extract information from timetables.

**Exercise**

1. The table below gives the timetable for a Bus that runs from Mbale to Kampala.

| Mbale | depart | 08:57 |
|-------|--------|-------|
| Iganga | depart | 10:06 |
| Jinja | arrive | 16:57 |
| Mukono | arrive | 17:23 |
| Kampala | arrive | 17:42 |

(a) At what time does the bus leave Mbale?
(b) At what time does the bus arrive at Kampala?
(c) Where does the bus arrive at 16:57?
(d) Mr Okot arrives in Mbale at five past nine. Can he catch the bus?

2. Mike is in Brussels and wants to return to Ashford. He looks at this train timetable:

**Brussels to Waterloo**

| Brussels Midi | 0856 | 1102 | 1302 | 1456 | 1702 | 1756 | 1856 | 2102 |
|---|---|---|---|---|---|---|---|---|
| Lille Europe | 0937 | 1142 | 1342 | 1536 | 1742 | 1836 | 1936 | 2142 |
| Ashford | 0938 | 1141 | 1341 | 1536 | 1741 | 1837 | 1938 | …. |
| International | 1047 | 1247 | 1447 | 1639 | 1843 | 1939 | 203 | 2239 |
| Waterloo | | | | | | | | |
| International | | | | | | | | |

a) At what time should he catch a train if he wants to arrive in Ashford at 17:41?
b) Which train should he avoid if he wants to go to Ashford?
c) If he catches the 14:56 train, at what time does he arrive in Ashford?
d) He catches the 14:56, but falls asleep and does not get off at Ashford. At what time does he get to Waterloo?

3. The Journey from Kabale (Uganda) to Kigali (Rwanda) takes 2 ½ hours. The time in Uganda is 1 hour ahead of Rwanda.
   a) If you leave Kabale at 10:00, what will be the local time when you arrive in Kigali?
   b) If you leave Kigali at 17:45, what will be the local time when you arrive in Kabale?

4. Jean earns UGX 4,000 per hour on weekdays, UGX 4,500 per hour on Saturdays and UGX 6,000 per hour on Sundays.

The table below lists the hours she worked on each day for one week:

| Day | No. hours worked |
|---|---|
| Monday | 4 |
| Tuesday | 2 |
| Wednesday | 8 |
| Thursday | 10 |
| Friday | 3 |
| Saturday | 5 |
| Sunday | 2 |

How much money did Jean earn that week?

**Situation of Integration:** A primary school has two sections, that is, lower primary (P1-P4) and upper primary (P5-P7). The head teacher of primary school needs to draw a timetable for both sections. The sections should start and end their morning lessons at the same time before break time, start and end their break time at the same time. The after break lessons should start at the same time. The lunchtime for both sections should start at the same time.

**Support:** The time to start lessons for the two sections is 8.00am. The duration of the lesson for the lower section is 30 minutes and that of the upper section is 40 minutes.

**Resources:** Knowledge of fractions, percentages, natural numbers, factors, multiples, lowest common multiples and of time.

**Task:** Help the head teacher by drawing the timetable up to lunch break for the two sections. How many lessons does each section have up to lunch break?

Express the total number of lessons for the lower primary as a fraction of the total number of lessons for the whole School. (Consider lessons up to lunch break)

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