1000 REVISION QUESTIONS IN UACE MATHEMATICS

For use in Holiday and weekend revision classes.

You can order for a printed hard copy with final answers at the back of the book at UGX 15,000 only. UGX 10,000 for those that attend our holiday program or our weekly revision classes in and around Fortportal City.

You can join our online discussion forum. THE MATH CLINIC to stay updated or to consult (students only).

Send **Hello, MATH CLINIC** to 0781226393 on Whatsapp to receive a link.

ATTENTION – FORTPORTAL MATHEMATICS SEMINAR!!!

In 2025, we shall have an expanded edition of the FORTPORTAL MATHEMATICS SEMINAR. The first edition was very successful with over 13 schools from Fortportal, Kabarole and Bunyangabu. We apologise for not invinting some schools. We wanted manageable numbers since it was a maiden edition.

Due to the demand from schools outside Fortportal, we are expanding the 2025 edition to cover the greater RWENZORI REGION i.e; Fortportal, Kabarole, Bunyangabu, Kyenjojo and Kasese. We shall have coordinating centres in each of those districts. Book a slot through 0781226393(Whatsapp) to receive a link to the seminar organizing group. (**Teachers only**)

EQUATIONS

Solve the equations;

1. $3^{2-x}-2(3^{x})+17=0$
2. $m^2-6=\frac{6}{m^2-6}-1$
3. $\sqrt{\frac{x-1}{3x+2}}+2\sqrt{\frac{3x+2}{x-1}}=3$
4. $x^4-2x^3-6x^2-2x+1=0$
5. $y^4-2y^3-2y^2+2y+1=0$
6. $9^{x-1}-3^{x+2}+162=0$
7. $\sqrt{x+1}+\sqrt{(5x+1)}=2\sqrt{x+5}$
8. $3^{x}+6(3^{-x})=5$
9. $x^2+2x=3+\frac{35}{x^2+2x}$
10. $x^4-25x^2+144=0$
11. $\log_{10}(x^2+9)+2\log_{10}x=1$
12. $\log_3 x+\log_x 3=\frac{10}{3}$
13. $\log_{10}\left(\frac{x^2+24}{x}\right)=1$
14. $2^{x}\times 3^{x+1}=5^{2x+1}$

Solve the following equations both by row reduction to achelon form and then by elimination method;

15. $p-2q-2r=0,\ 2p+3q+r=1, 3p-q-3r=3$
16. $p+q+r=0;\ p+2q+2r=2;\ 2p+3r=4$
17. $2x+3y+4z=8, 3x-2y-3z=-2$ and $5x+4y+2z=3$
18. $x-10y+7z=13;\ x+4y-3z=-3;\ -x+2y-z=-3$
19. Solve for x in: $\log_x(x+3)+\frac{1}{\log_x y}=2\log_y 2$.

Solve the simultaneous equations.

20. $\log_5(2x+y)=0$,
    $2\log_5 x=\log_5(y-1)$.
21. $2x+y=1$,
    $x^2+xy+3x-y=4$.
22. $3\log_8 xy=4\log_2 x$ and
    $\log_2 y=1+\log_2 x$.
23. $x^2-xy+7y^2=27$,
    $x^2-y^2=15$
24. $\frac{y}{x}+\frac{x}{y}=\frac{17}{4}$,
    $x^2-4xy+y^2=1$
25. $4x^2+25y^2=100; xy=4$
26. $x^2-y^2=24$,
    $\frac{1}{x+y}+\frac{3}{x-y}=\frac{11}{12}$
27. $xy=2;\ 2\log(x-1)=\log y$
28. $\frac{x-y}{4}=\frac{z-y}{3}=\frac{2z-x}{1}; x+3y+2z=4$

PARTIAL FRACTIONS

29. Given that $\frac{p}{2x+3}+\frac{q}{3x+2}\equiv\frac{1}{(2x+3)(3x+2)}$. Find the values of the constants p and q.

Resolve into partial fractions;

30. $\frac{2x^3-1}{x^2(2x-1)}$

31. $\frac{6-7x+6x^2}{(x-1)(1+x^2)}$

32. $\frac{20(x^2+6)}{(1-x)(1+x^2)}$

33. $\frac{x}{(1-2x)^2(1-3x)}$

34. $\frac{1+x}{(1-x)(1+x^2)}$

35. $\frac{x^4+3x-1}{(x+2)(x^2-2x+1)}$

LOGARITHMS,SURDS AND INDICES

36. Show that $(a+\sqrt{b})^2=a^2b+2a\sqrt{b}$. Hence evaluate the square root of $9+4\sqrt{5}$ in the form $c+\sqrt{d}$.

37. Find the square root of $4+3\sqrt{2}$.

Show that;

38. $\log_q(pr)=\log_q p+\log_q r$

39. $\log_q\left(\frac{p}{r}\right)=\log_q p-\log_q r$

40. $\log_q p=\frac{\log_r p}{\log_r q}$

41. If $p^2=qr$, show that $\log_q p+\log_r p=2\log_q p\log_r p$.

42. If $\log_a b=\log_b c=\log_c c$ show that $a=b=c$.

43. If $x=\sqrt[3]{p}+\frac{1}{\sqrt[3]{p}}$, $y=\sqrt{p}+\frac{1}{\sqrt{p}}$. Show that $y^2-2=x(x-3)$.

44. If $a=\log_b c$, $b=\log_c a$, $c=\log_a b$, prove that $abc=1$.

45. Without using tables, show that $\frac{\log\sqrt{27}+\log\sqrt{8}-\log\sqrt{125}}{\log 6-\log 5}=\frac{3}{2}$.

46. If $\log_a n = x$ and $\log_c n = y$, where $n \neq 1$, prove that $\frac{x-y}{x+y} = \frac{\log_b c - \log_b a}{\log_b c + \log_b a}$

.

Verify this result, without using any tables, when $a = 4$, $b = 2$, $c = 8$, $n = 4096$.

47. Show that $\log_a (a+b)^2 = 2 + \log_a\left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$.

48. If $2\log_8 N = p$, $\log_2 2N = q$, $q - p = 4$, find N.

49. If $a^2 + b^2 = 23ab$ show that $\log a + \log b = 2\log\left(\frac{a+b}{5}\right)$.

50. Show that $\sqrt{x} - \sqrt{a} = \frac{x-a}{\sqrt{x}+\sqrt{a}}$. Hence deduce that $\frac{1}{\sqrt{x}-\sqrt{a}} = \frac{\sqrt{x}+\sqrt{a}}{x-a}$.

51. Evaluate $\frac{x^{\frac{3}{2}} + xy}{xy - y^3} - \frac{\sqrt{x}}{\sqrt{x} - y}$.

52. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$, prove that $x + y + z = xyz - 2$.

53. $\alpha(\beta-\gamma)+\beta(\gamma-\alpha)+\gamma(\alpha-\beta)=0$, show that $\left(\frac{b}{c}\right)^{\log a} \cdot \left(\frac{c}{a}\right)^{\log b} \cdot \left(\frac{a}{b}\right)^{\log c} = 1$.

QUADRATICS AND POLYNOMIALS

54. Given that the roots of the equation $ax^2 + bx + c = 0$ are in the ratio $p : q$. Show that $ac(p+q)^2 = b^2pq$.

55. If $(x+1)^2$ is a factor of $2x^4 + 7x^3 + 6x^2 + Ax + B$, find the values of A and B.

56. Given that $2x^2 + 7x - 4$, $x^2 + 3x - 4$ and $7x^2 + ax - 8$ have a common factor, find the;

a) Factors of $2x^2 + 7x - 4$ and $x^2 + 3x - 4$.

b) Value of a in $7x^2 + ax - 8$.

57. Solve the equation $5x^3 - 111x^2 + 74x - 16 = 0$ given that the roots are in G.P.

58. Solve the equation $64x^3 - 240x^2 + 284x - 105 = 0$ given that the roots are in A.P.

59. If the roots of the equation $x^3-5x^2+qx-8=0$ are in G.P, show that $q=10$.

60. When a polynomial $p(x)$ is divided by $x^2-5x-14$, the remainder is $2x+5$. Find the remainder when $p(x)$ is divided by;

(a) $x-7$

(b) $x+2$

61. The roots of the equation $2x^2-3x+5=0$ are α and β. Find the equation whose root are $\frac{\alpha}{\beta-2}$ and $\frac{\beta}{\alpha-2}$.

62. Use remainder theorem to express x^3+2x^2+x-18 as a product of two factors.

63. Find the value of p for which the polynomial $x^4+x^3+px^2+5x-10$ has $x+2$ as a factor.

64. If the equations $x^2+ax+b=0$ and $cx^2+2ax-3b=0$ have a common root, prove that $b=\frac{5a^2(c-2)}{(c+3)^2}$.

65. The polynomial $P(x)$ is divided by $x-7$ the remainder is 19 and when divided by $x+2$ the remainder is 1. Find the remainder when the polynomial is divided by $(x-7)(x+2)$.

66. The polynomial $8x^3+ax^2+bx-1$, where a and b are constants is denoted by $P(x)$. It is given that $(x+1)$ is a factor of $P(x)$ and that when the polynomial is divided by $(2x+1)$, the remainder is 1.

a) Find the values a and b.

b) When a and b have these values, factorise $P(x)$ completely.

67. Find the condition for the equations $x^2+2x+a=0$ and $x^2+bx+3=0$ to have a common root.

68. Find the values of k and 1 for which $x^4-2x^3+5x^2+kx+1$ has factor $(x-1)^2$.

69. If α and β are roots of the equation $4x^2+5x-1=0$, find the equation whose roots are $\left(2-\frac{\beta}{\alpha}\right)$ and $\left(2-\frac{\alpha}{\beta}\right)$.

70. The roots of the equation $3x^2+kx+12=0$ are equal, find the value of k.

71. If the roots of the equation $x^2+bx+c=0$ are α, β and the roots of the roots of the equation γ,δ show that the equation whose roots are $\alpha\gamma+\beta\delta$ and $\alpha\delta+\beta\gamma$ is;

$$x^2-\lambda b^2x+2\lambda^2c(b^2-2c)=0$$

72. If the equations $x^2+bx+c=0$, $x^2+px+q=0$, have a common root, then; $(c-q)^2=(b-p)(cp-bq)^2$.

73. In the equation $ax^2+bx+c=0$, one root is the square of the other. Without solving the equation, prove that $c(a-b)^3=a(c-b)^3$.

74. The roots of the quadratic equation $ax^2+bx+c=0$ are α and β. Given that $\alpha:\beta=\lambda:\mu$, Show that $\lambda\mu b^2-ac(\lambda+\mu)^2=0$

75. If $4x^3+kx^2+px+2$ is divisible by $x^2+\lambda^2$, prove that $kp=8$.

PROGRESSIONS

76. Given $x+2x^3+4x^5+8x^7+...=\frac{3}{7}$.
   a) Find the value of x.
   b) Find the 20th term.

77. The sum of the first m terms of a progression is $m(2m+11)$.
   a) Show that the progression is an A.P
   b) Determine the nth term of the progression. (9+4n)

78. The ninth term of an AP is -1 and the sum of the first nine terms is 45. Find the common difference and the sum of the first twenty terms.

79. The common ratio of a G.P is -5 and the sum of the first seven terms of the progression is 449. Find the first three terms.

80. The coefficients of the first three terms of the expansion of $\left(1+\frac{x}{2}\right)^n$ are in arithmetic progression (AP). Find the value of n.

81. The second, third and ninth terms of an A.P form a G.P. Find the common ratio of the G.P.

82. The first, fourth and eighth of an A.P form a G.P. If the first term is 9, find;
   a) The common difference of the A.P.
   b) The common ratio of the G.P

c) The difference in the sums of the first 6 terms of the progressions.

83. In a geometric progression, the first term is 7 and the nth term is 448. The sum of the first n terms is 889, find the common ratio.

84. The first three terms of a geometric series are 1,p and q. Given also that 10, q and p are the first three terms of an arithmetic series. Show that $2p^2 - p - 10 = 0$. Hence find the possible values of p and q. (ANs: $p = -2$ and $q = 4$ and $p = \frac{5}{2}$ and $q = \frac{25}{4}$.

85. The arithmetic mean of a and b three times their geometric mean. Show that $\frac{a}{b} = 7 \pm 12\sqrt{2}$.

86. The sum of the first 10 terms of an AP is 120. The sum of the next 8 terms is 240. Find the sum of the next 6 terms. (Ans: 264)

87. The sum to infinity of a GP is 7 and the sum of the first two terms is $\frac{48}{7}$. Find the common ratio and the first term of the GP given that it has a positive common ratio. (Ans: 1/7 , 6)

88. The first, second and last term of an A.P are a, b and c respectively. Prove that the sum of all terms is $\frac{(a+b)(b+c-2a)}{2(b-a)}$.

89. Find the amount at the end of 8 years when Shs 100,000 is invested at 5% compound interest if;

a) the interest is added annually

b) the interest is added bi-annually

90. Four million shillings is invested is invested each year at a rate of 10% compound interest by a certain bank.

a) find how much he will receive at the end of 10 years.

b) how many years will it take to accumulate to more than 40m.

91. A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10?

92. Kiiza opened an account in the bank and deposited Shs 2,000,000 every year for ten years without withdrawing. Find how much money he accumulated after 10 years if the bank offered a compound interest of 20% per annum.

PROOF BY INDUCTION

Prove by mathematical induction that;

93. $\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$

94. $\sum_{r=1}^{n} ap^{r-1} = \frac{a(p^n - 1)}{p-1}$

95. $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$

96. $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{1}{4}n^2(3n+1)(5n+1)$

97. $1\times3 + 2\times4 + 3\times5 + ... + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$

98. $\frac{1}{3\times5} + \frac{1}{5\times7} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$.

99. $p + pq + pq^2 + ... + pq^{n-1} = p\left(\frac{1-q^n}{1-q}\right)$

100. $\sum_{1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$ for all positive integers n.

101. $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ hence evaluate $\sum_{r=1}^{20} r(r+1)$.

102. $6^n - 1$ is divisible by 5 for all integral values of n.

103. $3^{2n} - 1$ is a multiple of 8 for all positive integers n.

104. $10^{2n-1} + 1$ is divisible by 11 for $n \geq 1$

105. $\frac{1}{2} + \frac{1}{6} + ... + \frac{1}{n(n-1)} = 1 - \frac{1}{n}$

106. $4^{2n} - 1$ is divisible by 5 for $n \geq 1$

107. Prove by mathematical induction that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ hence evaluate $\sum_{r=1}^{20} r(r+1)$.

COMBINATIONS AND PERMUTATIONS

108. Compute the sum of four-digit numbers formed with the our digits 2,5,3,8 if each digit is used only once in each arrangement.

109. Find the possible number of ways of arranging the letters of the word DIFERENTIATION in a line.

110. In how many ways can the letters in the word BUNDESLIGA be arranged if;

a) There is no restriction.
b) The vowels are together.

111. A committee of 5 people is to be chosen from 4 men and 6 women. Wilson is one of the four men and Martha is one of the 6 women. Find he number of different committees that can be chosen if Wilson and Martha refuse to be on the committee together.

112. A committee of four pupils is to be selected from three boys and seven girls. How many committees are formed in order to have girls as the majority in the committee?

113. How many ways can the word SUCCEEDED be arranged when the vowels are not together.

114. A committee of six members is to be chosen from among five men and three women such that at least two members of each group serve on the committee. Find the number of possible committees that can be formed.

BINOMIAL EXPANSION

115. Expand $(3-4x)^5$ in ascending order of x up to and including the term in x^3. Hence evaluate $(4.96)^5$ correct to 2 d.p.

116. The binomial expansion of $(1+kx)^n$ is $1-6x+30x^2+...$
Find the value of k and n

117. Given that then ratio of the 3rd to the 4th term of the expansion $(2+3x)^n$ is 5:14, find the value of n when $x=\frac{2}{5}$. (n=16)

118. Expand $\sqrt{\left(\frac{1-x}{1+x}\right)}$ in ascending powers of x up to a term in x^2.

119. Expand $\sqrt{1+8x}$ up to the term in x^3. Using $x=\frac{1}{100}$, Show that $\sqrt{3}=1.73205$.

120. Expand $\sqrt{\left(\frac{1-2x}{1+2x}\right)}$ up to the term in x^3. Hence by letting $x=\frac{1}{16}$. Estimate $\sqrt{7}$ correct to three decimal places.

121. Use the Binomial theorem to expand $\sqrt[4]{(1+2x)}$ up to the term in x^3. Hence evaluate $\sqrt[4]{83}$ correct to three decimal places.

122. Expand $\sqrt{\left(\frac{1+2x}{1-2x}\right)}$ up to and including the term in x^3. Hence find the value of $\sqrt{\frac{1.02}{0.98}}$ to four significant figures. Deduce the value of $\sqrt{51}$ to three significant figures.

123. a) Using the binomial expansion of $\sqrt{1+x}$ up to the term in x^3. Hence find the value of $\sqrt{1.08}$ to four decimal places.

b) Express $\sqrt{1.08}$ in the form $\frac{a}{b}\sqrt{c}$. Hence evaluate $\sqrt{3}$ correct to 3 significant figures.

124. Expand $(1-x)^{\frac{1}{3}}$ in ascending powers of x up to the term in x^3. Use the expansion to evaluate $\sqrt[3]{998}$.

125. a) Show that $\frac{1}{\sqrt{4-x}}=\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$.

b) Write the first three terms in the binomial expansion of $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ in ascending powers of x stating the range for which the expression is valid.

c) Find the first three terms in the expansion of $\frac{2(1+x))}{\sqrt{4-x}}$ in ascending powers of x for small values of x.

126. Determine the term independent of x in the expansion $\left[2x-\frac{1}{x^2}\right]^{12}$

127. Five million shillings are invested each year at a rate of 15% per annum. In how many years will it accumulate to more than 50 millions?

128. A man deposits Shs 800,000 into his savings account on which interest is 15% per annum. If he makes no withdraws, after how many years will his balance exceed Shs 8 millions. (Ans: 16.5 years)

129. A man deposits Shs. 200,000 at the beginning of every year in a savings group for seven years. If the Savings group offers a compound interest of 5% per annum, how much will he receive if he withdraws his savings at the end of the seventh year?

DIFFERENTIATION

Differentiate the following from first principles.

130. $y = 3x^2 + 5$
131. $y = 3e^{2x}$
132. $y = e^{3x^2}$
133. $y = \tan 2x$
134. $y = \sec 2x$
135. $y = \ln(4x^2 + 3)$
136. $y = 4^x$
137. $y = \log_2 \tan x$
138. $y = \ln \tan 4x$
139. $y = \ln(2x - 5)$
140. $\ln(\sec 2x + \tan 2x)$
141. $y = \ln \cos kx$
142. $y = \ln \tan x^2$
143. $y = 5^{\sin x}$

144. If $y = 2\sin\theta$ and $x = \cos 2\theta$. Show that $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$

145. Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x.

146. Given that $e^x = \tan 2y$, show that $\frac{d^2y}{dx^2} = \frac{e^x - e^{3x}}{2(1+e^{2x})^2}$.

147. Given that $y = \frac{1}{x^2}$ find $\frac{dy}{dx}$ from first principles.

148. Given that $y = \ln\left[x\sqrt{(x+1)^3}\right]$, find $\frac{dy}{dx}$.

149. Given that $y = \sqrt{\frac{(x+1)^3}{x+2}}$, show that $\frac{dy}{dx} = \frac{(2x+5)\sqrt{(x+1)}}{2\sqrt{(x+2)^3}}$

150. Given that $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, show that $\frac{dy}{dx} = \frac{-1}{1-x^2}$.

151. Given that $y = \ln\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$, show that $\frac{dy}{dx} = \frac{x^2-1}{x^2-4}$.

152. If $x^2 + y^2 = 2y$, show that $(1-y)^3 \frac{d^2y}{dx^2} = 1$

153. If $y = \sec\beta$. Show that $\frac{d^2y}{d\beta^2} = y(2y^2 - 1)$

154. If $y = \sin m\theta$ show that $\frac{d^2y}{d\theta^2} + m^2 y = 0$

155. If $y = (\sec\theta + \tan\theta)^n$ where n is a positive integer. Show that $\frac{dy}{d\theta} = ny\sec\theta$.

156. If $y = \sqrt{1-\theta^2}\sin^{-1}\theta$ show that $(1-\theta^2)\frac{dy}{d\theta} = 1 - \theta^2 - \theta y$.

157. If $y = \tan x$, show that $\frac{d^2y}{dx^2} = 2y + 2y^3$.

158. If $y = \cot^2\theta$ show that $\frac{d^2y}{d\theta^2} = 2(1+y)(1+3y)$

159. If $y = \tan x + \frac{1}{3}\tan^3 x$, prove that $\frac{dy}{dx} = (1 + \tan^2 x)^2$.

160. If $y = \sqrt{(4 + 3\sin x)}$ prove that $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$.

161. If $y = \frac{\cos x}{x^2}$, prove that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 + 2)y = 0$.

162. If $z = \left[v + \sqrt{(1+v^2)}\right]^p$ show that $(1+v^2)\frac{d^2y}{dv^2} + v\frac{dz}{dv} - p^2 z = 0$.

163. Given that $y = e^{-2x}\cos 4x$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0$.

164. If $y = \frac{\log_e x}{x}$. Show that $\frac{dy}{dx}$ is zero when $x = e$.

165. If $y = ae^{px} + be^{-px}$ show that $\frac{d^2y}{dx^2} = p^2 y$.

166. If $y = e^{2x}$ show that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

167. Find m if $y = e^{mx}$ is such that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$.

168. If $y = \sin(\log_e x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

169. Show that if $y = xe^x$, $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

170. Show that if $y = e^{-2x}\sin 5x$, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 0$

171. If $y = 2e^{-4x} - e^{3x}$ show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = ke^{3x}$ where k is a constant, state the value of k.

172. If $y = \log_e(1+\cos x)$, show that $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \bullet \frac{dy}{dx} = 0$.

173. If $y = \exp(\tan^{-1} x)$, show that $(1+x^2)\frac{d^2y}{dx^2} - (1-2x)\frac{dy}{dx} = 0$.

174. If $y = e^{-2x}\cos 4x$. Show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx}\right)^2 \cot 2y$.

175. If $y = \tan^{-1}(e^x)$ show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0$

176. Given that $y = e^x \sin x$, show that and $\frac{dy}{dx} = \sqrt{2}e^x \sin\left(x + \frac{\pi}{4}\right)$ and $\frac{d^2y}{dx^2} = 2e^x \sin\left(x + \frac{\pi}{2}\right)$.

177. The period, t of a swing of a simple pendulum of length, l is given by the equation $T^2 = \frac{4\pi^2 1}{g}$ where g is the acceleration due to gravity. An error of 2% is made in measuring the length, l. Determine the resulting percentage error in the period, T.

178. A rectangular field of area 7200m^2 is to be fenced using a wire mesh. On one side of the field is a straight river. This side of the field is not to be fenced. Find the dimensions of the field that will minimize the amount of wire mesh to be used.

179. A right circular cylinder is inscribed inside a sphere of given radius a. Show that then volume of the cylinder is $\pi h\left(a^2 - \frac{3}{4}h^2\right)$, where h is the height of the cylinder.

180. A right circular cone is inscribed in a sphere of given radius a. Show that the maximum volume of the cone is $\frac{32\pi a^3}{81}$.

181. An open cylinder (closed at one end) is made from a 12 m^2 thin sheet of metal. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}m^3$.

182. A right circular cone of vertical angle 2θ is inscribed in a sphere of radius a. Show that the area of the curved surface of the cone is $\frac{8\pi a^2}{3\sqrt{3}}$.

183. A wire of given length is cut into two portions which are bent into the shape of a circle and a square respectively. Show that the sum of the areas of circle and the square will be least when the side of the square is equal to the diameter of the circle.

184. A hemi spherical bowl of radius r cm is being filled with water at a steady rate. When the depth of the water is h cm, the volume of the water is $\frac{\pi h^2(r-h)}{3}cm^3$. Find the rate at which the water level is rising when it is half way to the top, given that $r = 6cm$ and the bowl fills in 1 minute.

185. Show that for a right circular cylinder of s given total surface area and maximum volume is such that its height is equal to the diameter of its base.

186. A container is in the form of an inverted right circular cone. Its height is 100cm and base radius is 40cm. the container is full of water and has a small hole at its vertex. Water is flowing through a hole at a arate of 10 cm^3s^{-1}. Find the rate at which the water level in the container is falling when the height of water in the container is halved.

187. The area of a circle increases at a rate of $40\pi cm^2s^{-1}$ at an instant when its circumference is 10π cm. Find the rate at which the circumference increases at this instant.

188. A rectangular sheet is 50cm long and 40cm wide. A square of x cm by x cm is cut off from each corner. The remaining sheet is folded to form an open box. Find the maximum volume of the box.

189. A rectangle is known to be twice as long as it is wide. If the width is measured as 20cm ± 0.2cm. Find the area in the form of $A \pm b$.

190. Use Maclaurin's theorem to expand $f(x) = \frac{1}{\sqrt{(1+x)}}$ up tot the term in x^3.

191. Find the first three non-zero terms of the expansion $\log_5(1+e^x)$ by Maclaurin's theorem.

192. Use Maclaurin's theorem to expand $\ln\left(\frac{1-x}{1+x^2}\right)$ up tot eh term in x^2.

193. Given that $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$. Hence find the first four non-vanishing terms of the Maclaurin's expansion of y.

194. Using Maclaurin's expansion, obtain the first four non zero terms of the expansion of $\ln(1+\cos x)$

195. Use Maclaurin's theorem to expand $\ln(1+ax)$ in ascending powers up to the term in x^3. Hence expand $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right)$ and give the values of x for which the expansion is valid.

196. Use Maclaurin's theorem to expand $e^{4x}\cos 3x$ as far as up to the term in x^3.

197. Use Maclaurin's theorem to obtain the expansion for $\log_3(1+e^x)$giving the first four non-vanishing terms.

198. Given that $f(x) = e^{2x}\sin 2x$. Show that $f''(x) = 4(f'(x) - 2f(x))$. Hence or otherwise find the first four non-vanishing terms of the Maclaurin's expansion of $f(x)$.

199. Use Maclaurin's theorem to obtain the expansion for $e^{4x}\cos 3x$ up to the term in x^3.

200. Using Maclaurin's, obtain the expansion for $\tan^{-1}(3^x)$ stating the first two non-zero terms. Hence evaluate $\int_0^2 \tan^{-1}(3^x)dx$ leaving your answer in exact form.

201. Using Maclaurin's theorem show that the series expansion up to the term in x^3 for $e^{-x}\sin x$ is $\frac{1}{3}x(x^2 - 2x + 3)$. Hence evaluate $\exp(-\pi/3)\sin(\pi/3)$ to four decimal places.

202. Find the first four terms of the expansion $\frac{1}{1+x}$ using Maclauren's theorem.

203. Use Maclaurin's theorem to expand $\tan x$ in ascending powers of x up to up to and including the term in x^3.

204. Use Maclaurin's theorem to express $\ln(\sin x + \cos x)$ as a power series up to the term in x^2.

205. a) Given that $f(x)=\ln(1+ax)$. Expand $f(x)$ up to and including the term in x^3.

b) Hence expand $f(x)=\ln\left[(1-3x)^2/(1+2x)\right]$ up to x^3. For what range of values of x is the expansion va

INTEGRATION

Integrate;

206. $\int \sin 3x\sqrt{\sec 3x}\,dx$

207. $\int \frac{7x^2}{\sin^2(2x^3+5)}dx$

208. $\int \frac{1}{x^4\sqrt{x^2-1}}dx$

209. $\int \tan x\sqrt{\sec x}\,dx$

210. $\int \frac{\cos x}{4+\sin^2 x}dx$

211. $\int \frac{(\tan^{-1}2x)^4}{1+4x^2}dx$

212. $\int \frac{\cos^3 x}{\sqrt{\sin x}}dx$ using $u=\sin x$

213. $\int x\ln(2x)dx$

214. $\int \frac{\cos x}{4+\sin^2 x}dx$

215. $\int e^x \sin 2x\,dx$

Evaluate;

216. $\int_0^{\pi/3}(1+\cos 3x)^2 dx$

217. $\int_0^{\pi/2}\frac{4}{3+\cos\theta}dx$

218. $\int_4^8 \frac{\sqrt{x-2}}{x}dx$

219. $\int_{4/3}^{12/5}\frac{dx}{x(1+x^2)^{3/2}}$ using the substitution $u=+\sqrt{1+x^2}$

220. $\int_2^5 \frac{2(x+1)}{2x^2-3x+1}dx$

221. $\int_0^{\frac{\pi}{2}}\frac{\sin x}{3+5\cos x}dx$

222. $\int_0^1 \frac{2x-1}{(x-3)^2}dx$

223. $\int_3^5 x\sqrt{x-1}\,dx$

224. $\int_1^2 \frac{dx}{x^2\sqrt{x-1}}$ using the substitution $x=\sec^2\theta$.

225. $\int_1^2 \frac{1}{x^2+6x+5}dx$

226. $\int_0^{\sqrt{\frac{\pi}{2}}} x\cos x^2 dx$

227. $\int_0^{\frac{\pi}{2}}\tan^2\frac{x}{2}dx$

228. $\int_{1}^{2}\frac{dx}{x^2\sqrt{5x^2-1}}$,using;

a) using the substitution $x^2=\frac{1}{u}$

b) the sine substitution

229. $\int_{0}^{\sqrt{\frac{\pi}{2}}}\frac{x}{1+\sin(x^2)}dx$

230. $\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{(5x^3-12x+4)}{\sqrt{1-x^2}}dx$

231. $\int_{0}^{\frac{\pi}{3}}\frac{1}{1+8\cos^2 x}dx$ using the substitution $t=\tan x$

232. $\int_{0}^{4}\frac{dx}{x^2\sqrt{(25-x^2)}}$

233. $\int_{0}^{\pi/2}x\sin^2 2x\,dx$

234. $\int_{0}^{\pi/2}\frac{dx}{1+\sin x+\cos x}$ using the substitution $t=\tan\frac{x}{2}$.

Show that;

235. $\int_{1}^{2}\frac{2x^3-1}{x^2(2x-1))}dx=\frac{3}{2}+\frac{1}{2}\ln\left(\frac{16}{27}\right)$

236. $\int_{0}^{1/2}\frac{x^4dx}{\sqrt{1-x^2}}=\frac{(4\pi-7\sqrt{3})}{64}$

237. $\int_{0}^{1/2}(1-x^2)^{1/2}dx=\frac{(2\pi+3\sqrt{3})}{24}$

238. $\int_{0}^{1}\frac{x}{(x+1)^{\frac{1}{2}}}dx=\frac{2}{3}(2-\sqrt{2})$

239. Express $\frac{2-x+x^2}{(x+1)(1-x)^2}$ in partial fractions hence evaluate $\int_{0}^{\frac{1}{2}}\frac{2-x+x^2}{(x+1)(1-x)^2}dx$ to three decimal places.

240. Express $f(x)=\frac{5x^2-8x+1}{2x(x-1)^2}$ into partial fractions hence show that $\int_{4}^{9}f(x)dx=\ln\left(\frac{32}{3}\right)-\frac{5}{24}$.

241. Use partial fractions to evaluate $\int_4^6 \frac{dx}{x^2-2x-3}$.

242. Express $y=\frac{x^4+2x}{(x-1)(x^2+1)}$ into partial fractions hence evaluate $\int ydx$.

243. Express $\frac{x^2+5x+11}{(x+1)(x^2+4)}$ in partial fractions hence evaluate $\int_1^4 \frac{x^2+5x+11}{(x+1)(x^2+4)}dx$.

244. Express $f(x)=\frac{x^3+4x^2-5x-4}{(x-2)^2(1+x^2)}$ into partial fractions, hence evaluate $\int_3^5 f(x)dx$.

245. Express $\frac{x^2}{x^4-1}$ into partial fractions, hence find $\int \frac{x^2}{x^4-1}dx$.

246. Using the substitution $u=\tan^{-1}x$, show that $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx=\frac{\pi^2}{32}$.

247. Find the volume of the solid of revolution generated when the area under $y=\frac{1}{x-2}$ from $x=3$ to $x=4$ is rotated through four right angles about the x-axis.

248. The area enclosed by the curve $x^2+y^2=a^2$, the y-axis and the line $y=-\frac{1}{2}a$ is rotated through 90° about the y axis. Find the volume of the solid generated.

249. Find the volume of the solid generated by rotated about the y-axis, the area enclosed by the curve $y^2+4x=9$, the y-axis and $y=-2$.

250. The curve $y=x^3+8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B and C and hence find the area enclosed between the curve and the line.

251. A hemispherical bowl of internal radius 15 cm contains water to a depth of 7 cm. Find the volume of the water in the bowl correct to 1 decimal place.

Show by integration that the volume of;

252. a sphere of radius r is $\frac{4}{3}\pi r^3$.

253. a right circular cylinder of height h and radius r is $\pi r^2 h$.

254. a right circular cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$ .sss

DIFFERENTIAL EQUATIONS

Solve the differential equation:

255. $xy\frac{dy}{dx} = x^2 + y^2$.

256. $x\frac{dy}{dx} + y = xe^{-2x}$

257. $\frac{dy}{dx} = 4x - 7$, given that $y(2) = 3$.

258. $\operatorname{cosec}x\frac{dy}{dx} = e^x \operatorname{cosec}x + 3x$, given $y\left(\frac{\pi}{3}\right) = 3$.

259. $\frac{dy}{dx} + y\tan x = 1$, given that $y = 2$ when $x = 0$.

260. $\frac{dr}{d\theta} + 2r\tan\theta = \sec^2\theta$

261. $\frac{dy}{dx} - y\tan x = \cos x$, given $y = 0$ at $x = \frac{1}{2}$.

262. $x\frac{dy}{dx} - 2y = (x-2)e^x$

263. $\frac{dy}{dx} + 2(y+1)\cot x = \sin 2x$,

264. $\frac{dx}{dt} = \frac{1}{x+2} - \frac{3}{3x+5}$ given that $x = 1$ when $t = 2$.

265. $x\frac{dy}{dx} + 2y = x^2$ if $y(1) = 1$.

266. $x^2\frac{dy}{dx} = x^2 + xy + y^2$

267. $x\frac{dy}{dx} = y + kx^2\cos x$ given that $y = 2\pi$ when $x = \pi$.

268. By eliminating the constants A and B, form a differential equation for which $Ae^{3t} + Be^{-2t}$ is a solution.

269. A disease is spreading at a rate proportional to the product of the number of people already infected and those who haven't yet been

infected. Assuming that the total number of people exposed to the disease is N;

a) Write down a differential equation

b) Initially 20% of the population is infected. Two months later 40% of the population is infected. Determine how long it takes for only 25% of the population to remain uninfected.

270. Find the equations of the two curves which pass through the point $(1,1)$ and which satisfy the differential equation $\tan(y-1)\frac{dy}{dx}=\tan(x-1)$.

271. The tangent at any point $P(x,y)$ on the curve, cuts the x-axis at A and the y-axis at B. Given that $2AP=PB$ and that the curve passes through the point $(1,1)$, find the equation of the curve.

272. In established forest fires, the proportion of the total area of the forest which has been destroyed is denoted by x and the rate of change of x with respect to time t hours is called the destruction rate. Investigations show that the destruction rate is directly proportional to the product o x and $(1-x)$. A particular fire is initially noticed when half of the forest is destroyed and it's found out that the destruction rate at this time is such that if it remained constant thereafter, the forest would be destroyed completely in a further 24 hours.

Show that $12\frac{dy}{dx}=x(1-x)$. and deduce that approximately 73% of the forest I destroyed 12 hours after it was noticed.

273. According to Newton's law of cooling, the rate at which a hot object cools is proportional to the difference between the temperature of the surrounding air (assume to be constant). If an object cools from 100°C to 80°C in 10 minutes in the surrounding air temperature of 20°C, find the rate at which the object's temperature is falling when the time elapsed is $\frac{10\ln 2}{\ln(4/3)}$ minutes.

274. The rate at which a liquid runs out of a container is proportional to the square root of the depth of the outlet below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank, originally full is found to drop by 20 cm in 1 hour, and by 19 cm in the next hour. Find the depth at which the leak is located.

275. The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time. Initially he had V_o supporters and t weeks later, he had V supporters.

a) Form a differential equation connecting V and t.

b) Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in (a) above.

c) Find how long it will take for the candidate to remain with 20% of the initial supporters.

276. A research to investigate the effect of school fees on the students enrollment in schools revealed that the number of students enrolled increased with a reduction in school fees and was proportional to the number of student enrolled. A school charging 80 dollars per term had 300 students while the one charging 60 dollars had 450 students. Estimate the number of students in school X whose school fees was 40 dollars per term, given that all schools are assumed to provide the same quality of education.

277. A liquid is being heated in an oven maintained at a constant temperature of 180°C. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180-\theta)$ where θ°C is the temperature of the liquid at time, t, minutes. If the temperature of the liquid rises from 0°C to 120°C in 5 minutes. Show that $\frac{180}{180-\theta}=3^{t/5}$.

Hence find the temperature of the liquid after a further 5 minutes.

278. An election survey revealed that during parliamentary campaigns in a certain constituency with total electorate, V_o , Wesley, a parliamentary candidate was gaining support at a rate proportional to the number of voters, V, already supporting him at time, t days and those not yet supporting him.

a) Write down a differential equation for the given findings.

b) The total electorate was 2,000,000 and a candidate could win with at least 51% of the votes. Given that initially, Wesley had 100,000 supporters and was gaining 5,000 voters per day, determine the least number of days he would require to campaign in order to be sure of winning.

279. An infectious disease spreads through a farmers' heard at a rate which is proportional to the product of the number of cows that are infected

and that one of those which are not yet infected. A farmer has a heard of 900 cows. Given that initially 15 cows are infected and after 3 days, 90 cows are infected,

a) Write down a differential in terms of the number N of infected cows at any time t in days.
b) Solve the differential equation in (a) above and give your solution in terms of t.
c) How long (in days) does it take for 90% of the heard to be infected.

280. A perfect elastic spherical balloon I being deflated at a rate proportional to its surface area. Given that within 4 hours, its volume reduces half way. How long will it take for it to be fully deflated?

281. A moth ball evaporates at a rate proportional to it volume, loosing half of its volume every after four weeks. If the volume of the moth ball is initially 15 cm^3 and becomes ineffective when its volume reaches 1 cm^3, how long is the moth ball effective?

282. A liquid cools in the environment of a constant temperature of 21^0C at the rate proportional to the excess temperature. Initially the temperature of the liquid is 100^0C and after 10 minutes the temperature dropped by 16^0C. Find how long it takes for the temperature of liquid to cool to 20^0C.

283. A certain chemical reaction is such that the rate of transformation of the reacting substance is proportional to its concentration. If initially the concentration of the reagent was 9.5 grams per litre and if after 5 minutes the concentration was 3.5 grams per litre, find what the concentration was after 2 minutes.

284. A police patrol on Jinja road found a dead body lying in the middle of the road at Banda at 7:00 am and its body temperature was 30°C. Ten minutes 28.5°C, the air temperature was 20°C. The body temperature loses heat at a rate proportional to the difference between the body temperature T and the surrounding temperature T_o. If the normal body temperature is 37°C, estimate the time when the man was killed.

285. The rate at which the temperature of a body placed in an oven increases at any instant is proportional to the amount by which the temperature of the oven exceeds the temperature of the body at that instant. The temperature of the oven is 120°C. Given that the temperature of the

body rises from 50°C to 80°C in 6 minutes, how long does the temperature of the body take to rise from 90°C to 99°C?

286. A sample of a radioactive substance loses mass at a rate, which is proportional to the amount present. If M is the mass after time t, where t is in years;

a) Form a deferential equation connecting M, t and a constant k.

b) If initially the mass of the substance is M_o deduce $M = M_o e^{-kt}$

c) Given that its initial mass halved in 1600 years, determine the number of years it takes 15g of the substance to reduce to 13.6g?

VECTORS

287. Find the angle between the lines $\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z+3}{1}$ and $\mathbf{r} = (2+2\lambda)\mathbf{i} + (1+3\lambda)\mathbf{j} + (6\lambda-1)\mathbf{k}$.

288. Given that A(1,3,2), B(2,-1,1), C(-1,2,3) and D(-2,6,4) are vertices of a parallelogram, find the area of the parallelogram ABCD.

289. The directional vectors $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} + p\mathbf{k}$ and $\mathbf{c} = 9\mathbf{i} + 9\mathbf{j}$ are such that $\mathbf{a}$ is perpendicular to $\mathbf{b}$. Find the;

a) value of the scalar p.

b) angle between $\mathbf{b}$ and $\mathbf{c}$.

290. The points A(a,-3,6) , B(2,b,2) and C(3,3,0) lie on a straight line. Find the values of a and b.

291. The points P, Q and R have position vectors $\mathbf{p} = 5\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $\mathbf{q} = -\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ respectively. Show whether or not P, Q and R are vertices of a triangle.

292. Find the acute angle between the lines $\frac{x-3}{5} = \frac{y+1}{-4} = \frac{z}{2}$ and

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

293. Find the point of intersection of the lines $\frac{x-25}{9} = \frac{y}{7} = \frac{z+13}{2}$ and $\frac{x+26}{-6} = \frac{y-7}{7} = \frac{z-13}{8}$.

294. The line $r=\begin{pmatrix}1\\2\\g\end{pmatrix}+\lambda\begin{pmatrix}h\\1\\-2\end{pmatrix}$ is parallel to the plane $x+2y+2z=2$ and the perpendicular distance of the line from the plane is 3 units. Find the value of h and the possible values of g.

295. Find the shortest distance from the point (2,-1,3) to the line $\frac{\mathbf{x}-2}{-1}=\frac{\mathbf{y}+1}{3}=\frac{\mathbf{z}+4}{2}$.

296. Find the point of intersection of the planes $\mathbf{x}+\mathbf{y}-\mathbf{z}=-4$, $2\mathbf{x}-3\mathbf{y}+2\mathbf{z}=15$ and $5\mathbf{x}+2\mathbf{y}+\mathbf{z}=1$.

297. Find the equation of the plane that passes through (1,-2,2) and containing the line $\frac{\mathbf{x}-9}{4}=\frac{\mathbf{y}-6}{-1}=\frac{\mathbf{z}-8}{1}$.

298. Find the acute angle between the line $\frac{\mathbf{x}-6}{5}=1-\mathbf{y}=1+\mathbf{z}$ and the plane $7\mathbf{x}-\mathbf{y}+5\mathbf{z}+5=0$ giving your answer to the nearest degree.

299. Show that the three planes; $2\mathbf{x}+3\mathbf{y}+\mathbf{z}=8$, $\mathbf{x}+\mathbf{y}+\mathbf{z}=10$ and $3\mathbf{x}+5\mathbf{y}+\mathbf{z}=6$ contain a common line. Find the equation of the line.

300. Find the coordinates of the point where the line $\frac{\mathbf{x}-3}{5}=\frac{3-\mathbf{y}}{-2}=\frac{\mathbf{z}-4}{3}$ meets the plane $2\mathbf{x}-3\mathbf{y}+7\mathbf{z}-10=0$

301. M is a point which divides line AB externally in the ratio of 4:3. A is (1,4,1) and B is (-1,-1,3). Find the Cartesian equation of the line through M and N(2,1,0).

302. Given $\mathbf{OT}=\begin{pmatrix}5\\3\\-1\end{pmatrix}$ and $\mathbf{OS}=\begin{pmatrix}2\\0\\7\end{pmatrix}$**.** Find the coordinates of R such that $TR:TS=2:3$

303. Show that the line $\frac{\mathbf{x}-2}{4}=\frac{\mathbf{y}-1}{9}=\frac{\mathbf{z}+3}{5}$ is parallel to the plane $3\mathbf{x}+2\mathbf{y}-6\mathbf{z}+9=0$. Hence find the shortest distance between the lie and the plane.

304. The line L has its vector equation $\mathbf{r}=\mathbf{i}-\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}-2\mathbf{k})$. A plane P has its scalar product equation $\mathbf{r}.(6\mathbf{i}-2\mathbf{j}+\mathbf{k})=-3$.

a) Show that the line L is parallel to plane P.

b) Determine the Cartesian equation of another plane M that is parallel to plane P and contains the line L.

c) Find the perpendicular distance from the line to the plane P.

305. The plane $\mathbf{P}_1$ has equation $3\mathbf{x}-4\mathbf{y}+2\mathbf{z}=5$ and plane $\mathbf{P}_2$ has equation $\mathbf{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\5\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$.

a) find the Cartesian equation of plane $\mathbf{P}_2$.

b) obtain the acute angle between the planes.

c) find the vector equation of intersection of the two planes.

306. The lines $\mathbf{x}=\frac{\mathbf{y}-1}{2}=\frac{\mathbf{z}-21}{-3}$ and $\mathbf{r}=(12+3\lambda)\mathbf{i}+(7-3\lambda)\mathbf{j}+(1-\lambda)\mathbf{k}$ lie in the same plane.

a) Show that the lines intersect and calculate the angle between them.

b) Find the equation of the plane where both lines lie.

307. **A** and **B** are the points (3,1,1) and (5,2,,3) respectively, and **C** is a point on the line $\mathbf{r}=\begin{pmatrix}2\\4\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$. If angle BAC = 90^0 , find the coordinates of C.

308. The vector $\begin{pmatrix}4\\1\\2\end{pmatrix}$ is perpendicular to the plane containing the line $\frac{\mathbf{x}-3}{-2}=\frac{\mathbf{y}+1}{a}=\frac{\mathbf{z}-2}{1}$, find the;

(i) value of a.

(ii) Cartesian equation of the plane.

309. A plane $\mathbf{P}_1$ passing through the points (1,-1,0) and (1,0,-3) is perpendicular to the plane $\mathbf{P}_2$ having the equation: $\mathbf{x}+\mathbf{y}-6\mathbf{z}=0$. Find;

a) The equation of $\mathbf{P}_1$

b) The acute angle between $\mathbf{P}_1$ and another plane $\mathbf{P}_3$ with equation; $\mathbf{x}-\mathbf{y}+\mathbf{z}=7$.

310. Determine equation of the plane equidistant from the points A(1,3,5) and B(2,-4,4).

311. (a). Find the coordinates of the point, P, in which the plane $4\mathbf{x}+5\mathbf{y}+6\mathbf{z}=87$ intersects the line $\frac{\mathbf{x}-2}{3}=\frac{\mathbf{y}-3}{4}=\frac{\mathbf{z}+4}{2}$.

(b). Calculate the angle between the line and the plane in (b) (i) above.

312. Find , in vector form, the equation of a line passing through the point (1,1,3) and perpendicular to the plane $2\mathbf{x}+3\mathbf{y}+3\mathbf{z}=7$.

313. Find the position vector of the point of intersection of the lines $\mathbf{r}_1=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}5\\-3\\2\end{pmatrix}$ and $\mathbf{r}_2=\begin{pmatrix}9\\2\\2\end{pmatrix}+\mu\begin{pmatrix}-3\\5\\-1\end{pmatrix}$. Write down the vector equation of the plane containing the lines $\mathbf{r}_1$ and $\mathbf{r}_2$ hence or otherwise find the Cartesian equation of the plane containing lines $\mathbf{r}_1$ and $\mathbf{r}_2$.

314. a) Calculate the angle between the line $\frac{2-\mathbf{x}}{3}=\mathbf{y}=\frac{6+3\mathbf{z}}{-6}$ and the plane $\mathbf{r}=\begin{pmatrix}1\\1\\7\end{pmatrix}+\lambda\begin{pmatrix}5\\-4\\8\end{pmatrix}+\mu\begin{pmatrix}-2\\0\\3\end{pmatrix}$.

b) Find the Cartesian equation of the plane passing through the midpoint of AB with A(-1,2,-5) and B(3,0,-1) which is perpendicular to the line in a) above. Hence, find the line of intersection of this plane and the plane in a) above.

315. a) The lines $\mathbf{L}_1$ and $\mathbf{L}_2$ are given by the equation $\frac{\mathbf{x}-1}{\mathbf{k}}=\frac{\mathbf{y}}{-1}=\frac{\mathbf{z}+3}{1}$ and $\frac{\mathbf{x}-4}{1}=\frac{\mathbf{y}+3}{1}=\frac{\mathbf{z}+3}{2}$ respectively. Find the;

i) value of k for which the two lines intersect.

ii) point of intersection of the lines and the obtuse angle between them.

b) Find the equation of the plane that contains both lines in a) above.

COMPLEX NUMBERS

316. Express $\frac{(4\sqrt{3}-4\mathbf{i})(4+4\mathbf{i}\sqrt{3})}{1-\mathbf{i}}$ in the form $x+iy$.

317. Use De Moivre's theorem to evaluate $(3-4i)^{2/3}$

318. Given $z=-1+i\sqrt{3}$, find the value of the real number p such that $\operatorname{Arg}\left(z^2+pz\right)=\frac{5\pi}{6}$.

319. Express $z=\frac{3+i}{1-i}$ in the form $a+bi$, where a and b are integers and hence find the argument of $\overline{Z}$.

320. If $\mathbf{z}_1$ and $\mathbf{z}_2$ are complex numbers, solve the simultaneous equations; $4\mathbf{z}_1+3\mathbf{z}_2=23$ $\mathbf{z}_1+\mathbf{i}\mathbf{z}_2=6+8\mathbf{i}$, giving your answer in the form $x+iy$.

321. Solve for x and y values in the equation $\frac{x}{2+3i}+\frac{y}{3-i}=\frac{6-13i}{9+7i}$

322. Use De Moivre's theorem to show that $\cos 7\theta=64\cos^7\theta-112\cos^5\theta+56\cos^3\theta-7\cos\theta$

323. Use De Moivre's theorem to simplify $\frac{(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)}{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}$

324. If $a+ib$ is a root of the quadratic equation $x^2+cx+d=0$, show that $a^2+b^2=d$ and $2a+c=0$.

325. Find x and y if $(x+2i)(1-yi)=(3-i)^2$

326. Solve $Z\overline{Z}-5iZ=5(9-7i)$, where $\overline{Z}$ is the complex conjugate of Z.

327. Use De Moivre's theorem to show that $16\sin^5\theta=\sin 5\theta-5\sin 3\theta+10\sin\theta$

328. Use De Moivre's theorem to find the cube roots of $6\mathbf{i}-4$.

329. a) A complex number z has modulus 1 and argument $\frac{2\pi}{3}$. Find the fourth roots of Z.

330. Use De-Moivre's theorem to show that: $\tan 4\theta=\frac{4t-4t^3}{1-6t^2+t^4}$ where $t=\tan\theta$. Hence, find the roots of the equation $t^4+4t^3-6t^2-4t+1=0$. Correct to 3.s.f**.**

331. If $z=x+iy$, determine the Cartesian equation of the locus given by $\left|\frac{z-1}{z+1-i}\right|=\frac{2}{5}$

332. Use De Moivre's theorem to show that $\tan 5\theta=\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}$

By considering $\tan 5\theta=0$, show that $\tan^2\left(\frac{\pi}{5}\right)=5-2\sqrt{5}$.

333. If $z=\dfrac{4}{1+\cos 2\theta - i\sin 2\theta}$ where $-\dfrac{\pi}{2}\le\theta\le\dfrac{\pi}{2}$, find $|\mathbf{z}|$ and $\arg\mathbf{z}$ as functions of θ

b) Given that z is a complex number, find the equation of the locus $|z-2+i|=|z+1-2i|$,hence sketch the locus.

334. If $\dfrac{|w-1|}{|w+2|}<2$ where $w=x+iy$, represent the locus of w on an Argand diagram.

335. Solve $z^3+1=0$

336. If $\mathbf{z}$ is a complex number. Find the locus of $\mathbf{z}$ and illustrate it on an Argand diagram if $1\le\dfrac{2|z|}{|z-3+6i|}$.

337. Given that $z=\dfrac{1+2i}{1-3i}$. Express z in polar form hence represent it on a complex plane**.**

338. Use De Moivre's theorem to show that

$$\frac{(\cos 3\theta+i\sin 3\theta)^5(\cos\theta-i\sin\theta)^3}{(\cos 5\theta+i\sin 5\theta)^7(\cos 2\theta-i\sin 2\theta)^5}=\cos 13\theta-i\sin 13\theta$$

339. Given the complex numbers $z_1=\dfrac{1+i\sqrt{3}}{2}$ and $z_2=\dfrac{1-i\sqrt{3}}{2}$.

i) Express z_1 and z_2 in polar form.

ii) Find the value of $\mathbf{z}_1{}^5+\mathbf{z}_2{}^5$. (

340. Given that $z=x+iy$, show that the locus $\arg\left(\dfrac{\mathbf{z}-1}{\mathbf{z}+1}\right)=\dfrac{\pi}{4}$ describes a circle . State the centre and radius of the circle.

341. Find the square root of $1-i\sqrt{3}$

342. Show that $2-3i$ is a root of the equation $z^4-5z^3+18z^2-17z+13=0$. Hence find the other roots of equation.

343. Use De Moivre's theorem to show that $\cos 4x=\dfrac{\tan^4 x-6\tan^2 x+1}{\tan^4 x+2\tan^2 x+1}$ and

$$\tan 4x=\frac{4\tan x-4\tan^3 x}{1-6\tan^2 x+\tan^4 x}$$

344. Sketch the loci defined by the equations:

i) $\arg(\mathbf{z}+2)=-\dfrac{2\pi}{3}$

ii) $\arg\left(\frac{\mathbf{z}-3}{\mathbf{z}-1}\right)=\frac{\pi}{4}$

345. Find the values of z in $z^3-8i=0$

346. The arguments of the complex numbers $z-2$ and $z-2i$ differ by $\frac{\pi}{2}$. Find the locus of the point P(x,y) which represents $z=x+iy$ and describe the locus.

347. Given that $z_1=3+2i$ and $z_2=2-i$. Find z_1+z_2, graphically.

348. If $z_1=x+iy$ is a complex number, describe and illustrate on the Argand diagram the locus of $\left|\frac{z+2}{z}\right|=3$

349. Given that $z_1=3-2i$, $z_2=2+i$ and $z_3=4+3i$

a) Express $\frac{z_1+z_3}{z_1z_3}$ in the form $a+bi$ where a and b are real numbers.

b) Find a polynomial $P(x)$ of degree four where the roots of $P(x)=0$ are z_2 and z_3.

350. If $(a+bi)^2=-5+12i$, find a and b given that they are both real. Give the two square roots of $-5+12i$.

351. Express $\frac{(4\sqrt{3}-4i)(4+4i\sqrt{3})}{1-i}$ in the form $x+iy$.

352. Sketch the locus of a point P which is represented by $|z-3+2i|<2|z+2|$ and show the required region.

353. Prove that $\cos^6 x+\sin^6 x=\frac{1}{8}(3\cos^4 x+5)$

354. Solve the equation $2z-i\bar{z}=5-i$ where $z=x+iy$

355. Express the complex number $z=\frac{(3i+1)(i-2)^2}{i-3}$ in the form $a+bi$ where a and b are integers. Hence find;

a) the modulus of z

b) the principle argument of z.

356. Given that the complex number *z* and its conjugate $\bar{z}$ satisfy the equation $z\bar{z}-2z+2\bar{z}=5-4i$ Find the possible values of *z*.

357. Sketch the locus $|z+1-4i|\geq|z-2-i|$ where z is a complex number.

FURTHER CURVE SKETCHING AND INEQUALITIES

Solve the inequalities below;

358. $\frac{x+4}{x+1} < \frac{x-2}{x-4}$

359. $\frac{x^2+x-2}{x^2+4} > \frac{1}{2}$

360. $\frac{6}{1-x} \geq x+4$

361. $\frac{x-2}{x+1} \geq \frac{x+1}{x+3}$

362. $\frac{x-1}{x} > \frac{2}{3-x}$

363. $x^4 - 10x^2 + 9 > 0$

364. $\left|\frac{2x-4}{x+1}\right| < 4$

365. $\frac{2x^2-7x-4}{3x^2-14x+11} > 2$

366. $\left|\frac{2x+5}{x^2-4}\right| \geq \frac{1}{5}$

367. $6 - x > |3x-2|$

SECTION B – TYPE QUESTIONS

368. Show that the curve $y = \frac{2(x-2)}{(x-1)(x-3)}$ has no real turning points. Hence sketch the curve.

369. Given the curve $y = \frac{12}{x^2-2x-3}$ .Determine the;

a) range of values for y in which the curve does not lie and hence find the coordinates of the turning point.

b) asymptotes and hence sketch the curve.

370. The parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ represent a curve.

a) find the Cartesian equation of the curve.
b) determine the turning points of the curve and their nature.
c) state the asymptotes and the intercepts of the curve.
d) hence sketch the curve.

371. Sketch the curve $y = \frac{x+1}{(x-1)(2x+1)}$, showing clearly the asymptotes and turning points.

372. A curve is given by $y = \frac{x^2-6x+5}{x^2-4x+4}$.

a) Find the range of values of y for which the curve does not exist.
b) Sketch the curve.

373. Show that for real x, $0<\frac{4}{x^2+2x+2}\le 4$. Sketch $y=\frac{4}{x^2+2x+2}$.

374. Given the curve $y=\frac{x^2-x-2}{x^2+x-2}$

a). Find the;

i) equations of the three asymptotes of the curve.

ii) stationary point of the curve and determine its nature.

b). Sketch the curve.

375. Show that for real x, the function $f(x)=\frac{x^2-x-6}{x-1}$ can take all real values. Hence, sketch curve $y=f(x)$.

376. A curve is given by the parametric equations; $x=3t$ and $y=\frac{2t^2}{1-t}$.

a) Find the Cartesian equation of the curve.

b) Sketch the curve, showing clearly the asymptotes and turning points.

377. Given the curve $y=\frac{4x-10}{x^2-4}$.

a) Find the range of values of y within which the curve does not lie.

b) Determine the stationary points of the curve.

c) State the equations of the asymptotes and sketch the curve.

Sketch the curve;

378. $y=\frac{x(x-2)}{(x-1)(x+2)}$

379. $y=\frac{x(x+1)}{1+x^2}$

380. $y=\frac{(x+3)^2}{x}$

381. $y=\frac{3x^2+5x-2}{x^2+2}$

382. $y=\frac{9x^2+8x+3}{x^2+1}$

TRIGONOMETRY

Solve the equation;

383. $5\sin 2x-10\sin^2 x+4=0$ for $-180^\circ\le x\le 180^\circ$.

384. $2\sin 2\theta=7\sin\theta$ for $0^\circ\le x\le 360^\circ$.

385. $\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$

386. Express $\cos(\theta+30^\circ)-\cos(\theta+48^\circ)$ in the form $R\sin P\sin Q$. Where R is a constant. Hence solve the equation $\cos(\theta+30^\circ)-\cos(\theta+48^\circ)=0.2$.

387. Prove that in any triangle ABC $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$. Hence solve the triangle in which $a=9, b=5.5$ and $C=57^\circ$.

388. Show that $\sin(x+\alpha)=P\sin(x-\alpha)$ then $\tan x=\left(\frac{P+1}{P-1}\right)\tan\alpha$. Hence solve the equation $\sin(x+20^\circ)=2\sin(x-20^\circ)$for $0^\circ\le x\le 180^\circ$.

389. a) Express $7\cos x-24\sin x$ in the form $R\cos(x+a)$.

b) Write down the maximum and minimum value of the function $f(x)=12+7\cos x-24\sin x$

390. Show that $\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$. Hence solve $\frac{2\tan\theta}{1+\tan^2\theta}=3\cos 2\theta$ for $0\le\theta\le\pi$

391. Prove that $\cos^5 x=\frac{\cos 5x+5\cos 3x+10\cos x}{16}$.

392. Prove that $\tan(A+B+C)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan A\tan C-\tan B\tan C}$.Hence prove that is A, B and C are angles of a triangle, then $\tan A+\tan B+\tan C=\tan A\tan B\tan C$.

393. Prove that $4\cos\theta\cos 3\theta+1=\frac{\sin 5\theta}{\sin\theta}$. Hence find all the values of $\cos\theta\cos 3\theta=-\frac{1}{2}$.

394. Show that $\tan^{-1}(1/3)+\sin^{-1}(1/\sqrt{5})=\pi/4$.

395. Show that $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left[xy-\sqrt{\{(1-x^2)(1-y^2)\}}\right]$

396. Prove that $4\tan^{-1}(1/5)+\sin^{-1}(1/239)=\pi/4$.

397. Solve the equation $\tan^{-1}2x+\sin^{-1}3x=\pi/4$

398. Given that $2A+B=\frac{\pi}{4}$, show that $\tan B=\frac{1-2\tan A-\tan^2 A}{1+2\tan A-\tan^2 A}$.

399. Prove that $\cos^6 x+\sin^6 x=1-\frac{3}{4}\sin^2 2x$.

400. If $a=x\cos\theta+y\sin\theta$ and -+, prove that $\tan\theta=\frac{bx+ay}{ax-by}$.

401. Solve the equation $\tan 4\beta+\tan 2\beta=0$ for $0^\circ\le\beta\le 360^\circ$.

402. Solve the equation $5\tan x+\sec x+5=0$for $0^\circ\le\beta\le 360^\circ$.

403. Solve $3\tan^3 x - 3\tan^2 x = \tan x - 1$ for $0 \le x \le \pi$.

404. Solve the equation $\cos(2\theta + 45^\circ) - \cos(2\theta - 45^\circ) = 1$ for $0^\circ \le x \le 360^\circ$.

405. Given that $\sin(x+\beta) = 2\cos(x-\beta)$, prove that $\tan x = \dfrac{2-\tan\alpha}{1-2\tan\alpha}$.

406. Prove that $(\sin 2\alpha - \sin 2\beta)\tan(\alpha+\beta) = 2(\sin^2\alpha - \sin^2\beta)$.

407. If the roots of $ax^2 + bx + c = 0$ are $\tan\alpha$ and $\tan\beta$. Express $\sec(\alpha+\beta)$ in terms of a, b and c.

408. Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$. Hence, solve the equation $10\sin x\cos x + 12\cos 2x = -7$ for $0^\circ \le x \le 360^\circ$.

For questions If A, B and C are angles of a triangle, prove that;

409. $\sin B + \sin C - \sin A = 4\cos\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

410. $\sin^3 A + \sin^3 B + \sin^3 C = 3\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} + \cos\dfrac{3A}{2}\cos\dfrac{3B}{2}\cos\dfrac{3C}{2}$

411. $\cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2} = \cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2}$

412. $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

413. $\sin A + \sin B + \sin C = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

414. $\cos^2 2A + \cos^2 2B + \cos^2 2C = 1 + 2\cos 2A\cos 2B\cos 2C$

415. $\cos A + \cos B + \cos C = 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

416. $\sin^2\dfrac{1}{2}A + \sin^2\dfrac{1}{2}B + \sin^2\dfrac{1}{2}C = 1 - 2\sin\dfrac{1}{2}A\sin\dfrac{1}{2}B\sin\dfrac{1}{2}C$

417. If $\tan\theta = \dfrac{1}{p}$ and $\tan\phi = \dfrac{1}{q}$ and $pq = 2$ show that $\tan(\theta+\phi) = p+q$.

418. A and B are acute angles such that $\cos A = \dfrac{2}{3}$, and $\operatorname{cosec} B = 5$. Find the value of $\tan(A-B)$. Leave your answer in surd form.

419. Show that $\sin 3A = 3\sin A - 4\sin^3 A$. Deduce that $\sin^3 A + \sin^3(120^\circ + A) + \sin^3(240^\circ + A) = -\dfrac{3}{4}\sin 3A$.

420. Show that $\dfrac{1+\sin x - \cos x}{1+\sin x + \cos x} + \dfrac{1+\sin x + \cos x}{1+\sin x - \cos x} = \dfrac{2}{\sin x}$.

421. Give that $\sin 2\alpha + \sin 2\beta = p$, $\cos 2\alpha + \cos 2\beta = q$ prove that $p/q = \tan(\alpha+\beta)$. Prove also that $\frac{4p}{p^2+q^2+2q} = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$ and deduce an expression for $\tan\alpha\tan\beta$ in terms of p and q.

422. Given that $\tan 3\theta = 2$ evaluate without using tables $\frac{\sin\theta + \sin 3\theta + \sin 5\theta}{\cos\theta + \cos 3\theta + \cos 5\theta}$.

423. If $\sin\theta = \frac{1-x}{1+x}$ show that $\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{x}$.

424. If $\sin\alpha + \sin\beta = p$ and $\cos\alpha + \cos\beta = q$. Show that

i. $\sin(\alpha+\beta) = \frac{2pq}{p^2+q^2}$

ii. $\cos(\alpha+\beta) = \pm\left(\frac{q^2-p^2}{p^2+q^2}\right)$

425. Determine the maximum value of the expression; $6\sin x - 3\cos x$.

426. Show that;

i. $\frac{\sin x \sin y}{\cos x + \cos y} = \frac{2\tan\frac{x}{2}\tan\frac{y}{2}}{1 - \tan^2\frac{x}{2}\tan^2\frac{y}{2}}$

ii. $\frac{\cos x \cos y}{\cos x + \cos y} = \frac{\left(1-\tan^2\frac{x}{2}\right)\left(1-\tan^2\frac{y}{2}\right)}{2\left(1-\tan^2\frac{x}{2}\tan^2\frac{y}{2}\right)}$

427. Show that $\tan(B-C) + \tan(C-A) + \tan(A-B) = \tan(B-C)\tan(C-A)\tan(A-B)$.

428. If $\tan 2\phi - \sin 2\phi = x$ and $\tan 2\phi + \sin 2\phi = y$ show that ;

i. $\frac{x}{y} = \tan^2\phi$

ii. $(x^2 - y^2)^2 = 16xy$

429. Prove that $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

430. Show that $\frac{\cos(2P-3Q) + \cos 3Q}{\sin(2P-3Q) + \sin 3Q} = \cot P$.

431. Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$. Hence find the maximum value of $10\sin x\cos x + 12\cos 2x$.

432. Solve the equation $5\cos\theta - 3\sin\theta = 4$ for $0^\circ \le \theta \le 360^\circ$.

433. If $\sin\theta+\sin\phi=p$ and $\cos\theta+\cos\phi=q$ prove that;

i. $\tan\theta+\tan\phi=\dfrac{8pq}{(p^2+q^2)-4p^2}$

ii. $\cos 2\theta+\cos 2\phi=\dfrac{(q^2-p^2)(p^2+q^2-2)}{p^2+q^2}$

434. Given that $x=\sec A-\tan A$, prove that $\tan\dfrac{A}{2}=\dfrac{1-x}{1+x}$.

435. Given that $\sin 45^{o}=\dfrac{1}{\sqrt{2}}$. Show without a calculator or tables that $\sin 292.5^{o}=\dfrac{\sqrt{2-\sqrt{2}}}{2}$.

436. a) Given that $y=\dfrac{\sin x-2\sin 2x+\sin 3x}{\sin x+2\sin 2x+\sin 3x}$ prove that $y+\tan^2\dfrac{x}{2}=0$.

b) Hence express the exact value of $\tan^2 15^{o}$ in the form $p+q\sqrt{r}$ where p,q and r are integers.

c) hence, find the value of x in the range $0^{o}\le x\le 360^{o}$ for which $2y+\sec^2\dfrac{x}{2}=0$

437. Given that $p=2\cos 2x+3\cos 4x$ and $q=2\sin 2x+3\sin 4x$;

a) Find the greatest and least value of p^2+q^2.

b) Given that $p^2+q^2=19$, find x for $0^{o}\le x\le 90^{o}$.

c) Using the result in b) above, and without using a calculator or mathematical tables, show that $pq=\dfrac{-5\sqrt{3}}{4}$.

438. Express $4\sin\theta-3\cos\theta$ in the form $R\sin(\theta-\alpha)$; where R is a constant and α is an acute angle. Hence solve the equation $4\sin\theta-3\cos\theta+2=0$, for $0^{o}\le\theta\le 360^{o}$. (**Ans**: $5\sin(\theta-53.1^{o})$ hence 256.7^{o})

439. Show that $\sqrt{3}\cos\theta-\sin\theta$ maybe written as $2\cos(\theta+30^{o})$ or $2\sin(60^{o}-\theta)$. Find the maximum and minimum values of the expression, state the values of θ between 0^{o} and 360^{o} for which they occur.

440. Solve the equation $\tan^{-1}(2x+1)=\tan^{-1}(2)-\tan^{-1}(2x-1)$.

441. Solve the equation $3\cos^2\theta-4\cos\theta\sin\theta+\sin^2\theta=2$ for $0^o\le\theta\le 180^o$.

442. Solve $\cos 3\theta+\cos 2\theta+\cos\theta=0$. $0^o\le\theta\le 180^o$.

443. In any triangle ABC, show that $\frac{a+b-c}{a-b+c}=\tan\frac{B}{2}\cot\frac{C}{2}$.

444. Show that $\sin 3\theta=3\sin\theta-4\sin^3\theta$, hence find all solutions of the equation $8x^3-6x+1=0$. Correct to 3 decimal places.

445. Prove that $\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$. Hence solve the equation $t^3-6t^2-3t+2=0$ correct to 2.s.f.

446. Use the substitution $x=2\sin\theta$, to solve $3x^3-9x+2=0$ correct to 4 s.f.

447. Solve the equation $\sin t\cos 3t+\sin 3t\cos t=0.8$ for $0\le t\le 2\pi$

448. Without using tables or calculators, show that $\tan^2\left(\frac{\pi}{8}\right)=3-2\sqrt{2}$.

449. Solve $\sec^2(2\theta)-3\tan 2\theta+1=0$ for $0^\circ\le\theta\le 180^\circ$.

450. Express $3\cos\theta-4\sin\theta$ in the form $\mathrm{R}\cos(\theta+\alpha)$. Hence;

i) determine the maximum value of $\frac{2}{3\cos\theta-4\sin\theta+2}$.

ii) solve the equation $3\cos\theta-4\sin\theta+2=0$.

451. Show that $\cos 3\theta=4\cos^3\theta$. Hence if $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$, Prove that; $\cos 3\theta=\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

452. Given that $2A+B=135^\circ$, show that $\tan B=\frac{\tan^2 A-2\tan A-1}{1-2\tan A-\tan^2 A}$.

453. If α is an acute angle and $\tan\alpha=\frac{4}{3}$, show that $4\sin(\theta+\alpha)+3\cos(\theta+\alpha)=5\cos\theta$ hence solve for θ in the equation $4\sin(\theta+\alpha)+3\cos(\theta+\alpha)=\frac{\sqrt{300}}{4}$ for $-180^\circ\le\theta\le 180^\circ$.

454. Solve the equation $\cos(45^\circ-x)=2\sin(30^\circ+x)$ for $-180^\circ\le x\le 180^\circ$.

Prove that in any triangle ABC;

455. $\frac{b-c}{a}=\frac{\sin\frac{1}{2}(B-C)}{\cos\frac{1}{2}A}$

456. $\frac{b^2-c^2}{a^2}=\frac{\sin(B-C)}{\sin(A+B)}$

457. $\frac{a+b-c}{a+b+c}=\tan\frac{A}{2}\tan\frac{B}{2}$

458. $a=b\cos C+c\cos B$

459. $\frac{1}{a}\cos^2\frac{A}{2}+\frac{1}{b}\cos^2\frac{B}{2}+\frac{1}{c}\cos^2\frac{C}{2}=\frac{(a+b+c)^2}{4abc}$

460. $b+c=a\cos\frac{(B-C)}{2}\operatorname{cosec}\frac{A}{2}$

461. $(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=2c\operatorname{cosec}\frac{C}{2}$

462. If $\tan\theta=\frac{b+c}{b-c}\tan\frac{A}{2}$ then $a=(b-c)\cos\frac{A}{2}\sec\theta$.

463. If $2s=a+b+c$, then; $1-\tan\frac{A}{2}\tan\frac{B}{2}=\frac{c}{s}$.

464. If $2s=a+b+c$, then; $\sin\frac{A}{2}=\sqrt{\left\{\frac{(s-b)(s-c)}{bc}\right\}}$.

465. If $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{`13}$ then $\frac{\sin A}{7}=\frac{\sin B}{6}=\frac{\sin C}{5}$ and $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

466. Solve the equation $7\cos\theta+\sin\theta-5=0$.

467. Show that if $\tan^2\alpha-2\tan^2\beta=1$, then $2\cos^2\alpha-\cos^2\beta=0$

468. Solve the equation $\sin 3x+\frac{1}{2}=2\cos^2 x$ for $0\le x\le 360^o$.

COORDINATE GEOMETRY

469. Determine the equation of a straight line perpendicular to the line $2x+y=7$ and passes through the point of intersection of the lines $5x+7y+29=0$ and $11x-3y-65=0$.

470. The points C lies on the perpendicular bisector of the line joining the points A(4,6) and B(10,2). C also lies on the line parallel to AB through (3,11).

a) find the equation of the perpendicular bisector of AB.

b) calculate the coordinates of C.

471. A curve is defined by the parametric equations $x = t^2$, $y = \frac{1}{t}$. Find the equation of the tangent to the curve at the point the curve cuts the x-axis.

472. Find the coordinates of the point C on the line joining the points A(-1,2) and B(-9,14) which divides AB internally in the ratio 1:3. Find also the equation of the line through C which is perpendicular to AB.

473. Determine the equation of the circle with centre at (1,5) and has a tangent passing through the points A(-1,2) and B(0,-2).

474. Find the co-ordinates of the point of intersection of the common chord to the circles $x^2 + y^2 - 4y - 4 = 0$ and $x^2 + y^2 - x + y - 12 = 0$ and the line $y = 7 - 3x$.

475. Determine the equation of a circle which passes through the point (0,-1) and the intersection of the circles $x^2 + y^2 + 2x - y - 5 = 0$ and $x^2 + y^2 + 3x + 4y + 1 = 0$.

476. A and B are points on the x-axis and y axis respectively, and P is the midpoint of AB. Given that the area of triangle AOB is 8 square units show that the locus of P is $xy = 4$

477. Find the coordinates of the point C on the line joining the points A(-1,2) and B(-9,14) which divides AB internally in the ratio 1:3. Find also the equation of the line through C which is perpendicular to AB.

478. Find the equations of the tangents from the point (4,4) to the hyperbola $9x^2 - 9y^2 = 16$.

479. Determine the equation of the circle passing through the points A(-1,2) , B(2,4) and C(0,4).

480. If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m.

481. Show that $3x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity.

482. Show that the curve whose parametric equations are $x = 9\cos\theta$ and $y = 12\sin\theta$ represents an ellipse hence determine its eccentricity.

483. Find the coordinates on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is a horizontal line.

484. The normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ meets the axis of the parabola at G. If GP produced beyond P to Q such that $GP = PQ$, show that the equation of the locus of Q is $y^2 = 16a(x + 2a)$.

485. A focal chord PQ to the parabola $y^2 = 4x$, has a gradient $m = 1$. Find the coordinates of the midpoint of PQ.

486. The line $4x - 3y = 5$ and $y = 3$ are tangents to circles whose centres lie on the line $x = 7$. Find the distance between the centres of the circles.

487. Find the gradient of the curve $x^2 \tan x - xy - 2y^2 = -2$ at the point (0,1).

488. If the line $y = 2x + c$ is a tangent to the hyperbola $4x^2 - 6y^2 = 24$ show that $c = \pm 2\sqrt{5}$.

489. Find the equation of the circle which is tangent to the lines $3y = 4x$, $y = 8$ and $4x + 3y = 0$.

490. If the line $y = mx + c$ is a tangent to the ellipse $a^2y^2 + b^2x^2 = a^2b^2$, prove that $c^2 = b^2 + a^2m^2$. Hence determine the equations of the common tangents to the ellipse $4x^2 + 14y^2 = 56$ and $3x^2 + 23y^2 = 69$.

491. A circle C, has the equation $x^2 + y^2 - 2x - 8y - 8 = 0$. Find the; (i) coordinates of its centre. (ii) shortest distance of the point A(-5,-4) from the circle.

492. The equation of the normal to the curve $xy^2 + 3y^2 - x^3 + 5y - 2 = 0$ at the point (a,-2) is $15x - 8y - 46 = 0$. Find the value of a.

493. Find the gradients of the two tangents from the point $(3,-2)$ to the circle $x^2 + y^2 = 4$.

494. Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix. (05 marks)

495. a). Find the equation of the chord through the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ of the parabola $y^2 = 4ax$.

b). Show that chord in (a) cuts the directrix where $y = \frac{2a(pq-1)}{p+q}$

496. Find the equation of a chord of a parabola passing through $P(at^2, 2at)$ and $Q(ad^2, 2ad)$. If the chord also passes through $(0, 2a)$. Show that $t + d = td$.

497. Given that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Show that $c^2 = a^2m^2 + b^2$ hence, determine the equations of the tangents at a point $(-3,3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

498. A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find;

a) equation of the tangent.

b) equation of the line L parallel to the normal at $T(t^2, 2t)$ and passes through (1,0)

c) the point of intersection of the line L and the tangent.

d) A point $P(x, y)$ is equidistant from X and T in (b) above, show that the locus of P is $t^3 + 3t - 2(xt + y) = 0$

499. A circle with centre C, cuts another circle $x^2 + y^2 - 4x + 6y - 7 = 0$ at right angles and passes through the point (1,3). Find t hocus of the centre C.

500. Find the equations to the tangents to the parabola $y^2 = 6x$ which pass through the point (10,-8)

501. Find the coordinates of the foot of the perpendicular from the point (2,-6) to the line $3y - x + 2 = 0$.

502. Show that the equation $y^2 - 4y = 4x$ represents a parabola; hence determine the focus, vertex and directrix.

503. A circle touches both the x-axis and the line $4x - 3y + 4 = 0$. Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$

504. If the x-axis and the y-axis are tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, Show that $c^2 = g^2 = f^2$.

505. Find the locus of a point which moves such that the ratio of its distance from the point A(2,4) to its distance from the point B(-5,3) is 2:3.

506. A point P moves such that its distance from two points A(-2,0 and B(8,6) is in the ratio AP:PB = 3:2. Show that the locus of P is a circle.

507. Find the locus of the point $P(x, y)$ which moves such that its distance from the point A(5,3) is twice its distance from $x = 2$.

508. Show that $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$

509. Find the equation of the circle whose end diameter is the line joining the points A(1,3) and B(-2,5)

510. A chord to the parabola $4x - 3y^2 = 0$ is parallel to the line $2x - y = 4$ and passes through point (1,1). Find;

a) Equation of the chord
b) The coordinates of the point of intersection of the chord with the parabola.
c) The acute angle between the chord and the directrix of the parabola.

511. A circle whose centre is in the first quadrant touches the x- and y-axes and the line $8x - 15y = 20$. Find the;

a) Equation of the circle
b) Point at which the circle touches the y-axis

512. A point P on a curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$. Find the equation of the normal to the curve at the point where $\theta = \pi/3$

513. Show that the parametric equations $x = 9\cos\theta$ and $y = 16\sin\theta$ represents an ellipse. Hence determine the foci and the diretrices.

514. Show that the equation of the tangent t the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.Hence determine the tangents at the points where the line $2x + y = 3$ cuts the ellipse $4x^2 + y^2 = 5$.

515. If the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Express c in terms of a, b and m. Hence show that $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$ is the point of contact.

516. Find the equation of the line through the point (-4,5) and perpendicular to the line $4x - 13y = 7$.

517. The points C lies on the perpendicular bisector of the line joining the points A(4,6) and B(10,2). C also lies on the line parallel to AB through (3,11).

a) find the equation of the perpendicular bisector of AB.

b) calculate the coordinates of C.

518. Find the length and the equation of the perpendicular from the point P(3,4) to the line $3x + 4y + 6 = 0$.

519. Write down the equations of the bisectors between the lines $8x - 6y - 3 = 0$ and $12x + 5y - 8 = 0$.

520. Find the equation of the line through the point of intersection of the lines $3y - x - 13 = 0$ and $3x - 2y + 3 = 0$ and which is perpendicular to the line $x + 5y = 0$.

521. The points $A(3,4), B(2a,5), C(6,a)$ form a triangle whose area is $9\frac{1}{2}$ square units. Find the two possible values of a.

522. Find the points P and Q which divide the line joining A(3,2) and B(10,16) internally and externally in the ratio 3:4.

523. Find the ratios in which the line joining the points A(8,2) and B(-2,7) is divided by the points (12,0) and (0,6).

524. Find the equation to the straight line which lies mid-way between the point (2,-1) and the line $5x + 12y = 8$.

525. Find the coordinates of the point C on the line joining the points A(-1,2) and B(-9,14) which divides AB internally in the ratio 1:3. Find also the equation of the line through C which is perpendicular to AB.

526. Find the equation of the locus of a point which always moves so that its distance from the x-axis is twice its distance from the point (2,-3).

527. A point P moves in such a way that the sum of its distance from (0,2) and (0,-2) is 6. Find the equation of the locus of P.

528. Find the locus of points which is equidistant from the point $A(2,-3)$ and the line $12x-5y+4=0$

529. A and B are points on the x-axis and y axis respectively, and P is the midpoint of AB. Given that the area of triangle AOB is 8 square units show that the locus of P is $xy=4$.

530. Find the locus of a point which moves such that the ratio of its distance from the point A(2,4) to its distance from the point B(-5,3) is 2:3

531. Find the locus of the point $P(x,y)$which moves such that its distance from the point A(5,3) is twice its distance from $x=2$.

532. A is the point (0,-2), and B is the point (2,0). Find the locus of a point P which moves so that $PA-PB=3$.

533. M and N are points on the axes, and the line MN passes through the point (3,2). P is a variable point which moves so that the midpoint of the line joining P to the origin is the midpoint of MN. Find the locus of the point P

534. A straight line LM, of length 4 units, moves with L on the line $y=x$ and M on the x-axis. Find the locus of the midpoint of LM.

535. A line parallel to the y-axis meets the curve $y=x^2$ at P and the line $y=x+2$ at Q. find the locus of the midpoint of PQ.

536. Variable lines through the points O(0,0) and A(2,0) intersect at right angles at the point P. Show that the locus of the midpoint of OP is $y^2+x(x-1)=0$

537. Find the equation of the tangent and the normal to the curve $9x^2-16y^2=20$ at the point (2,-1)

538. Find the equation of the normal to the curve $y=x^2-2x-8$ at the point where it cuts the curve cuts the y-axis and the point where this normal meet the x-axis?

539. Find the equation of the tangent to the parabola $y^2=\frac{x}{9}$ at the point $\left(p^2,\frac{p}{3}\right)$.

540. Find the of the tangent to the hyperbola $x=4t, y=4/t$ which passes through (4,3).

541. Find the equation of the tangent and normal to the rectangular hyperbola $xy=c^2$ at the point $P(ct,c/t)$.

542. Show that if the line $y = 2x + c$ is tangent to the circle $x^2 + y^2 = 9$ then $c^2 = 45$.

543. Find the equation of the tangent to the curve whose points are of the parametric form $\left(3q, \frac{1}{q}\right)$.

544. Find the equations of the tangents to the curve $y = 2x^3 - x$ which are parallel to the line $2y - 12x + 1 = 0$.

545. A curve is defined by the parametric equations;

$$x = t^2 + 3t$$
$$y = 2t - 1$$

Find the equation of the tangent to the curve at (4,1).

546. Prove that $2x + y + 4 = 0$ is a tangent to a rectangular hyperbola whose parametric co-ordinates are of the form $\left(\sqrt{2}t, \frac{\sqrt{2}}{t}\right)$.

547. If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m.

548. Find the condition that the line $lx + my + n = 0$ should touch the ellipse.

549. Show that the circles $x^2 + y^2 - 4x + 2y - 8 = 0$ and $x^2 + y^2 + 6x - 13y - 22 = 0$ touch each other and find the equation of the tangent at the point of contact.

550. A circle A passes through the point $(t+2, 3t)$ and has a centre at $(t, 3t)$.

i. Determine the equations of the circles A and B in terms of t.

ii. If $t = 1$, show that , show that circles A and B intersect at $(2, 3 \pm \sqrt{3})$.

iii. Show that the area of the region of intersection of the two circles A and B is $8\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$.

551. Show that the parametric equations $(3 + 2\cos\theta, \sin\theta - 1)$ represents a circle. State its radius and centre.

552. Find the equation of the circle with A(1,2) and B(-3,4) as ends of its diameter.

553. the equation of the circle that passes through the points A(1,1) , B(2,0) and C(3,1).

554. a) Show that the equation of a circle passing through the points (-2,-4), (3,1) and (-2,0) is $(x-1)^2+(y+2)^2=13$.

b) with reference to the circle in (a) above, show that the tangent at point (3,1) is parallel to the diameter that passes through the point (-2,0).

555. a) PQRS is a square inscribed in a circle $x^2+y^2+4x-16y+4=0$. Find the;

i) the centre and radius of the circle.

ii) the length of the diagonals to and the area of the square.

b) i) Show that the circle $x^2+y^2-8x-20=0$ is orthogonal to the circle in a) above.

ii) Hence or otherwise find the equation of the common chord to the circles and the point where this chord meets the line joining their centres.

556. The equation of circle, centre O is given by $x^2+y^2+Ax+By+C=0$ where $4A=3B$, $3A=2C$ and $C=9$.

a) Determine the coordinates of the centre of the circle and its radius.

b) A tangent is drawn from the point Q(3,2) to the circle. Find;

i). The coordinates of P, the point where the tangent meets the circle.

ii) The area of the triangle QPO.

557. Find the equation of the circle which passes through the points (1,1), (1,-1) and is orthogonal to $x^2+y^2=4$.

558. A circle passes has centre C(4,-3) and touches the line $3x-4y+6=0$.

a) Find the equation of the circle.

b) Find the coordinates of the point Q where the tangent from the point P(5,6) touches the circle. Hence find the area of triangle QCP.

559. a) Find the Cartesian equation of the circle with parametric equations are $x=2+5\cos\theta$ and $y=1-5\sin\theta$. State the coordinates of its centre and radius.

b) Find the equations of the tangents to the circle in a) above from the point (15,-5)

Find the equation of a parabola;

560. focus at (-5,0) and directrix $x-5=0$

561. with vertex at (-2,3) and focus at (-7,3)

562. focus (-2,3) and directrix $y+1=0$

563. Find the focus, directrix and length of the latus rectum for the parabola $y^2 = 4x - 8$

564. A point P moves in a Cartesian plane such that it is always equidistant from the circle $x^2 + y^2 = 4$ and from the line $y = 6$. Show that the locus of P is a parabola. Hence find its locus and directrix.

565. The tangents at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ to the parabola $y^2 = 4ax$ meet at the point R. Find the coordinates of R. If R lies on the line $2x + a = 0$, find the equation of the locus of the midpoint of PQ.

566. Find the length of the Latus rectum for a standard parabola $y^2 = 4ax$, and hence find the length of the Latus rectum of $y^2 - 4y - 20 = 8x$.

567. Given the parabola whose equation is $y^2 - 4x = 4(y - 2)$. Find the vertex, focus and directrix of the parabola.

568. Show that the tangents drawn from the end points of a focal chord joining the points $T_1(at_1^2, 2at_1)$ and $T_2(at_2^2, 2at_2)$ intersect at 90^o at the directrix.

569. The points $P(ap^2, 2ap)$ and $Q(at^2, 2at)$ are on a parabola $y^2 = 4ax$. OP is perpendicular to OQ, where O is the origin. Show that $pt + 4 = 0$.

570. Show that the curve $y^2 - 8y = -4x$ is a parabola. Sketch the parabola and state its focus.

571. The points $P(at^2, 2at)$ and $Q(aT^2, 2aT)$ lie on the parabola with equation $y^2 = 4ax$. Determine the locus of the mid-point of the line segment PQ for when $tT = 2a$.

572. The tangent to the parabola $y^2 = 4ax$ at $T(at^2, 2at)$ meets the x-axis at P. the straight line through T parallel to the axis of the parabola meets the directrix at Q. If S is the focus of the parabola. Prove that TPQS is a rhombus.

573. $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$. if the chord passes PQ passes through the focus, show that $pq = -1$. If M is the midpoint of PQ, deduce that the locus of M is $y^2 = 2a(x - a)$.

574. Given that the point $P(1 + 4\cos\theta, 2 + 3\sin\theta)$ moves in the Cartesian plane. Show that the locus of P is an ellipse. Hence obtain the coordinates of its centre and foci. Find also the equation of the tangent to the locus at P.

575. A point moves so that its distance from (3,2) is half its distance from the line $2x+3y=1$. Explain why this locus is an ellipse, find the equation of the ellipse and state the equation of its major axis.

576. Determine the equations of the director and auxiliary circles to the ellipse $\frac{(x-5)^2}{4}+\frac{(y+3)^2}{9}=1$.

577. Express the equation of the ellipse $9x^2+25y^2-54x+100y-44=0$ in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$. Hence find the coordinates of its centre and foci.

578. An ellipse has a Cartesian equation $\frac{x^2}{16}+\frac{y^2}{4}=1$. The general point $P(4\cos\theta, 2\sin\theta)$ lies on the ellipse.

a) Show that the equation of the normal to the ellipse at P is $2x\sin\theta - y\cos\theta = 16\sin\theta\cos\theta$.

b) The normal to the ellipse at P meets the x-axis at the point Q and O is the origin. Show clearly that as θ varies, the maximum area of the triangle OPQ is $4\frac{1}{2}$.

579. Find the Cartesian equation of a curve whose polar equation is given by $r = a\tan\theta$.

580. The points $P(ap^2, 2ap)$ and $T(at^2, 2at)$ lie on the parabola $y^2 = 4ax$ and the tangents at P and T intersect at point S. Show that the area of triangle PTS is given by $\frac{1}{2}a^2(p-t)$.

581. A parabola has focus (1,2) and vertex (2,4).

a) Find the equations of

(i) its axis

(ii) its directrix

(iii) its curve

b) Find the length of its latus rectum.

c) the equation of the parabola.

582. Given the equation $y = 5x - 2x^2$

a) Show that this equation represents a parabola and state the length of its latus rectum.

b) Find the:

i) Coordinates of its focus
ii) Equation of its directrix
iii) Equation of its axis.

583. Obtain the equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, a\sin\theta)$. If the tangent cuts the x and y axes at points Q and R respectively, determine the locus of the midpoint of QR.

584. Show that the equation $xy-5x-2y+3=0$ represents a rectangular hyperbola. State the equations of its asymptotes and hence give its sketch.

585. Show that the equation $x^2+4y^2-4x+24y+36=0$ represents an ellipse. Hence find its:
a) Centre
b) Eccentricity
c) Foci.

586. P is a variable point given by parametric equations $x=\frac{a}{2}\left(t+\frac{1}{t}\right)$ and $y=\frac{b}{2}\left(t-\frac{1}{t}\right)$.
a) Show that the locus of P is a hyperbola with cartesian equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
b) Sate the asymptotes of the hyperbola.
c) Determine the coordinates where the tangent to the curve at P meets the asymptotes.

587. The point $P(ap^2, 2ap)$ lies on the parabola $y^2=4ax$. L is the point $(-a, 2a)$ not on the parabola and M is the midpoint of line PL. Show that as P moves on the parabola, the locus of M is given by $y^2=2a(x+y)$.

588. The equation of the auxiliary circle of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2$. State the equation of the auxiliary circle of the hyperbola $2(x-1)^2-3(y+2)^2=12$.

589. The normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point $R(a\cos\theta, b\sin\theta)$ cuts the x and y axes at point A and B respectively. Find the area of the triangle AOB.

590. Show that for the line $yl+x+m=0$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $m^2=a^2+b^2l^2$. Hence determine the four common tangents to the two ellipses $4x^2+14y^2=56$ and $3x^2+23y^2=69$.

591. Show that if the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on the parabola $y^2=4ax$ and that PQ is a focal chord, then the area of triangle OPQ is $a^2(p-q)$.

592. A curve is defined parametrically by $x=3\left(1+\frac{1}{p}\right)$ and $y=6\left(\frac{p}{1+p}\right)$. Show that the curve is a rectangular hyperbola and hence find the length of its latus rectum.

593. P is a variable point given by parametric equations $x=\frac{a}{2}\left(t+\frac{1}{t}\right)$ and $y=\frac{b}{2}\left(t-\frac{1}{t}\right)$.

a) Show that the locus of P is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

b) State the asymptotes

c) Determine the coordinates of the points where the tangent to the curve at P meets the asymptotes.

594. Given that the tangents tot eh parabola $y^2=4ax$ at the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ contain an angle of $3\pi/4$ rad, show that $q=\frac{p-1}{p+1}$.

595. a). Determine the equation of the tangent and normal at $P\left(ct, \frac{c}{t}\right)$ on the rectangular hyperbola $xy=c^2$. (Ans: $x+t^2y=2ct$: $ty+ct^4=t^3x+c$)

b) The normal at P meets the hyperbola again at $Q\left(cT, \frac{c}{T}\right)$. Show that $t^3T+1=0$.

c) If the tangents at P and Q in (b) above meets at R, show that the locus of R is $(x^2 - y^2)^2 + 4c^2xy = 0$.

596. Part of the line $x - 3y + 3 = 0$ is a chord of rectangular hyperbola $x^2 - y^2 = 5$. Find the length of the chord.

597. Find the equation of the chord joining the points $P(cp, c/p)$ and $Q(cq, c/q)$. (Ans: $x + pqy = c(p+q)$.)

598. A point A is the midpoint of PQ on the rectangular hyperbola $xy = c^2$ where P and Q are respectively $(cp, c/p)$ and $(cq, c/q)$. The line through P and Q meets the x-axis at B. The line through B parallel to OA meets the hyperbola at $R(cr, c/r)$ and $S(cs, c/s)$. Show that $rs + pq = 0$ and $r + s = p + q$.

599. The tangent to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the x-axis at point T. A straight line through P parallel to the axis of the parabola intersects the directrix at Q.

a) If S is the focus of the parabola, show that PQTS is a rhombus.

b) If M is the midpoint of PT and N is the midpoint of PM, find;

(i) The locus of M.

(ii) The locus of N.

600. a) Find the equation of the tangent to the parabola $y^2 = \frac{x}{16}$ at the point $(t^2, t/4)$.

b) if the tangents to the parabola in (a) above at the points $P(p^2, p/4)$ and $Q(q^2, q/4)$ meet on the line $y = 2$

(i) show that $p + q = 16$.

(ii) deduce that the midpoint of PQ lies on the line $y = 2$.

601. The normal to the rectangular hyperbola $xy = 8$ at a point (4,2) meets the asymptotes at M and N. Find the length of MN.

602. Given the hyperbolic curve $xy = c^2$

a) Determine the equation of the tangent to the curve at point $A\left(ct_1, \frac{c}{t_1}\right)$

b) Determine the equation of the normal to the curve $xy = c^2$ at point $B\left(ct_2, \frac{c}{t_2}\right)$.

c) If the tangent at A above meets the normal at B above intersecting on the y – axis. Show that $2t_2 = t_1(1 - t_2^4)$.

APPLIED MATHEMATICS
LINEAR INTERPOLATION AND EXTRAPOLATION

603. The table below shows the experimental variation of quantities x and y.

x	0.022	0.146	0.209	0.311
y	14.9834	9.0763	7.6348	4.3131

Using linear interpolation or linear extrapolation, find
a) y when x = 0.252
b) x when y = 2.1473

604. The table shows the value s of x and y.

x	0.022	0.146	0.209	0.311
y	14.9834	9.0763	7.6348	4.3131

Use linear interpolation or extrapolation to find;
a) y when x = 15 b) x when y = -3.2

605. The table below shows the relationship between x and $\ln x$

x	0.8	1.2	1.6
lnx	-0.24	0.18	0.48

Use linear interpolation to estimate;

a) $\ln(0.5)$
b) x when $\ln x = -0.12$

606. The table below shows extracts of tangents of angles:

x°	45.0	45.1	45.2	45.3
tanx	1.0000	1.0035	1.0070	1.0105

Using linear interpolation and extrapolation, find:
a) $\tan^{-1}(1.0052)$
b) $\tan(45.32)$

607. Given that for a function $f(x)$, $f(0.9)=0.2661, f(1.0)=0.2420$ and $f(1.1)=0.2179$, use linear interpolation or linear extrapolation to estimate;
a). $f(0.96)$ b) $f^{-1}(0.2082)$

608. The table below shows the velocity of a particle during the course of its motion

Time(s)	5	9	12
Velocity (ms^{-1})	10	13	17

Use linear interpolation or extrapolation to estimate the;
a) velocity when time = 7s.

b) time when velocity = 19 ms^{-1}.

609. The heights of a sample of seedlings when 200grams, 300grams,350grams and 450grams of fertilizers were applied to similar seedlings of the same initial heights were 1.6cm, 1.9cm, 2.4cm and 2.8 cm respectively. Use linear interpolation or linear extrapolation to estimate the;

a) Length of a seedling when 272 grams of fertilizer is applied,

b) Amount of fertilizer required if the height of the seedling is to be 3.1 cm.

610. The quantity of water (in litres) remaining in a leaking drum carried through distances (in km) of 3,5,10,15 and 20 from a well are 54,39,25,12 and 8 litres respectively. Find the;

a). quantity of water remaining in the drum after a distance of 24km from the well.

b). distance covered when 30 litres remains in the drum.

611. A physics-Mathematics teacher is confident that there is a linear relationship between his class in performance in Physics and Mathematics. He realized that a student who scored 59% in physics scored 72% in Mathematics and the one who scored 76% in physics scored 81% in mathematics.

Find the teacher's prediction for the;

a) mathematics marks for a student with 66% in physics.

b) physics marks for a student with 90% in mathematics.

612. A faulty computer system in a certain school was used to print students marks on report cards. When the teacher fed in 70%, the computer printed 50% and instead of 60% , it printed 40%. Find the;

a) true mark if the computer printed 47%.

b) value printed by the computer if the true mark is 52**%.**

TRAPEZIUM RULE

613. Using trapezium rule with five strips evaluate $\int_0^{\pi}\sqrt{\sin x}\,dx$ correct to four decimal places. **(05 marks)**

614. a) Use trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}dx$, correct to three decimal places.

b) Find the exact value of $\int_{3}^{4} \frac{1}{\sqrt{(x-1)^2 - 3}} dx$.

c) Find the percentage error in the approximation in a) above and suggest how this error can be reduced.

615. a) Use trapezium rule with five strips to estimate $\int_{0}^{4} 3^{2x} dx$, correct to two decimal places.

b) Find the exact value of $\int_{0}^{4} 3^{2x} dx$ correct to two decimal places.

c) Calculate the relative error made in (a) above and state how you can reduce on such a relative error.

616. Use trapezium rule with 6 ordinates to estimate $\int_{1}^{2} \tan^{-1} x dx$ correct to 4 decimal places.

617. a) Use trapezium rule with 6 ordinates to estimate $\int_{0}^{\frac{3}{4}} \sqrt{(1-x^2)} dx$ correct to 3 decimal places.

b) Find the exact value of $\int_{0}^{\frac{3}{4}} \sqrt{(1-x^2)} dx$ correct to 3 decimal places.

Hence find the error made in using the estimate instead of the exact. State how the error may be reduced.

618. Use trapezium rule with 6 ordinates to estimate the value of $\int_{0}^{1} \frac{dx}{1+x^2}$.

Give your answer correct to 3 decimal places.

619. a) Use trapezium rule with six ordinates to estimate $\int_{0}^{1} \sin^2 x dx$, correct to **three** significant figures.

b) Determine the percentage error made in your calculation in (a) above and suggest how this error may be reduced.

620. a) Use the trapezium rule with 6-ordinates to find the value of $\int_{0}^{\pi/4} (t + \sin t) dx$ correct to four places.

b) Find the percentage error made in the calculation in (a) above and suggest how this error may be reduced.
c) suggest how the error can be reduced.

621. a) Use the trapezium rule with 7 ordinates to evaluate;

$$\int_0^{\frac{\pi}{2}} (2x + \cos x)dx$$

b) Calculate the percentage error made in the evaluation in (a) above. Suggest how this error can be minimized.

622. a) Use trapezium rule with four strips to show that

$$\sum_0^{\frac{\pi}{2}} (3x \sin 2x)dx \approx k\pi^2(\sqrt{2}+1).$$

b) Find the exact value of $\sum_0^{\frac{\pi}{2}} (3x \sin 2x)dx$ correct to three decimal places.

c) Find the percentage error in the approximation in a) above and suggest how this error can be reduced.

623. Use the trapezium rule with 6 ordinates to evaluate $\int_0^1 e^{-x^2} dx$ correct to 3 decimal places.

624. a) Use the trapezium rule with six ordinates to estimate $\int_1^3 x^2 \ln x dx$

Give your answer correct to three decimal places.
b) Hence find the percentage error in your estimate and suggest how it can be reduced.

LOCATION OF ROOTS, ITERATIONS AND FLOW CHARTS

625. a) Show that there is a real root of the equation $x^3 + 2x - 1 = 0$ between $x = 0$ and $x = 1$.
b) Use linear interpolation once to find the first estimate of the root of the equation, correct to two decimal places.
c) Using Newton Raphson iterative formula and your approximate root in (b) above as the initial value, calculate the root of the given equation correct to two decimal places.

626. a) Show graphically that the equation $2\sin x - \ln x = 0$ has a root between 2 and 3.

b) Use Newton Raphson's methyod to find the root of the equation in a) above correct to four significant figures.

627. a) Show that the equation $\ln x = \sin x + 2$ has a root between $x = 3$ and $x = 4$ b) Use linear interpolation to estimate the initial approximation x_o to one decimal place.

c) Using the x_o above and the Newton Raphson method find the root correct to 3 decimal places.

628. Find the consecutive integers within which the root of the equation $x^3 + 4x^2 - 16 = 0$ lies. Use Newton-Raphson Method to find the root of the equation correct to four decimal places.

629. a) Show that the equation $x\sin x = 1$ has a root lying between 1 and 1.5.

b) Use linear interpolation once to find the first approximation, x_o of the root of the equation. Hence use the Newton-Raphson Method to compute the root correct to 4 decimal places.

630. a) Show graphically that the equations $y = e^{-3x}$ and $y = \cos x$ have a root in the interval $1.2 \le x \le 2$.

b) Use your graph to estimate the root of the equation $e^{-3x} - \cos x = 0$ correct the initial approximation (x_o) to one decimal place.

c) Using the initial approximation (x_o) from b) above and the Newton Raphson method, find the root correct to three decimal places.

631. Derive the Newton-Raphson formula for solving the equation $20\cos x - x = 0$. Taking $x_o = \frac{\pi}{2}$, show that $x_1 = \frac{10\pi}{21}$.

632. By plotting graphs of $y = \sin x$ and $y = \ln x$ on the same exes. Show that the equation $\sin x = \ln x$ has a root between 2 and 3.

633. a).Show that the root of the equation $x^2e^{-x^2} + 3x - 6$ lies between 1 and 2.

b) Derive the Newton Raphson formular for finding the fifth root of a number N. Hence find $\sqrt[5]{72}$ to 4 decimal places.

634. a) Obtain graphically the root of the equation $x^3 - 3x + 4 = 0$.

b) Use Newton Raphson method to find the root of equation $x^3 - 3x + 4 = 0$ correct to 2 decimal places.

635. a) By drawing graphs x^3 and $3x - 4$ on the same axes, show that the root of the equation $x^3 - 3x + 4 = 0$ lies between -3 and -2.

b) Use linear interpolation two times to find the root of the equation $x^3 - 3x + 4 = 0$ correct to 2 decimal places.

636. a). Given the equation $px^2 + qx + r = 0$, show that the simplest interative formula based on Newton Raphson method for finding a better approximation to the root of the equation is

$$\frac{px_n^2 - r}{2px_n + q}, n = 0,1,2,...$$

b). Construct a flow chart that;

(i). reads the values of r,p,q and the initial approximation a,

(ii). computes and prints the root and number of iterations with an error of less than 0.0001.

c). Use your diagram to estimate the positive square root of 20, for a=4.

637. a) Obtain graphically the root of the equation $x^3 = 4$

b) Derive the simplest iterative formula based on Newton Raphson method that can be used to find a better approximation to the root of the equation in (a) above.

c) Using the value from the graph in (a) above as the initial approximation, find the root of the equation $x^3 = 4$ correct to four significant figures.

638. The sum, S_n, of the first n terms of a certain series is given by $S_n = Ax^{n-1} + A\frac{(1-x^{n-1})}{(1-x)}$ where A and x are constants and $n = 1,2,3,...$

Draw a flow chart that reads A and x and prints S_n.

If $A = 5$ and $x = 2$, perform a dry run for the flow chart.

639. a) Derive the simplest iterative formula based on Newton Raphson method for finding the fourth root of a given number A.

b) Draw a flow chart that;

(i) reads A and the initial approximation x_o

(ii) computes and prints the fourth root of A correct to three decimal places.

c) Perform a dry run for $A = 150.10$ and $x_o = 3.2$

640. The iterative formulae below are used for calculating the positive root of the equation $f(x) = 0$.

$$A : x_{n+1} = \frac{1}{3}\left(\frac{2x_n^{\ 3} + 12}{x_n^{\ 2}}\right)$$

$$B: x_{n+1} = \sqrt{\left(\frac{x_n{}^3 + 12}{2x_n}\right)}$$

a) Taking $x_o = 2$, use each formula twice and hence deduce the most suitable for solving $f(x) = 0$.

b) Find the root of the equation $f(x) = 0$ correct to three decimal places.

c) Find the equation whose root is in b) above.

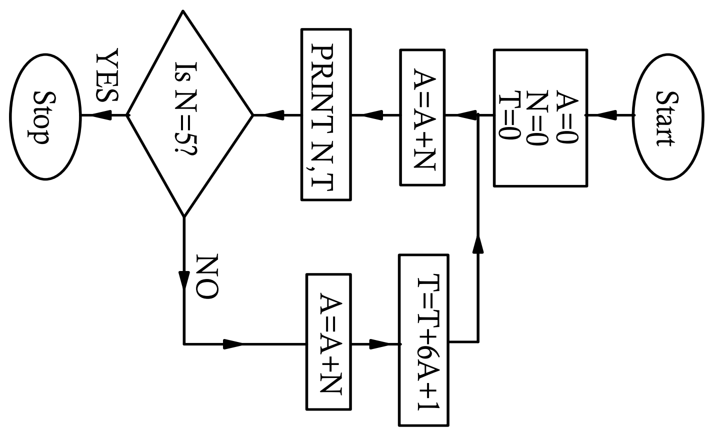
641. a). Show that the Newton Raphson Formular for finding the 4^{th} root of a number K is ;

$$X_{n+1} = \frac{3}{4}\left(X_n + \frac{K}{3x_n^3}\right); n = 0,1,2,...$$

b). i). Draw a flow chart that reads and computes the 4^{th} root of number K.

ii). Perform a dry run for finding the fourth root of 45 correct to 3 d.ps.

642. Perform a dry run for the flow chart below and state the relationship between N and T.



(05 marks)

ERRORS

643. The numbers $A = 4.2$, $B = 16.02$ and $C = 2.5$ are rounded off with corresponding errors of 0.5, 0.45 and 0.02. Calculate the absolute error in $\frac{A}{B-C}$ correct to 3 decimal places.

644. Given that $y = \sec 45^\circ \pm 10\%$. Find the limit within which the exact value of y lies.

645. Given that $x = 4.23$, $y = 2.1$ and $y = 3.2$ have percentage errors of 2, 3 and 4 respectively. Determine the;

(a) Errors in x, y, and z.

(b) Maximum value of $\frac{xy}{z}$

646. a) The quantities a and b were measured with errors Δa and Δb respectively. Show that the maximum relative error in calculating $z = a\sqrt{b}$ is $\left|\frac{\Delta a}{a}\right| + \frac{1}{2}\left|\frac{\Delta b}{b}\right|$

(b) Given that a = 2.5 and b = 0.16 were estimated with percentage errors of 4 and 5 respectively. Calculate the absolute error in evaluating $a\sqrt{b}$.

647. Find the maximum possible error made in the expression $6.3 - 3.1 - \frac{2.5 \times 4.1}{5}$ correct to three significant figures.

648. Two positive decimal numbers x and y were approximated with errors Δx and Δy respectively. Show that the maximum possible relative error in approximating X^2Y as x^2y is $2\frac{|\Delta x|}{x} + \frac{|\Delta y|}{x}$.

649. Given that $x = 3.10$, $y = 3.21$ and $z = 12.1$, rounded off to the given number of decimal places, find the;

a) Range within which $\frac{x-y}{z}$ lies.

b) Percentage error made in the approximation of $\frac{x-y}{z}$.

650. Given that $a = 1.50$, $b = 1.-13.3$ and $c = 9.200$. All are rounded off to the given number of decimal places, find the ,minimum value of;

a) $\frac{a+b}{c}$ b) $\frac{a-b}{c^2}$ c) $\frac{1}{a} - \frac{1}{b} + \frac{1}{c}$

651. The numbers $x = 1.5$, $y = 2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies.

652. Given that $y = x \sin x$ and $x = 2$, find the absolute error in y giving your answer correct to 3 significant figures.

653. a). i). Round of 6.00213,

ii). truncate 5415000 to 3 significant figures

b). Given that $x=2.79$, $y=1.375$ and $z=1.4$, find the limits within which $\frac{y}{z}-x$ lies. Correct to 4 significant figures.

654. The positive decimal numbers X and Y were approximated with errors E_1 and E_2 respectively. Show that the maximum possible relative error in the approximation of the product X^3Y^2 is $3\left|\frac{E_1}{X}\right|+2\left|\frac{E_2}{Y}\right|$.

b) Given that $X=5.64$ and $Y=10.0$, rounded off to the given number of decimal places. Find the;

(i) Maximum possible errors in X and Y.

(ii) Percentage error made in the approximation of X^3Y^2.

655. Given that $x=2.876$, $y=2.31$ and $z=8.6$ are rounded off to the given number of decimal places. Find the interval within which the exact value of $x-\frac{y}{z}$ lies correct to 4 significant figures.

656. Find the range within which the value of the expression $\frac{2.471-38.2^2}{-49.3252\times 23.17}$ lies.

657. The dimesions of a rectangle are 8cm and 4.26cm .

a) State the maximum possible error in each dimension.

b) Find the range within which the area of the rectangle lies. (correct to 2 decimal places)

658. Given that $x=1.25$(2 dps) and $y=1.600$(3 dps), calculate the interval within which the exact value of xy lies. Deduce the maximum error in xy.

659. Given that $x=2.45$ and $y=5.250$ are rounded off to the given number of decimal places. Determine the interval within which the exact value of $\frac{y-x}{y+x}$ lies. Give your answer to 4 decimal places.

660. The volume of a cone is given by $V=\frac{1}{3}\pi r^2 h$ with errors Δr and Δh in the radius and the height respectively. Show that the maximum possible relative error in the volume is given by $2\left|\frac{\Delta r}{r}\right|+\left|\frac{\Delta h}{h}\right|$

661. The numbers $x = 4.8$, $y = 4.905$ and $z = 2$ are rounded off to the nearest number of decimal places. Find the range within which the exact value of $\frac{z}{y-x}$ lies.

662. Given that the number p and q are rounded off with errors e_p and e_q respectively. Show that the maximum relative error in $\sqrt{pq}$ is given by $\frac{1}{2}\left(\left|\frac{e_p}{p}\right| + \left|\frac{e_q}{q}\right|\right)$ hence find the interval within which the exact value of $\sqrt{(1.20)(2.8)}$ is expected to lie.

663. Given that $a = 1.50$ and $b = 13.3$ and $c = 9.1000$ are rounded off to the given number of decimal places. Find the range of values within which the exact value of $\frac{a-c}{b^2}$ is expected to lie. Give your answer to four decimal places.

664. Numbers X and Y were estimated by x and y with maximum relative errors of E_x and E_y respectively. Show that the maximum relative percentage error in xy is given by; $\left(\left|\frac{E_x}{x}\right| + \left|\frac{E_y}{y}\right|\right) \times 100$

665. The numbers x and y are approximated by X and Y with errors Δx and Δy respectively. Show that the maximum relative error in $\frac{x}{y}$ is given by; $\left(\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|\right) \cdot \left|\frac{X}{Y}\right|$

CORRELATION

666. The table below shows the concentration of roots of a certain tree with respect to depth

Root concentration	80	75	86	60	75	92	86	50	64	75
Soil depth	60	58	60	45	68	68	81	48	50	70

Calculate the correlation coefficient between the root concentration and soil depth and comment.

667. The table below shows the scores in two subjects Biology (x) and chemistry (y) for ten students.

x	82	78	86	72	91	80	95	72	89	74
y	75	80	93	65	87	71	98	68	84	77

a) Plot a scatter diagram. Draw a line of best fit and find its equation in the form $y = \alpha x + \beta$ where α and β are constants.

b) Calculate the coefficient of rank correlation between x and y. Comment on the significance of Biology on chemistry. (Spearman's rank correlation coefficient $|\rho| = 0.79$ based on ten observations at 1% level of significance)

668. The following table gives the marks (x) obtained by 12 students A,B,C,D,…,L in an examination in statistics at the end of a term together with the marks (y) obtained at the beginning of the term.

Students	A	B	C	D	E	F	G	H	I	J	K	L
Marks(x)	53	74	48	71	66	60	47	72	48	65	80	40
Marks(y)	41	50	44	38	41	48	45	57	36	46	50	47

Calculate the rank correlation coefficient and comment on the results.

669. The table below shows results scored by twelve students in physics(x) and mathematics (y)

Physics	28	20	40	28	21	22	31	36	29	30	24	25
Mathematics	30	20	40	28	22	25	45	35	27	31	23	27

a) Construct a scatter diagram, draw a line of best fit and comment.

b) Calculate the rank correlation co-efficient and comment on its significance at 5%

670. The table below shows the marks obtained by 10 students in two mathematics tests.

Student	A	B	C	D	E	F	G	H	I	J
Test 1	61	69	69	68	67	58	60	62	69	67
Test 2	68	78	73	75	70	63	67	68	70	68

a) i) Illustrate the two sets of marks by means of a scatter diagram plotting the Test 1 marks on the x-axis.

ii) Calculate the mean marks for each test $(\bar{x}, \bar{y})$ and plot the result on the scatter diagram.

b) Draw the line of best fit and use it to determine:

i). the mark of Test 1 given that the student scored 65 in Test 2.

ii). the mark of Test 2 given that the student scored 70 in Test 1.

c) Find the Rank correlation coefficient between the performance in Test 1 and Test 2 and comment on the result.

671. The heights and corresponding masses of 7 tourists were taken, and ranked as shown below.

	Ranks

Height	1	2	3	4	5	6	7
Mass	2	1	4	3	7	5	6

The heights and corresponding masses of 7 tourists were taken, and ranked as shown.

Calculate the spearman's rank correlation coefficient for this data. Comment on your result.

672. The mock examination and average final examination marks of a certain school are given in the table below.

Mock marks (x)	28	34	36	42	48	52	54	60
AV. Final marks (y)	54	62	68	70	76	66	76	74

a) (i) Plot the marks on the scatter diagram and comment on the relationship between the two marks.

(ii) Draw a line of best fit and use it to predict the average final mark of a student whose mock mark is 50.

b) Calculate the rank correlation coefficient between the marks and comment on your result.

673. The data below shows the ages in years, x of patients and the number of days, y taken by the patient to recover from a particular disease.

x	55	51	62	66	72	59	78	55	62	70
y	34	44	49	49	48	43	51	41	46	51

a) Calculate the rank correlation coefficient for the data.

b) Comment on the significance of the age on the number of days taken by the patients to recover fully at 1% level of significance.

674. Ten boys compete in throwing a ball, and the table below shows the height of each boy (x cm) and the distance (y m) to which he can throw the ball.

675.

Boys	A	B	C	D	E	F	G	H	I	J
Height (xcm)	122	124	133	138	144	156	158	161	164	168
Distance(ycm)	41	38	52	56	29	54	59	61	63	67

Calculate the rank correlation coefficient and comment on the result at 5% level of significance.

676. The table below shows the distribution of wages of employees and the time taken to do the job.

Time (hours)	5	6	13	7	9	22	14	7	6	8
Wage (,000) UGX	1	12	15	12	13	16	13	10	9	11

Calculate the rank correlation coefficient and comment on the relationship between wage and hours.

677. The following table shows results scored by twelve students in building construction drawing (BCD) and studio (s) tests.

BCD	28	20	40	28	21	22	31	36	29	30	24	25
s	30	20	40	28	22	25	45	35	27	31	23	27

a) Draw a scatter diagram to represent the performance of students in the two course units and comment on the relationship.

b) Calculate the rank correlation coefficient between the marks of the two subjects. Hence comment on your answer.

678. The following table gives the order in which six candidates were ranked in midterm exams and final exam.

Mid term score	E	C	B	F	D	A
Final exam score	F	A	D	E	B	C

Calculate the coefficient of rank correlation and comment on your results.

INDEX NUMBERS

679. The table below shows the price indices of beans, maize, rice and meat with corresponding weights.

ITEM	Price index 2008 (2007 = 100%)	Weight
Beans	105	4
Maize	X	7
Rice	104	2
Meat	113	15

Calculate the;

a) Value of X given the price indices of 2007 and 2008 using 2006 as the base year are 112 and 130 respectively.

b) Weighted price index for 2008 using 2007 as the base year.

680. The table below shows the expenditure (in UGX) of a student during the first and second terms.

ITEM	Expenditure		Weight
	First term	Second term	
Clothings	46,500	49,350	5

Pocket money	55,200	57,500	3
Books	80,000	97,500	8

Using the first term expenditure as the base, calculate the average weighted price index to one decimal place.

681. The table below shows the price (Shs) and amount of items bought weekly by a restaurant in 2002 and 2003.

Items	Price (Shs.)		Amount
	2002	**2003**	
Milk per litre	400	500	200
Eggs per day	2500	3000	18
Cooking oil per litre	2400	2100	2
Flour per packet	2000	2200	15

Calculate;

a) The weighted aggregate price index taking 2002 as the base year.

b) In 2003, the restaurant spent Shs. 450,000/=. Using the weighted aggregate price index, find how the restaurant could have spent in 2002.

682. The following information relates to three products sold by a company in the year 2001 and 2004.

Product	**2001**		**2004**	
	Quantity in thousands	Selling price per unit	Quantity in thousands	Selling price per unit
A	76	0.60	72	0.18
B	52	0.75	60	1.00
C	28	1.10	50	1.32

Calculate:

a) Percentage increase in sales over the period.

b) Corresponding percentage increase in income of the period.

683. The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005.

Components	**Weight**	**Price(USD)**	
		1998	**2005**
A	6	35	60
B	5	70	135
C	3	43	105
D	2	180	290

E	1	480	800

a) Taking 1998 as the base year, calculate the;
   i) Simple price aggregate price index
   ii) Price relative of each component
   iii) Weighted price index

b) Using the price index in (a) (i) estimate the cost of the engine in 1998 if the cost of the engine in 2005 was 1600 USD.

684. The table below shows the prices of items per kg in the year 2005 and 2007.

Item	Posho	Beans	Rice	Beef	Chicken
Price in 2005	1200	2000	1200	4000	8000
Price in 2007	1600	2500	1600	6000	9500

Calculate for 2007 using 2005 as the base year;

a) Simple price index
b) Simple aggregate price index

685. The price index of an article in 2000 based on 1998 was 130. The price index for the article in 2000 was 80. Calculate;

a) Price index of the article in 2005 based on 1998.
b) Price of the article in 1998 if the price of the article was UGX 45,000 in 2005.

686. The table below shows the prices and quantities of four items in the year 2020 and 2021.

Item	**Price per unit**		**Quantities**	
	2020	**2021**	**2020**	**2021**
A	100	120	36	42
B	110	100	96	88
C	50	65	10	12
D	80	85	11	10

a) Calculate the price index
b) Simple aggregate price index
c) Weighted aggregate price index
d) A,B,C and D are ingredients used to make chapaptti. If in 2020, the price of chapatti was 600. Calculate the price of chapatti in 2021using the index in (c) above.

687. The table below shows the prices of items per kg in the year 2001 and 2002.

Item	**2001=100**	**2002**

	Price (Shs.)	Qty (kg)	Price (Shs.)	Qty (kg)
Rice	2,800	20	3,200	30
Millet	1,500	10	1,900	10
Beans	2,000	5	2,500	70

Calculate for 2002

a) Price index

b) Simple aggregate price index

c) Single aggregate quantity index

d) Weighted aggregate price index

688. The table below shows the prices in 2010 and 2018.

Item	Price in 2010	Price in 2018
Flour per kg.	3,000	5,400
Eggs per dozen	5,000	7,800

Calculate for 2018 using 2010 as the base year

a) Simple price index

b) Simple aggregate price index

DESCRIPTIVE STATISTICS

689. The table below shows the frequency distribution of marks obtained in a mathematics test by a group of students.

a) Calculate the:

i) Mean mark,

ii) Standard deviation ,

iii) Number of students who scored above 54%

b) Draw a cumulative frequency curve and use it to estimate the;

i) 5^{th} decile,

ii) Number of students who passed the test if the pass mark was 40%

iii) Least mark for a distinction if 10% of the students scored a distinction.

690. The time, to the nearest seconds taken by 100 students to solve a problem are as shown below.

Time in seconds	30-49	50-64	65-69	70-74	75-79
No. of students	10	30	25	20	15

Determine the;

(a) mean .

(b) number of students who took less than 64.5 seconds.

691. The table below shows the speeds in ms^{-1} for the vehicles crossing a certain bridge.

Time in seconds	20-<30	30-<40	40-<60	60-<80	80-<100
No. of students	2	7	20	16	5

Calculate the;

a) 40^{th} percentile speed

b) Number of vehicles whose speed exceeds the 40^{th} percentile mark.

692. The frequency distribution below shows the ages of 240 students admitted to Makerere University.

Age(years)	18-<19	19-<20	20-<24	24-<26	26-<30	30-<32
No.of Stds	24	70	76	48	16	6

a) Calculate the mean age of the students.

b) i). Draw a histogram for the given data.

ii). Use the histogram to estimate the modal age.

693. A certain frequency distribution with standard deviation 2.5 has the following results: $\sum f = n$, $\sum fx = 177$ and $\sum fx^2 = 5259$. Find the value of n.

694. The table below shows the length of leaves in (cm) of a certain type of plant sampled from a forest reserve.

5.	5.	8.	6.	6.	8.	6.	7.	7.	7.
5.	5.	5.	6.	6.	6.	7.	6.	7.	6.
7.	7.	5.	6.	8.	6.	7.	6.	7.	5.
6.	7.	6.	5.	6.	6.	7.	6.	7.	6.
7.	6.	8.	6.	6.	6.	5.	6.	8.	8.

a) Form a frequency table with class intervals of 0.5 starting with 5.0 as the lowest.

b) Calculate the;

i). mean

ii). standard deviation of the leaves

c) Draw a histogram to represent the above data and use it to estimate the modal length.

695. Study the frequency table below;

Marks	0 – 10	10 –	15 –	25 –	40 –	50 – 60
Number of children	8	10	25	15	4	2

a) Calculate the;
1. mean mark
2. modal mark

b) Plot a cumulative frequency curve and use it to estimate the 10^{th} to 80^{th} percentile deviation.

696. The table below shows the number of children in 100 families in a certain village during the 2024 national population census.

Number of children	1	2	3	4	5	6	7	8
Number of families	8	9	16	25	20	12	6	4

Calculate the;

a) Mean number of children per family

b) Standard deviation.

697. Show that the variance of n one's, 6 two's and 7 threes is a factor of the reciprocal of $(n+13)$.

698. The table below shows the distribution of a random sample of 100 sales of unleaded petrol at a petrol station.

Volume (in litres) of petrol	Number of sales
0 – 9	15
10 – 19	38
20 – 29	22
30 – 39	15
40 – 49	8
50 – 59	2

Draw a cumulative frequency curve for the data and use it to estimate the;

a) Median volume of the unleaded petrol sold.

b) 40^{th} percentile of the volume of unleaded petrol sold.

699. The table below gives a survey data for land ownership is square metres owned in Shisakulu trading center.

Landsize (m^2)	Frequency

$21-25$	2
$26-30$	8
$31-35$	17
$36-40$	26
$41-50$	24
$51-65$	16
$66-75$	6

a) Draw a cumulative frequency curve and use it to estimate the semi-inter-quartile range.

b) Find the;

i). mode

ii). standard deviation of the sizes.

700. The table below the marks scored by some students in mathematics test at a certain contest.

Marks	**No. of students**
$10-<15$	2
$15-<20$	8
$20-<30$	17
$30-<35$	26
$35-<40$	24
$40-<50$	16
$50-<60$	6
$60-<65$	1

a) Calculate the mean and median

b) Draw a histogram and use it to estimate the modal mark

c) Find the number of students who passed, given that the pass mark was 37.

PROBABILITY THEORY

701. Two events A and B are such that $P(A)=0.7$, $P(B)=0.4$ and $P(A/B)=0.3$. Determine;

a) the probability that either A or B occurs.

b) $P(A/(A\cup B))$.

702. Two events are such that $P(A)=0.7$, $P(B)=0.2$ and $P(A/B)=0.1$. Find;

a) $P(A\cup B)$ b) $P(A\cap B')$

703. Events A and B are such that $3P(A\cap B)=2P(\overline{A}\cap B)=P(\overline{A}\cap\overline{B})=x$ and $P(A)=\frac{3}{5}$. Use a venn diagram to find;

a) The value of x.

b) P(A or B but not both A and B)

704. Events A and B are such that $P(A)=0.4$, $P(A/B)=0.8$ and $P(A\cap B)=0.25$, find;

a) $P(A\cap B')$ b) $P(A'\cup B)$ c) $P(B'/A')$

705. Two events M and N are such that $P(M'\cap N)=2y$, $P(M\cap N')=y$ and $P(M)=\frac{6}{7}$. Use a venn diagram to find;

a) Value of y. b) $P(M\cap N)$

706. Given that $P(A\cup B)=0.8$, $P(A/B)=0.2$ and $P(A'\cap B)=0.4$. Find;

a) $P(A\cap B)$ b) $P(A)$

707. Events A and B are such that $P(B)=\frac{7}{20}$, $P(\overline{A}/B)=\frac{3}{7}$ and $2P(A)=3P(A\cap\overline{B})$. Find;

a) $P(A\cap B)$ b) $P(A)$

708. Given that A and B are independent events such that:

a) $P(A\cup B')$ b) $P(A'\cup B')$

709. The events A and B are such that $P(A/B)=0.4$. $5P(A)=8P(B)$ and $P(A\cup B)=0.12$. Find;

a) $P(B)$ to three d.p.s. b) $P(A\cap\overline{B})$.

710. Events A and B are such that $P(A)=\frac{3}{5}$, $P(B/\overline{A})=\frac{1}{3}$ and $P(A\cap B)=\frac{9}{20}$.

Find: (a). $P(B)$ (b). $P(A\cup B)$

711. Given that $P(B/A)=\frac{1}{3}$, $P(B/A')=\frac{5}{8}$ and $P(A'\cap B')=\frac{3}{20}$. Find;

$P(A)$ (b) $P(A\cap B')$

712. Given that $P(A)=\frac{3}{5}$, $P(A/B)=\frac{5}{7}$ and $P(B/A)=\frac{2}{3}$;

a) State with reasons whether or not A and B are;

i). independent events

ii). mutually exclusive events

b) Find;

i) $P(A \cap B)$

ii) $P(B)$

iii) $P(A/\overline{B})$

713. Two events X and Y are such that $P(X)=\frac{2}{5}$, $P\left(X/Y\right)=\frac{1}{2}$ and $P\left(Y/X\right)=\frac{2}{3}$, find;

a) $P(X \cap Y')$

b) $P\left(Y'/X'\right)$

714. A box P contains 3 red and 5 black balls, while another box Q contains 6 red and 4 black balls. A box is chosen at random and from it a ball is picked and put in another box. A ball is then randomly drawn from the later. Find the probability that;

a) Both balls are red

b) First ball drawn is black given that the balls picked were of different colors.

715. Three machines A,B and C produce solar bulbs in the ratio 30%, 60% and 10%. Of those produced by machine A, 25% are colored, that of B is 30% and that of C is 70%. Find the probability that a bulb selected at random is;

a) colored.

b) produced by C given that it is not coloured.

716. Dafy, Eli and Fabio are members of an amateur cycling club that holds a time trial each Sunday during summer. The independent probabilities that Dafy, Eli and Fabio take part in any one of these trials are 0.6, 0.7 and 0.8 respectively. Find the probability that, on a particular Sunday during the summer:

a) none of the three cyclists takes part.

b) exactly one of the three cyclists takes part.

717. Box A contains 3 red and 4 black balls, and box B contains 3 red and 2 black balls. One ball is selected from A at random and placed into B. A ball is then selected at random from B and placed into A. if thereafter, a ball is randomly picked from A, find the probability that both balls picked are black.

718. The table below shows the number of apples put in boxes A,B and C.

Apples	**Boxes**

	A	B	C
Green	4	7	3
Red	7	5	11

A box is randomly selected and two apples are selected from it without replacement. Box A is twice as likely to be picked as B. While A and C have the same chance of being picked.

Determine the probability that both apples are;

a) Of the same colour.

b) From box B, given they are of the same colour.

719. The probability that John listens to a certain radio station is 0.52 and the probability that he listens to A and not to another radio station B is 0.45. the probability that he listens to neither of the radio stations is 0.20. Find the probability that he listens to radio station B.

720. Calculate the the probability of arranging the letters of the word "**MATHEMATICIAN**" in a row such that:

a) the A's are separated.

b) each word begins and ends with "**M**".

c) every word strictly begins with "**HE**" in that order.

721. A bag initially contains 2 red balls and 3 black balls. A trial consists of selecting a ball at random noticing its colour and replacing it together with an additional ball of the same colour.

Given that three trials are made. Find the probability that

a) At least two black are drawn

b) The last ball picked in the second black.

DISCRETE RANDOM VARIABLES

722. A discrete random variable has probability density function;

$$P(X=x)=\begin{cases}\frac{x}{k}; & x=1,2,3,\ldots n\\ 0; & \text{elsewhere}\end{cases}$$

where k is a constant.

If $E(X)=3$

Find

a. the value of n.

b. the value of k

723. A random variable X takes the integer value x with $P(x)$ defined by $P(X=1)=P(X=2)=P(X=3)=kx^2$, $P(X=4)=P(X=5)=P(X=6)=k(7-x)^2$

Find the;

a) value of the constant k, hence sketch the graph of $f(x)$.

b) $E(Y)$ and $Var(Y)$ where $Y=4X-2$.

724. The table below shows the number of apples put in boxes A,B and C.

Apples	**Boxes**		
	A	B	C
Green	4	7	3
Red	7	5	11

A box is randomly selected and two apples are selected from it without replacement. Box A is twice as likely to be picked as B. While A and C have the same chance of being picked.

If X is the number of green apples taken, construct the probability density function of X, hence find the mean and standard deviation.

725. The discrete random variable, X, has a probability function $P(X=x)$, defined by

$$P(X=x)=\begin{cases} 0.15; & x=-3,-2, \\ k; & -1,0 \\ 0.1; & x=1,2, \\ 0; & \text{elsewhere} \end{cases}$$

where k is a constant

Determine the:

a. value of k,

b. cumulative distribution of X and sketch its graph.

726. The discrete random variable X has a probability function;

$$P(X=x)=\begin{cases} \dfrac{kx}{(x^2+1)}; & x=2,3 \\ \dfrac{2kx}{(x^2-1)}; & x=4,5 \\ 0; & \text{otherwise} \end{cases}$$

a) Show that the value of k is $\dfrac{20}{33}$.

b) Find the probability that X is less than 3 or greater than 4.
c) Find $F(3.2)$.
d) Find $E(X)$ and $Var(X)$.

727. The discrete random variable X takes integer values only and has p.d.f;

$$P(X=x)=\begin{cases} kx; & x=1,2,3,4,5 \\ k(10-x); & x=6,7,8,9 \\ 0; & \text{elsewhere} \end{cases}$$

Find;
e) The value of the constant k.
f) $E(X)$
g) $Var(X)$
h) $E(2X-3)$
i) $Var(2X-3)$

728. A random variable X that takes on only integral values has a p.d.f defined by;

$$f(x)=\begin{cases} \left(\frac{1}{2}\right)^x; & 1\le x\le 5 \\ c; & x=6 \\ 0; & \text{otherwise} \end{cases}$$

where c is a constant.
Determine the value of c and hence the mode and the mean of X**.**

729. The discrete random variable X has a p.d.f $P(X=x)=k|x|$, where x takes the values -3,-2,-1,0,1,2,3.
Find;
a) the value of the constant k. (**Ans**: 1/12)
b) $E(X)$ (**Ans**: 0)
c) the standard deviation of X. (**Ans**: 2.45)

730. The discrete random variable X has a distribution function $F(X)$ where

$$F(X)=1-\left(1-\frac{1}{4}x\right)^x \text{ for } x=1,2,3,4$$

a) Show that $F(3)=\frac{63}{64}$ and $F(2)=\frac{3}{4}$
b) Obtain the probability distribution of X
d) Find $E(X)$ and $Var(X)$ (**Ans**: $2\frac{1}{64}$ and 0.547)

e) Find $P(X > E(X))$ (**Ans**: 1/4)

731. A discrete random variable X has a probability function

$$P(X=x)=\begin{cases}\frac{x}{k}; & x=1,2,3,\ldots,n\\ 0; & \text{otherwise}\end{cases}$$

Where k and n are real numbers.

Given that the expectation of X is 3, find

a) The values of n and k. (**Ans**

b) The median and variance of X

c) $P(X=2/X\geq 2)$ (Ans: 2/9)

732. A discrete variable X has its p.d.f given by

$$P(X=x)=\begin{cases}\frac{k}{x}; & x=1,2,3\\ 0; & \text{elsewhere}\end{cases}$$

Find:

a) The value of constant k,

b) $E(X+1)^2$.

c) Median,

d) 3rd decile.

733. A discrete random variable X, has the following probability distribution.

x	0	1	2	3
$P(X=x)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{10}$

Find;

a) $E(4X+9)$

b) $Var(4X+9)$

BINOMIAL DISTRIBUTION

734. A biased coin is tossed six times. The coin is such that the ratio of the likelihood of the tail to the head occurring is 1:2. Determine the probability of getting:

a) At least 5 heads.

b) Between 1 and 3 tails.

735. The probability that a person vaccinated against COVID-19 falls sick is 0.4. if a group of 25 persons are checked at random, find the probability that not more than 20 patients are found to be vaccinated.

736. A die is tossed 15 times and the probability of getting a four on any one toss is 0.25. Estimate the probability of getting;

a) between 3 and 8 fours.

b) at most 7 fours.

737. Three fair tetrahedral, dice are tossed at once. If x is the number of times the figure 4 appears at the bottom, find the standard deviation of x.

738. A hotel has 50 single rooms, 16 of which are on the ground floor. The hotel offers guests a choice of full English breakfast, a continental breakfast or no breakfast. The probabilities of these choices being made are 0.45, 0.25 and 0.30 respectively. It may be assumed that the choice of breakfast is independent from guest to guest.

a) On a particular morning there are 16 guests, each occupying a single room on the ground floor. Calculate the probability that exactly 5 of these guests require a full English breakfast.

b) On a particular morning when there are 50 guests, each occupying a single room, determine the probability that;

c. at most 12 of these guests require a continental breakfast.

d) more than 10 but fewer than 20 of these guests require no breakfast.

739. In any trial, the probability that a head occurs when a coin is tossed is three times the probability that a tail occurs. The coin is tossed 15 times, calculate the probability that a head will occur at least 7 times.

740. The probability that a marksman aims and hits a target with a single shot is 0.4. if the marksman is given 25 bullets, find the probability that he hits the target:

a) exactly 8 times,

b) between 9 and 15 times inclusive.

741. A biased coin is tossed 12 times, the coin is such that the ratio of the head to the tail to land on top is 1:3. Find the probability of getting;

a) at most 4 heads.

b) between 6 and 10 heads.

742. A batch of 10 nails is drawn from a machine output of which is 40% defective. Find the probability that the batch contains;

a) No defective nails,

b) More than five defective nails.

743. In a certain paper, there are 8 questions each of which has five suggested answers with only one of them correct. If a candidate answers all the

questions by guess work such that she is equally likely to chose any of the five alternatives, Find;

a) The probability that he answers at least 3 of the questions correctly.

b) The expected number of questions that he will fail.

CONTINUOUS RANDOM VARIABLES

744. A continuous random variable X has a probability density function given by ;

$$f(x)=\begin{cases}\frac{x^2}{27}; & 0\le x<\alpha\\ \frac{1}{3}; & \alpha\le x<\beta\\ 0; & \text{otherwise}\end{cases}$$

Find the value of α and β.

745. A continuous random variable X is uniformly distributed over the interval $\alpha \le x \le \beta$. Given that $E(X)=2$ and $P(X\le 3)=\frac{5}{8}$. Find the;

a) values of α and β.

b) p.d.f of X.

746. The probability density function $f(x)$ of a random variable is defined by;

$$f(x)=\begin{cases}c(x+3); & 0<x<2\\ c(7-x); & 2<x<4\\ 0; & \text{elsewhere}\end{cases}$$

a) Sketch the function f(x) and hence use your sketch to find the value of c.

b) Determine the expectation, $E(X)$.

c) Find $P(1\le X<3)$

747. A continuous Random variable X has probability density function $f(x)$ that is as indicated below.

$$f(x)=\begin{cases}\frac{x}{3}-\frac{2}{3}; & 2\le x\le 3\\ a; & 3\le x\le 5\\ 2-bx & 5\le x\le 6\\ 0; & \text{otherwise}\end{cases}$$

where a and b are constants.

Determine the;

a) values of a and b,

b) cumulative distribution function, $F(x)$.

c) hence from (b) above $P(2.5 < X < 3.5)$

748. The distribution function of a continuous random variable is as follows:

$$F(x)=\begin{cases} 0; & x \le 1 \\ \frac{1}{2}(x-1)^2; & 1 < x \le 3 \\ \frac{1}{24}(\beta x + \lambda - x^2); & 3 < x \le 7 \\ 1 & x > 7 \end{cases}$$

Find the;

a) Values of β and λ.

b) Probability density function of X.

c) Mean, μ of the distribution.

749. A continuous random variable X has a cumulative probability function given by;

$$F(x)=\begin{cases} \log_2(x^k); & 0 \le x \le e \\ 1; & x \ge e \end{cases}$$

a) Show that $k = \ln 2$

b) Obtain the p.d.f of X.

750. The random variable X has a p.d.f given by

$$f(x)=\begin{cases} kx(1-x^2); & 0 < x < 1 \\ 0; & \text{elsewhere} \end{cases}$$

where; k is a constant.

a) Find the cumulative distribution function, $F(x)$.

b) Using your answer in (a) above, find the;

i). value of k

ii). median of X.

c) Calulate the mean of X.

751. X is a continuous random variable whose distribution function is given by;

$$F(x)=\begin{cases} a(x^2-1); & 1 \le x \le 2 \\ a+bx & 2 \le x \le 3 \\ 1 & x \ge 3 \end{cases}$$

a) Find the constants a and b; hence sketch the graph of $F(x)$.

b) Compute; $P(X<2.5/X>1.5)$

c) Calculate, $E(X)$ the mean of X.

752. The continuous random variable X is distinguished between the values $x=0$ and $x=2$ and has a probability density function ax^2+bx with the mean 1.25. Find;

a) the values of a and b and hence $f(x)$.

b) the mode of X.

753. A continuous random variable X has a p.d.f given by;

$$f(x)=\begin{cases} a(1-\cos x); & 0\le x\le \frac{\pi}{2} \\ a\sin x; & \frac{\pi}{2}\le x\le \pi \\ 0; & \text{elsewhere} \end{cases}$$

Find the;

a) the value of a.

b) the mean

c) $P\left(\frac{\pi}{3}<x<\frac{3\pi}{4}\right)$.

754. The cumulative distribution function of a continuous random variable X is given by;

$$F(x)=\begin{cases} 0; & x<-1 \\ \frac{1+x}{8}; & -1\le x<0 \\ \frac{1+3x}{8} & 0\le x<2 \\ \frac{5+x}{8} & 2\le x\le 3 \\ 1; & x>3 \end{cases}$$

a) Sketch the graph of the probability density function f(x).

b) Determine the expectation and the variance of X.

c) Determine $P(X<1/X>-0.5)$

755. A continuous random variable has a probability distribution given by;

$$F(x)=\begin{cases} 0; & x<0 \\ \frac{x^2}{4}; & 0\le x\le 1 \\ qx-\frac{1}{4}; & 1\le x\le 2 \\ p(5-x)(x-1); & 1\le x\le 2 \\ 1; & x\ge 3 \end{cases}$$

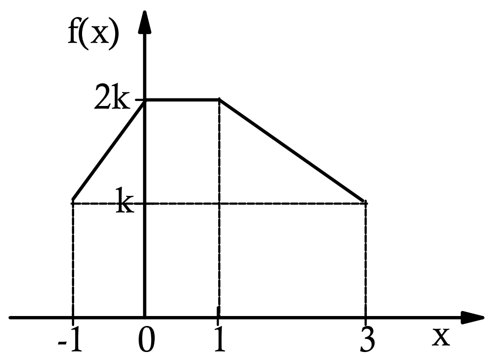
a) Find the values of p and q.

b) Determine;

(i) $f(x)$

(ii) $E(X)$

756. A continuous random variable has a p.d.f illustrated by;



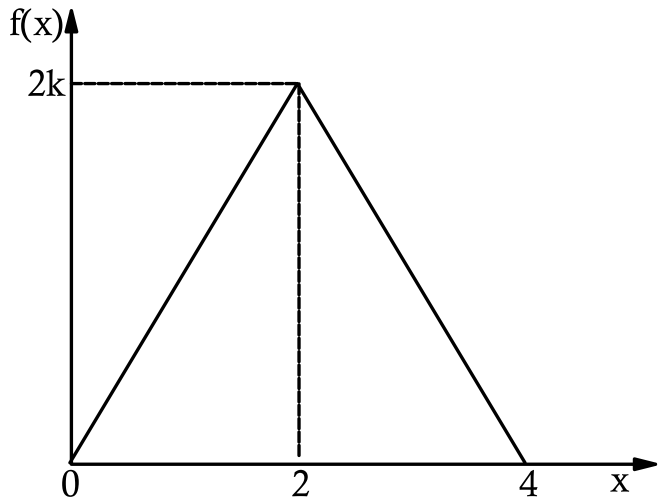
a) Determine the expression for f(x) hence or otherwise find the value of k.

b) Find the;

i). mean of X.

ii). $P(0<X<1/X>0)$.

757. The probability density function of a random variable X is as shown in the graph below.



Find the;

a) Value of k, and the p.d.f f(x).

b) Cummulative probability function F(x) and use F(x) to find $P(1 \le x < 3)$

758. The weekly demand for petrol in thousands of units in a university is a continuous random variable with a probability density function of the form;

$$f(x) = \begin{cases} ax^2(d - x); & 0 \le x \le 1 \\ 0; & \text{elsewhere} \end{cases}$$

a) Given that the average demand per week is 600 units, determine the values of a and d.

b) Find $P(0.9 < x < 1)$

759. The continuous random variable *Y* has a cumulative distribution function given by;

$$F(x) = \begin{cases} 0; & y < 1 \\ Ay^2(y^2 - 1); & 1 < y < 2 \\ 1; & y > 2 \end{cases}$$

Find the:

a) value of A.

b) 90^{th} Percentile

c) $f(x)$, probability density function of Y.

d) $E(X)$

e) $Var(2Y + 3)$

760. A random variable X has a cumulative distribution function given below.

$$F(x)=\begin{cases} 0; & x \le 0 \\ ax; & 0 \le x \le 1 \\ \frac{x}{3}+b; & 1 \le x \le 2 \\ 1; & x \ge 2 \end{cases}$$

Find;

a) The value of a and b, (Ans: 2/3, 1/3)

b) $P(x < 1.5 / x > 1)$

c) Mean of X.

761. A continuous random variable X has a probability density function; function;

$$f(x)=\begin{cases} k; & 0 \le x < 2 \\ k(2x-3); & 2 \le x < 3 \\ 0; & \text{otherwise} \end{cases}$$

Find

a) the value of k.

b) the cumulative mass function $F(x)$;

c) the median

d) sketch $F(x)$

762. The continuous random variable X has a cumulative distribution function

$$F(x)=\begin{cases} 0; & x < 2 \\ \frac{x-3}{8}; & 3 \le x \le 11 \\ 1; & x \ge 11 \end{cases}$$

Find

a) $E(X)$

b) $Var(X)$

763. A continuous random variable X, is such that $X \sim R(a,b)$.

Show that;

a) $E(2X+b)=a+2b$

b) $Var\left(\frac{X}{a+b}\right)=\frac{1}{12}\left(\frac{a-b}{a+b}\right)^2$

c) The cumulative mass function of X, $F(x)=\frac{x-a}{b-a}; a < x < b$

764. In a certain school, the response time of students to the assembly bells is uniformly distributed with mean 13 minutes and standard deviation $\frac{7}{\sqrt{3}}$ .Find the probability that a student chosen at random will respond between 8 and 14 minutes after the bell is rung.

765. In a particular month, the volume of rainfall ml followed a uniform distribution between 4 ml and b ml. if the mean volume of rain for that month is 4.5, find the;

a) value of b.

b) standard deviation of the volume of rain.

766. The continuous random variable X is uniformly distributed in the interval $a \leq x \leq b$. The lower quartile mark is 5 and the upper quartile is 9. Find the;

a) Values of a and b

b) $P(6 \leq X \leq 7)$

c) Cummulative distribution function of X.

767. The number of patients visiting a certain hospital is uniformly distributed between 150 and 210.

a) Write down the probability density function for the number of patients.

b) Find $P(170 < x < 194)$.

768. A continuous random variable has cumulative distribution function;

$$F(x) = \begin{cases} 0; & x < 1 \\ \frac{x^2 - 1}{2} - x; & 1 \leq x \leq 2 \\ 3x - \frac{x^2}{2}; & 2 \leq x \leq 3 \\ 1; & x \geq 3 \end{cases}$$

Find the;

a) Probability density function, $f(x)$ and sketch it.

b) $P(1.2 < x < 2.4)$

c) Mean of X.

769. The continuous random variable X has a probability density function given by;

$$f(x) = \begin{cases} k(x+3); & -3 \leq x \leq 3 \\ 0; & \text{elsewhere} \end{cases}$$

where k is a constant;

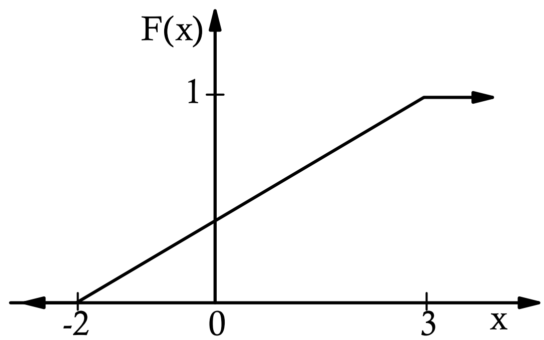
a) Show that $k = \frac{1}{18}$

b) Find $E(X)$ and $Var(X)$

c) Find the lower quartile of X.

d) Let $Y = aX + b$, where a and b are constants with $a > 0$. Find the values of a and b for which $E(Y) = 0$ and $Var(Y) = 1$.

770. A continuous random X has a cumulative distribution $F(x)$ illustrated as follows;



Find;

a) The probability density function $f(x)$.

b) The standard deviation of X.

c) The inter-quartile range.

d) The 20^{th} percentile.

771. A continuous random variable has a p.d.f

$$f(x) = \begin{cases} k; & 0 \le x < 2 \\ k(2x-3); & 2 \le x < 3 \\ 0; & \text{otherwise} \end{cases}$$

a) Sketch $f(x)$

b) Find;

(i) the value of k

(ii) the semi-interquartile range.

(iii) $P[(0 < X \le 2)/X \ge 1]$

NORMAL DISTRIBUTION, NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTION AND ESTIMATION

772. The number of people infected by a certain type of disease is normally distributed with mean 5.6 people and variance 2.25. if a random sample of 100 people is taken, find the probability that the mean number of people infected

a) lies between 5 and 6.1

b) is greater than 5.7

773. The diameter of a sample of oranges to the nearest cm were:

Diameter (cm)	8	9	10	11	12	13	14
Frequency	9	15	21	32	19	13	11

a) Calculate the mean and standard deviation. (**11,1.6633**)

b) Assuming the distribution is normal, find the minimum diameter if the smallest 10% of the oranges are rejected for being too small.

774. The random variable X is distributed normally with mean μ and standard deviation σ such that $P(X < 35) = 0.2$ and $P(35 < X < 45) = 0.65$. Find;

a) values of μ and δ

b) $P(1 \leq X < 3)$

775. The marks in an examination were normally distributed with mean μ and standard deviation σ. 30% of the candidates scored more than 60 marks and 55% scored between 35 and 60 marks. Find the;

a) values of μ and σ

b) percentage of candidates who scored more than 50 marks.

776. A total population of 700 students sat an examination for which the pass mark was 50. The marks were normally distributed. 28 students scored below 40 marks while 30 scored above 60.

a) Find the mean mark and standard deviation of the students.

b) What is the probability that a student chosen at random passed the examination?

c) Suppose the pass mark is lowered by 2 marks, how many more students will pass?

777. An examination has 100 questions. A student ha 60% chance of getting each question correct. A student fails the examination for a mark less than 55. A student gets a distinction for a mark 68 or more. Calculate the probability that a student:

a) Fails the examination

b) Gets a distinction

778. In a school of 800 students their average weight is 54.5 kg and standard deviation 6.8kg. If the weights of all the students in the school assume a normal distribution, find the;

a) Probability that a student picked at random weighs 52,8 or less kg.

b) Number of students who weigh over 75 kg.

c) Weight range of the middle 56% of the students of the school.

779. A die is tossed 40 times and the probability of getting a six on any one toss is 0.122. Estimate the probability of getting between 6 and 10 sixes.

780. The marks in an examination were normally distributed with mean μ and standard deviation σ. 20% of the candidates scored less than 40 marks and 10% scored more than 75 marks. Find the;

a) Value of μ and σ.

b) Percentage of the candidates who scored more than 50 marks.

781. The time required to complete a certain car journey has been found from experience to have mean of 2 hours 20 minutes and standard deviation of 15 minutes. What is the probability that on one day chosen at random the journey requires between 1 hour 50 minutes and 2 hours 40 minutes?

782. The marks in an examination were found to be normally distributed with mean 53.9 and standard deviation 16.5. 20% of the candidates who sat for the examination failed. Find the pass mark for the examination.

783. The weights of ball bearings are normally distributed with mean 25grams and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample lies between 24.12 grams and 26.73 grams.

784. A random sample of 76 electrical components produced by a certain manufacturer has resistance $r_1, r_2, \ldots r_{76}$ ohms. Where $\sum r_i = 740$ and $\sum {r_i}^2 = 8,216$. Calculate the;

a) Unbiased estimate of the population variance.

b) 91.86% confidence interval for the mean resistance of the electrical components produced

785. The germination time for a certain species of seeds is known to be normally distributed. If for as given batch of the seeds, 20% take more than 6 days to germinate and 10% take less than 4 days to germinate.

a) Determine the mean and standard deviation of the germination time.

b) Find the 99% confidence limit of the mean germination time.

786. A random sample of ten packets is taken. These have masses (measured in kg) of $x_1, x_2, ..., x_{10}$ such that; $\sum_{i=1} x_i = 2.57$ and $\sum_{i=1} x_i^2 = 0.6610$.

Calculate the 95% confidence limits for the mean.

787. Packets of a poultry drug are normally distributed. If 63%of the packets are found to be above 200g while 54% of the packets are below 250g. Find the;

a) Mean and standard deviation

b) Percentage of packets exceeding 195g.

788. The chance that a tree is eaten up by the army caterpillars in one day is 0.72. if 100 trees are chosen at random, find the 95% confidence limits for the mean number of tree leaves that will be eaten up.

789. The lengths of iron sheets produced in a certain factory are normally distributed. Given that 10% of the sheets are of lengths less than 2.4m and 20% are of lengths more than 2.68m, find the;

a) the mean and variance of the distribution.

b) the percentage of iron sheets that would be expected to be of lengths less than 2.55m.

790. A random sample of 10 newly born babies in a certain hospital had amean weight of 3.36kg and standard deviation 0f 0.96kg. Construct a 99% confidence interval for the mean weight of the babies in the hospital.

791. It is estimated that on average one match in five in the football league is drawn. If ninety matches are selected at random. What is the probability that between 13 and 20 inclusive of the matches are drawn.

792. The mean and standard deviation of a random sample of size 100 is 900 and 60 respectively. Given that the population is normally distributed, find a 95% confidence interval of the population mean.

793. On day one of the Coachella music festival, the height of the revelers can be modeled into a normal distribution with mean 1.75 m and variance 0.0064 m^2. A draw is to be carried out and it is decided that one should have a height greater than 1.67 m but less than 1.83m to participate.

(a) Find the:

i. Percentage of the people who qualify to take part in the draw.
ii. Fraction that is rejected because they are too tall.

(b) By day three of the event, the heights of the people present are normally distributed with mean μ, and standard deviation 0.085m. When the criteria used to select participants is not altered, 3.5% of the revelers are rejected because they are too short. Find the:

i. value of μ
ii. probability that a reveler whose height exceeds the mean qualifies to take part in the draw.

794. A random sample of size 9 drawn from a normally distributed population has the following values: 297.5, 298.7,596.5,300,297.4,596.6,297.5,300.5,300. Determine a 99% confidence interval for the population mean. $[245.3841, 484.5493]$

LINEAR MOTION AND PROJECTILES

795. Find the angle of projection of a ball which is thrown at $20ms^{-1}$, and is at its greatest height when it just passes over the top of a building that is 16m high.

796. A ball is projected from level ground with a speed of $25\sqrt{2}$ ms^{-1} at an elevation of 45^{o} passes just above the top of two vertical posts each of height 30m. Find the distance between these posts.

797. A ball is projected from a point A and falls at a point B which is in level with A and at a distance of 160m from A. the greatest height of the ball attained is 40m. find the;

a) Angle and velocity at which the ball is projected.
b) Time taken for the ball to attain its greatest height.

798. Two stations A and B are a distance of $6x$ m apart along a straight track. A train starts from rest at A and accelerates uniformly to speed v m/s covering a distance of x m. The train then maintains this speed until it has travelled a further $3x$ m. it then retards uniformly to rest at B. Make a sketch of the velocity- time graph for the motion and show that if T is the time taken for the train to travel from A to B, then $T = \frac{9x}{v}$ seconds.

799. A boy throws a ball at an initial sped of 40 ms^{-1} at an angle of elevation, α. Show, taking g to be 10 ms^{-2}, that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$.

800. A stone projected at an angle α to the horizontal with speed, u ms^{-1} just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the;

a) Angle of projection

b) Time taken to reach that point.

801. A projected particle achieves the greatest range of 0.12km. Find the ;

a) Speed of projection.

b) Greatest height attained.

802. A particle is projected with a speed of 36 ms^{-1} at angle of 40^o to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70metres on the horizontal plane from the point of projection. Find the:

b) (i) time taken fro the particle to reach the wall.

(ii) Height of the wall.

b) Maximum height reached by the particle from the point of projection.

803. A particle is projected vertically upwards with velocity of 19.6 ms^{-1}. Find how far it takes the particle to be at the height of 10m above the point of projection.

804. A stone is projected from the top of a cliff of height 25m with an initial speed of 12 ms^{-1} at angle of $60^o$to the vertical. Find the time it takes the stone to hit the sea level.

805. A particle is projected horizontally from a point 2.5m above a horizontal surface. The particle hits the surface at a point which is horizontally 10m from the point of projection. Find the initial speed of projection.

806. A particle is projected upwards with a velocity of $20ms^{-1}$. Find the:

c) greatest height attained.

d) Time taken by the particle to reach maximum height

e) Time of flight.

807. A car is travelling at $20ms^{-1}$, the driver observes an obstacle that is a distance of 30m away and starts braking. If he manages to stop just at the obstacle, determine the;

a) retardation

b) time it takes to stop

808. A driver of a car travelling at 72 km h^{-1} notices a tree which has fallen across the road, 800m ahead and suddenly reduces the speed to 36 km

h^{-1} by applying the brakes. For how long did the driver apply the brakes?

809. A ball is thrown vertically upwards from a point A, with initial speed of $21ms^{-1}$, and is later caught again at A. find the length of time for which the ball was in air.

810. A bullet is fired vertically upwards at a speed of $147ms^{-1}$. Find the length of time for which the bullet is at least 980m above the level of projection.

811. A taxi accelerates uniformly from rest at 1 ms^{-2}. At the same time, a passenger is 4m behind the taxi runs with constant speed after the taxi and just manages to catch up with it. Find the speed of the passenger.

812. Points A,B and C lie along a straight line. A particle accelerating uniformly at 0.5 ms^{-2} is projected from A towards C with a velocity of 2 ms^{-1} and after 1 second another particle is projected from C towards A with a velocity of 6 ms^{-1} and a constant retardation of $2ms^{-2}$. Given that the particles passed each other at B while moving with the same speed, find the distance BC.

813. Two points P and Q are x metres apart, a particle starts from rest at P and moves directly to Q with uniform acceleration a ms^{-2} until it acquires a speed of v ms^{-1}. It maintains this speed fro T seconds and then brought to rest at Q under retardation a ms^{-2}. Prove that $T = \frac{x}{v} - \frac{v}{a}$.

814. A smooth inclined plane of length L and height h is fixed on a horizontal plane. Show that the velocity with which a particle must be projected down the plane from the top in order that it may reach the horizontal plane in the same time as a particle let fall from the top is;

$$u = \frac{L^2 - h^2\left(\sqrt{\frac{g}{2h}}\right)}{L}$$

815. A stone is thrown vertically upwards with a speed $u\,ms^{-1}$. A second stone is also projected from the same point with the same speed but T seconds later. Prove that they collide at a distance $\left(\frac{4u^2 - g^2T^2}{8g}\right)$ above the point of projection.

816. There are two possible angles of projection α and β for which a particle projected with speed of 30ms^{-1} from point (0,0) to pass through another point (40,10). Show that $\tan(\alpha+\beta)=\frac{36}{25}$.

817. Two stations P and Q are 2.5km apart. A train passes P at a speed of 14ms^{-1} and accelerates uniformly for 20s to a speed v_1. Over the next 720m covered in 15s, its acceleration alters to a speed v_2. It travels at this speed for 13s and then over the next 500m covered in 10s with uniform deceleration its speed at Q is v_3. Find the:

a) Values of v_1 , v_2 , and v_3
b) Acceleration for the second part of the motion
c) Fraction of the whole distance covered with constant speed.

818. A gun is fired from the top of a cliff of height h and the shot attained a maximum height of $(h+b)$ above sea level and strikes the sea level a distance a from the foot of the cliff. Prove that the angle of elevation of the gun is given by the equation $a^2\tan^2\alpha-4ab\tan\alpha-4bh=0$.

819. A and B are two points such that B is h m vertically above A. From A, a particle is projected vertically upwards with velocity U. At the same time another particle is projected with velocity V vertically upwards from B. If the particles collide at C above B, prove that $\overline{AC}=\frac{Uh}{U-V}=\frac{gh^2}{2(U-V)^2}$.

820. A ball is projected so as just to clear two walls, the first of height a and at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on horizontal plane is $\frac{a^2+ab+b^2}{a+b}$ and that the angle of projection exceeds $\tan^{-1}3$.

821. A particle is projected at an angle of elevation 60° with a speed of 20 ms^{-1} . If the point of projection is 8m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground.

822. A ball is projected from a point A and falls at a point B which is level with A and at a distance of 160m from A. the greatest height of the ball attained is 40m. Find the:

a) Angle and velocity at which the ball is projected.
b) Time taken for the ball to attain its greatest height.

823. A particle is projected from level ground at an angle of elevation θ with initial speed u ms^{-1}. Show that the equation of its path is given by

$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$$

824. A ball kicked from level ground with a speed of 30 ms^{-1} just clears a vertical wall 9m high and 72m away. Calculate the possible angles of projection.

825. The horizontal and vertical components of the initial velocity of a particle projected from a point O on a horizontal plane are p and Q respectively.

a) Express the vertical distance y, travelled in terms of the horizontal distance x. find the components of p and q.

b) Find the greatest height, h, attained and the range ,R, on the horizontal plane through O. Hence show that $y = \frac{4Hx(R-x)}{R^2}$.

Given that the particle passes through the point (20,80) and H = 100m find the velocity of projection.

826. A particle is projected at an angle of elevation 30^{o} with a speed of 21 ms^{-1}. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground.

827. A ball is projected at an angle with a speed of $14\sqrt{10}ms^{-1}$ from the top of a tower 200m high hits the ground at a point 200m away from the foot of the tower.

a) Show that the two possible directions of projection are at right angles to each other

b) Find the two possible times of flight.

828. A particle is projected from the origin O with velocity $u = (9.8i + 29.4j)ms^{-1}$ and moves freely under gravity.

a) Find the particle's velocity and position vector after t seconds

b) Show that the particle's equation of path is given by $y = 3x - \frac{3x^2}{98}$

c) Find the direction in which the particle is moving after t seconds

d) Find the two times when the direction in which the particle is moving is at right angles with the line joining the position of the particle to O.

829. A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find:

a) its initial speed and angle of projection

b) the distance beyond the pole where the particle will fall.

830. A and B are two points on level ground. A vertical tower of height 4h has its base at A . Another tower of height h has its base at B. A stone is thrown horizontally with speed V from the top of the taller tower towards the smaller tower, it lands at a point X where $AX = \frac{3}{4}AB$.

When a stone is thrown horizontally with speed U from the top of the smaller tower towards the taller tower, it also lands at X. Show that $3U = 2V$.

FORCE AND RESULTANTS OF FORCES

831. Forces of 7N, 6N , 10N , 13N and 15N act along lines BA,BC,DC, DA and diagonal AC respectively of a rectangle ABCD. Their directions are in the order of their letters with AD as the horizontal if $\overline{AB} = 3a$, $\overline{BC} = 4a$. Find the resultant of the system of forces and the distance from A at which its line of action cuts AD.

832. a) ABC is an isosceles triangle right angled at A with $\overline{AB} = 2m$. Forces of 8N, 4N and 6N act along the sides BA, CB and CA respectively. Find the magnitude and direction of the resultant force.

833. In a square ABCD, three forces of magnitude 4N, 10N and 7N act along AB, AD and CA respectively. Their direction are in the order of the letters. Find the magnitude of the resultant force.

834. ABCD is a square of side 4m. Forces of magnitude 7N, 3N , $5\sqrt{2}$ N and $2\sqrt{2}$ N act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{BD}$ respectively. Show that the system of these forces reduce to a couple.

835. A square ABCD of side 4m has forces of magnitude 8N, 3N, 3N, 4N and $2\sqrt{2}$N acting along AB, CB, DA, CD and BD respectively. Taking AB and AD as x and y axes respectively,

a) Find the distance from A where the line of action of the resultant crosses AB.

b) When a couple of magnitude M is introduced, the force system is reduced to a single force passing through B. Find M and its direction.

836. A particle of weight 50N is supported by two light inextensible strings of lengths 8m and 13m attached to two fixed points 15m apart on a horizontal beam. Find the tensions in each string.

837. The ends P and Q of a light inextensible string PBCQ are fastened to two fixed points on a horizontal beam. Particles of mass 3kg and 4kg are attached to the string at the points B and C respectively. If PB is inclined at 45^o to the horizontal and $\angle PBC = 150^o$, find the;

a) tension in each portion of the string

b) angle CQ makes with the horizontal

838. Three boys are pulling a heavy trolley by means of three ropes. The boy in the middle is exerting a pull of 100N. The other two boys, whose ropes both make an angle of 30^o with the centre rope, are pulling with forces 80N and 140N. Determine the magnitude of the resultant pull on the trolley.

839. ABCDEF is a regular hexagon of side 2 m. Forces of 3.5N, 4N, 6N, 1.5N, 3N and 2N act along the sides AB, BC, CD, ED, FE and FA respectively with the direction of the forces being indicated by the direction of letters. Find the;

a) magnitude and direction of the resultant force.

b) Equation of line of action of the resultant force and hence or otherwise find where it cuts AB.

840. ABCDE is a regular hexagon of side 4m. forces of magnitude 1N, 2n, 4N, 3N, 1N and 2N act along the sides AB,BC,CD,ED,FE and AF respectively, the order of the letters indicating the direction of the forces. Taking AB and AE as x and y reference axes respectively, find

(a) The magnitude and direction of the resultant force.

(b) The equation of the line of action of the resultant force hence find the distance from A where the resultant force cuts side AB.

841. The forces $\begin{pmatrix}0\\2\end{pmatrix}$, $\begin{pmatrix}0\\4\end{pmatrix}$, $\begin{pmatrix}0\\3\end{pmatrix}$ and $\begin{pmatrix}0\\a\end{pmatrix}$N act at points $(p,1)$, $(2,3)$, $(4,5)$ and $(6,1)$ respectively. The resultant is $\begin{pmatrix}0\\3\end{pmatrix}$N at point (1,1). Find the values of a and p.

842. ABCDE is a regular pentagon of side 4m. Forces of magnitude 2N, 3n 5N and 7N act along AB,BC,CD and EB respectively. The resultant of this system of forces cuts AB produced at H. Taking A as the origin and AB as the x-axis,

a) find magnitude and direction of the resultant force

b) show that the length AH = 15.34 m correct to 4.s.f.

c) find the perpendicular distance from A to the line of action of the resultant force.

843. A square ABCD of side 4m has forces of magnitude 8N, 3N, 3N, 4N and $2\sqrt{2}$N acting along AB, CB, DA, CD and BD respectively. Taking AB and AD as x and y axes respectively,

a) find the distance from A where the line of action of the resultant crosses AB.

b) when a couple of magnitude M is introduced, the force system is reduced to a single force passing through B. find M and its direction.

844. Coplanar forces $(3i+3j)N$, $(4i-5j)N$, $(-5i+2j)N$ and $(2i+3j)N$ act at points with position vectors $(3i+j)m$, $(i+3j)m$, $(-2i+j)m$ and $(-2i-2j)m$ respectively.

a) Find the resultant force and find where its line of action cuts the x-axis.

b) A couple of moment bNm acting anticlockwise and a force $(pi+qj)N$ acting at a point with position vector $(2i+j)m$ are now added to the above system. If these reduce the system to equilibrium, find the values of p,q and b.

845. Four forces $ai+(a-1)j$, $3i+2aj$, $5i+6j$ and $-i+-2j$ Newtons have their resultant acting in the direction making an angle of 45° with the horizontal. Find the value of a. Hence determine the magnitude of the resultant force.

846. Forces $F_1=\begin{pmatrix}3\\-5\end{pmatrix}N$, and $F_2=\begin{pmatrix}-3\\5\end{pmatrix}N$ act at points (6,1) and (4,1) respectively. Show that the forces reduce to a couple and find the moment of a couple.

847. ABCDEF is a rectangular hexagon of side 2m. Forces of 2N, 3N, 4N and 5n act along AC, AE, AF, and ED respectively. Taking AB as the horizontal, find the magnitude and direction of the resultant force.

848. A system of forces consists of two forces, F_1 and F_2 acting on a rigid body. $F_1=(-2i+j-k)N$ and acts at the point with position vector, $r_1=(i-j+k)m$. $F_1=(3i-j+2k)N$ and acts at the point with position vector, $r_2=(4i-j-2k)m$.

Given that the system is equivalent to a single force, FN, acting at the point with position vector $(5i+j-k)$m, together with a couple G NM, find:

a) F.

b) The magnitude of G.

849. Two inextensible strings AC and BC are used on a body of 30N at C. if the strings make 3.6^{o} and 31.2^{o} respectively with the downward vertical, find the tensions in the strings.

850. Forces PN, 4PN, 2PN and 6PN act along the sides AB, BC, CD and AD of a square ABCD of side a. Find the equation of the line of action of the resultant referred to AB as x-axis. Find also where the resultant cuts the axis.

NEWTON'S LAWS OF MOTION AND FRICTION

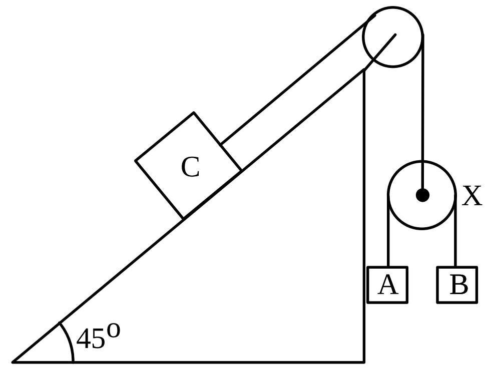
851. Two particles of mass A and B are connected by a light inextensible string passing over a smooth fixed pulley. Show that if $A > B$, the acceleration , a, of the system is given by $\frac{g}{A+B}(A-B)ms^{-2}$.

852. A mass of 4kg in contact with a smooth plane of 5 in 6 is connected by a light inelastic string passing over a smooth light pulley at the top of the plane and under a light moveable pulley carrying a mass of 14kg. the other end of the string is fixed to a point above the fixed pulley. If the parts of the string supporting the moveable are parallel and the system is released from rest, find the

a) Tension in the string

b) Acceleration of the 14kg mass.

853. The diagram below shows tow masses A and B of 0.5kg and 1kg respectively connected by a light inextensible string passing over a smooth pulley X of mass 0.5kg. Pulley X is connected to a mass C of 2kg lying on a smooth plane inclined at 45^{o} to the horizontal by a light inextensible string passing over a fixed pulley.



Find the:

a) Acceleration of masses B and C

b) Tension in the strings when the system is released.

854. A light inextensible string has a mass of 2 kg attached to one end. The string passes over a fixed pulley, under a moveable pulley of 6 kg, over another fixed pulley and has a 3 kg mass attached to its other end. If the system is released from rest and moves in a vertical position, find the;

a) acceleration of the masses.

b) tension in the string.

c) distance moved by the moveable pulley in 2 seconds.

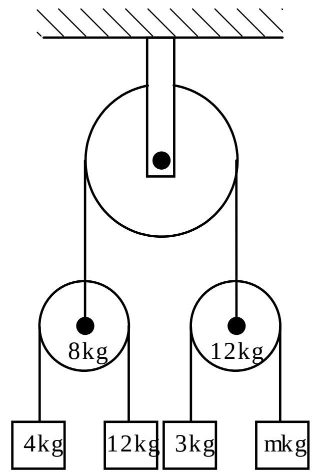
855. A string with one end fixed passes under a pulley A of mass M_1 then over a fixed pulley, then under a pulley B of mass M_2 and its other end is attached to the axle of A. the string is taut and its hanging parts are vertical. Find the ratio of the velocities of A and B when the system is in motion and show that the acceleration of A is $\frac{(4M_1 - 2M_2)g}{4M_1 + M_2}$.

856. To one end of a light inelastic string is attached a mass of 1kg which rests on a smooth wedge of inclination 30°. the string passes over a smooth fixed pulley at the edge of the wedge, under a second smooth moveable pulley of mass 2kg and over a third smooth fixed pulley, and has a mass of 2kg attached to the other end. find the accelerations of the masses and the movable pulley and the tension in the string. (Assume the portions of the string are vertical)

857. A box of mass 6.5kg is placed on a rough plane inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. The coefficient of friction between the box and the plane is 0.25. Find the least horizontal force required:

a) To move the box up the plane
b) To prevent the box from sliding down the plane.

858. The diagram below shows two pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.

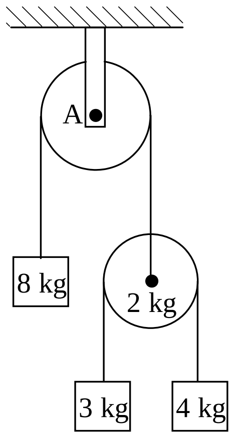


The acceleration of 4kg and 12kg masses are $\frac{g}{2}$ upwards and $\frac{g}{2}$ downwards respectively. The accelerations of the 3kg and mkg masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the:

a) Tensions in the strings.
b) Value of m.

859. A particle of mass M_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass M_2, its other end being fixed so that the parts of the string beyond the table are vertical. Show that M_2 descends with acceleration $\frac{M_2 g}{4M_1 + M_2}$.

860. The figure shows a light inextensible string passing over a smooth fixed pulley A, to one end of which is attached a mass of 8 kg and to the other end is attached pulley B of mass 2 kg. over B passes a second light inextensible string which carries masses of 3 kg and 4 kg at its free ends.



The system is released from rest.

Determine the;

a) Acceleration of the movable pulley, 3kg mass and 4kg mass.

b) Tensions in the strings.

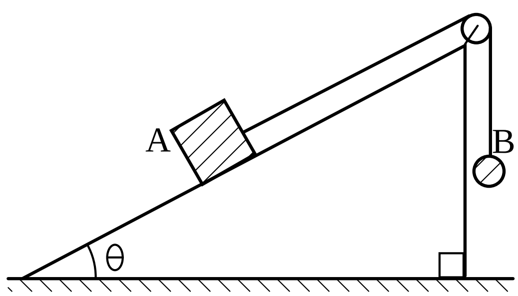
861. A mass of 4kg in contact with a smooth plane of 5 in 6 is connected by a light inelastic string passing over a smooth light pulley at the top of the plane and under a light moveable pulley carrying a mass of 14kg. The other end of the string is fixed to a point above the fixed pulley. If the parts of the string supporting the moveable pulley are parallel and the system is released from rest. Find th4e;

a) tension in the string

b) acceleration of 14kg mass.

862. A block of weight 20N rests on a rough plane of inclination 30° above the horizontal, the coefficient of friction being 0.25. find the horizontal force required to prevent it from just sliding down.

863.



a) A particle B of mass m kg keeps particle A of mass 10 kg at rest on a rough inclined plane of angle $\tan^{-1}\frac{4}{3}$. If the coefficient of friction between A and the inclined plane is 0.5, calculate the minimum and maximum values of m.

b) If the mass of A is doubled, calculate the magnitude of the accelerations of the particles.

864. A body of mass 4kg lies on a rough plane which is inclined at 30° to the horizontal. The angle of friction between the plane and the body is 15°. Find the magnitude of the least horizontal force that must be applied to the body to prevent motion down the plane.

865. A mass of 3kg is at rest on a smooth horizontal table. It is attached by a light inextensible string passing over a smooth fixed pulley at the edge of the table to another mass of mass 2kg which is hanging freely. The system is released from rest. Calculate the acceleration of the system and the tension in the string.

866. A light string ABCD has one end fixed at A and passing under a movable pulley of mass M_1 at B and over a fixed pulley at C, carries a mass M_2 at D. The portions of the strings not in contact with the pulleys are vertical. Show that M_1 descends with acceleration $\frac{(M_1 - 2M_2)g}{M_1 + 4M_2}$.

867. A mass of 12kg rests on a smooth inclined plane which is 6m long and 1m high. The mass is connected by a light inextensible string, which passes over a smooth pulley fixed at the top of the plane, to a mass of 4kg which is hanging freely. With the string taut, the system is released from rest. Find the acceleration of the system.

868. A particle of mass 2m rests on a rough plane inclined to the horizontal at an angle of $\tan^{-1} 3\mu$, where μ is the coefficient of friction between the particle and the plane. The particle is acted upon by a force of P Newtons.

a) Given that the force acts along the line of greatest slope and that the particle is on the point of slipping up, show that the maximum force possible to maintain the particle in equilibrium is $P_{max} = \frac{8\mu mg}{\sqrt{1+9\mu^2}}$.

b) Given that the force acts horizontally in a vertical plane through a line of greatest slope and that the particle is on the point of sliding down the plane, show that the minimum force required to maintain the particle in equilibrium is $P_{min} = \frac{4\mu mg}{\sqrt{1+3\mu^2}}$.

869. A particle of mass m is placed on a rough plane of inclination 30°. Given that the angle of friction. Show that the minimum force required

to move the up the plane is given by $\frac{1}{2}mg\left(\cos\lambda+\sqrt{3}\sin\lambda\right)$. If this is three times the least force that would cause the body to move down, show that $\lambda=\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

870. An object of mass 2000g is at rest on a plane inclined at 25^{o} to the horizontal. The coefficient of friction between the object and the plane is $\frac{2}{5}$. What minimum force applied parallel to the plane would move the object up the plane?

871. The force P acting along a rough inclined plane is just sufficient to maintain a body on the plane, the angle of friction λ being less than α, the angle of the plane. Prove that the least force, acting along the plane, sufficient to drag the body up the plane is $\frac{P\sin(\alpha+\lambda)}{\sin(\alpha-\lambda)}$.

872. A box of mass 2kg is at rest on a plane inclined at 25^{o} to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane?

873. A body of mass m kg is placed on a rough plane inclined at 30^{o} to the horizontal. Given that the angle of friction, λ exceeds 30^{o}.

a) Show that the minimum force, p, needed to move the body up the plane is $0.5mg\left(\cos\lambda+\sqrt{3}\sin\lambda\right)$ where g is the acceleration due to gravity.

b) Show also that the minimum force, P_1, required to cause the body to slide down the plane is $0.5mg\left(\sqrt{3}\sin\lambda-\cos\lambda\right)$.

874. A particle of mass ½ kg is released from rest and slides down a rough plane inclined at 30^{o} to the horizontal. It takes 6 seconds to go 3 metres.

a) Find the coefficient of friction between the particle and the plane.

b) What minimum horizontal force is needed to prevent the particle from moving.

875. A body of mass m kg rests on rough plane inclined at $\tan^{-1}2\mu$ to the horizontal. If the coefficient of friction is μ, show that the least horizontal force which will hold the body in equilibrium is $\frac{mg\mu}{\left(1+\mu^{2}\right)}$.

876. A particle of mass 5 kg is placed on a smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ to the horizontal. Find the magnitude of the force acting horizontally, required to keep the particle in equilibrium and the normal reaction to the plane.

877. A 2kg body lies on a plane of inclination 60^{o}. The coefficient of friction between the body and the plane is 0.25. Find the least horizontal force which prevents the body from sliding down the plane.

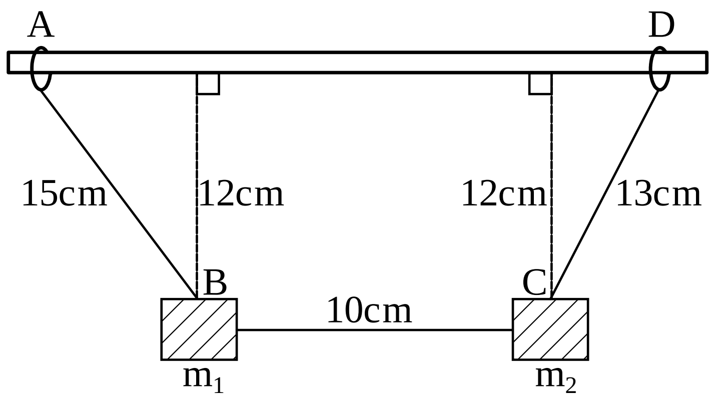
878. A particle of mass 12 kg slides down a plane inclined at 50^{o} to the horizontal. If the coefficient of friction between the particle and the plane is 0.4, calculate the acceleration of the particle.

879. A particle of weight 20N is placed on a rough plane inclined at an angle of 40^{o} to the horizontal. The coefficient of friction between the plane and the particle is 0.25. When a horizontal force P is applied on the particle, it rests in equilibrium. Calculate the value of P.

880. A body of mass 3kg is released from a rough surface which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If after 2.5s the body has acquired a velocity of $4.9ms^{-1}$ down the surface. Find the coefficient of friction between the body and the surface.

881. A car projected with a speed of $12ms^{-1}$ to move in a straight line on a rough horizontal surface comes to rest in 5 seconds. Calculate the distance it covers in its last second of motion.

882. The diagram below shows three strings. $\overline{AB}=15cm, \overline{BC}=10cm$ and $\overline{CD}=13cm$. A and D are fixed to small rings each of mass 2kg which can slide on a rough horizontal rail AD. Masses ,m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance 12cm below AD.



(a) Show that $9m_1 = 5m_2$.

(b) If the coefficient of friction between each ring and the rail is 0.25 and the ring A is on the point of slipping, determine the value of m_1

WORK, ENERGY AND POWER

883. a) A lorry of mass 800kg is pulling a trailer of mass 200kg up a hill of 1 in 14. When the total force of 1kN is exerted by the engine, the lorry and the trailer move up the hill at a steady speed. Find the total frictional resistance to the motion of the lorry and the trailer during the motion.

b) When the lorry and the trailer are travelling at a speed of $10ms^{-1}$ up the hill, the power exerted by the engine is instantaneously changed to 2kW. Calculate the

i). instantaneous acceleration.

ii). instantaneous tension in the coupling between the trailer and the lorry given that the total frictional resistance to the trailer is 70N.

884. A train of mass 250,000 kg starts from rest and moves along a straight level track against resistance of 0.06 Nkg^{-1}.

a) Find the driving force of the train if it attains a velocity of 40 ms^{-1} after $\frac{5}{3}$ minutes.

b) Calculate the power necessary to drive the train at a constant speed of $15ms^{-1}$ up an incline of 1 in 200.

885. A wooden block of mass 112kg is dragged across a rough horizontal floor by a force F Newtons inclined at 30^o above the floor at a constant speed. If the coefficient of friction between the block and floor is $\frac{2}{7}$. Find the;

a) Value of F.

b) Work done by dragging force in moving the block through 5.5m under the above condition.

886. A particle of mass 0.5 kg is projected with kinetic energy 750J up a plane inclined at 45^o above the horizontal. If the coefficient of friction is 0.25, find the work done against frictional force before the particle comes to rest.

887. A vehicle of mass 1200kg tows a trailer of mass 250kg up along an incline of $\sin^{-1}\left(\frac{1}{49}\right)$ above the horizontal. If the engine of the car is working at a constant rate of 4.2kn and that the resistance to motion of the car is four times that of the trailer, find the;

a) Resistance to motion of the vehicle when moving with a steady speed of 12 ms^{-1}.

b) Tension in the tow bar.

888. A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5m long and inclined at $\sin^{-1}\frac{4}{7}$ to the horizontal. The acceleration of the particle is 0.98 ms^{-2}.

Determine the:

a) Tension in the string.

b) Work done against gravity when the particle reaches the end of the slope.

889. A car of mass 1200kg pulls a trailer of mass 300kg up a slope of 1 in 100 against a constant resistance of 0.2N per kg. Given that the car moved at a constant speed of 1.5ms^{-1} for 5 minutes, calculate the;

a) tension in the tow bar.

b) work done by the car engine during this time a car has an engine that can develop 15kW. If the maximum speed of the car on a level road is 120kmh^{-1}, calculate the total resistance at this speed.

890. A water pump raises 40kg of water a second through a height of 20 m and ejects it with a speed of 45 ms^{-1}. Find the kinetic energy and the potential energy per second given to the water and the effective rate with which the pump is working.

891. Two constant forces F_1 and F_2, are the only forces acting on a particle, P, of mass 2 kg.The particle is initially at rest at a point A with position vector $(-2i-j-4k)$m.Four seconds later, P is at the point B with position vector $(6i-5j+8k)$m Given that $F_1=(12i-4j+6k)N$, Find;

a) F_2

b) The work done on P as it moves from A to B.

892. A car of mass 750kg moves along a horizontal road against a total resistance of 240N. If the car engine is working at a constant rate of 12kN, find the;

a) maximum velocity of the car.

b) acceleration of the car when its velocity is 30 ms^{-1}.

EQUILIBRIUM AND ACCELERATION UNDER CONCURRENT FORCES

893. A body of mass 60kg in contact with a smooth plane inclined at $\sin^{-1}\left(\frac{1}{40}\right)$ to the horizontal is released from rest. If the resistance to the motion is $\frac{1}{15}$N per kg, calculate the;

a) Acceleration of the body

b) Speed of the body 6 seconds after release.

894. A particle of mass 5kg is placed on a smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ to the horizontal. Find the magnitude of the force acting horizontally required to keep the particle in equilibrium and the normal reaction to the plane.

GENERAL EQUILIBRIUM OF A RIGID BODY & JOINTED RODS

895. A uniform ladder weighing 30kg and 8m long rests on a smooth wall and a rough ground with coefficient of friction 0.3 making an angle of 40° with the ground. A boy of mass 40kg climbs the ladder carrying a pan of sand weighing 5kg. If the ladder is on the verge of sliding.

a). Calculate the reaction on the ladder.

b). Show that the boy climbs approximately 0.69 m before it slides.

896. A heavy uniform rod of weight W is hung from a point by two equal strings. One attached to each end of the rod. A body of weight w is hung half between A and the centre of the rod. Prove that the ratio of the tensions in the string is $\frac{2W+3w}{2W+w}$.

897. A uniform ladder PQ of length *2a* and weight *w* is inclined at an angle of $\tan^{-1}2$ to the horizontal with its end Q resting against a smooth

vertical wall and end P on a rough horizontal ground with which the coefficient of friction is $\frac{5}{12}$. If a boy of weight W can safely ascend x up this ladder before it slips,

a) Show that $x = \frac{a(2w + 5W)}{3W}$.

b) Deduce that the boy can only reach the top of the ladder if $W = 2w$.

898. A uniform rod PQ of length 8m and weight 18N is freely hinged at P and carries a mass of 3kg at Q and to a point C distant 6m vertically above P. Find the:

a) Tension in the string.

b) Magnitude and direction of the reaction at the hinge.

899. A uniform bar AB of weight 2W and length L is free to turn about a smooth hinge at its upper end A, and a horizontal force is applied to the end B so that the bar is in equilibrium with B at a distance a from the vertical through A, Prove that the magnitude of the reaction at the hinge is $W\left(\frac{4L^2 - 3a}{L^2 - a^2}\right)^{\frac{1}{2}}$.

900. A non uniform ladder AB, 10 m long and off mass 8kg lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the centre of gravity of the ladder is 3m from the foot of the ladder, and the ladder makes an angle of 30^o with the horizontal, find the:

a) Coefficient of friction between the ladder and the ground.

b) Reaction at the wall.

901. A uniform ladder of weight W rests in limiting equilibrium with its top end against a rough vertical wall and its lower end on a rough horizontal floor. If the coefficients of friction at the top and foot of the ladder are $\frac{2}{3}$ and $\frac{1}{4}$ respectively.

a) Find the angle which the ladder makes with the floor.

b) Suppose a man of weight 3W ascends the ladder above, find how far he can ascend before the ladder slips.

902. A uniform rod AB of mass 5kg rests on a smooth horizontal floor at A and is supported at 4m from A. If the length of the rod is 6m with B

above A. Determine the reaction at the support, when the rod is resting at 60° above the floor.

903. A uniform ladder AB of mass 10kg stands on rough horizontal surface at A, and leans against a rough vertical wall at B, the coefficients of friction at A and B being $\frac{1}{2}$ and $\frac{1}{3}$ respectively. The angle of inclination of the ladder to ground is $\tan^{-1}\frac{3}{4}$. A boy of mass 40kg starts to climb the ladder.

Calculate the;

a) distance he climbs before the ladder starts to slide.

b) minimum horizontal force that should be applied at A so that the boy just reaches the top of the ladder.

904. A uniform ladder of mass 25kg is placed with its base on a rough horizontal floor (angle of friction $=\tan^{-1}\left(\frac{1}{5}\right)$), and its top against a rough vertical wall (angle of friction $=\tan^{-1}\left(\frac{1}{3}\right)$, with the ladder making an angle of 60° with the floor. Calculate the minimum horizontal force that could be applied at the base without slipping occurring.

905. A rod AB 1m long has a weight of 20N acting at a point 60cm from A. It rests horizontally with A against a rough vertical wall. A string BC is fastened to the wall at C, 75cm vertically above A. Find the;

a) Tension in the string

b) Frictional force at A.

906. A person whose weight is 7W climbed a uniform ladder AB of weight 2W whose end A is in contact with a rough horizontal ground and B resting against a smooth vertical wall. When he is a fifth of the way up, the ladder is about to slide. If the coefficient of friction between the ladder and the ground is $\frac{4}{15}$. Find the angle that the ladder makes with the ground.

907. A uniform ladder of length 2L and weight W rests in a vertical plane with one end against a rough vertical wall and the other end on a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\left(\frac{1}{3}\right)$ and $\tan^{-1}\left(\frac{1}{2}\right)$ respectively.

a). Find θ, the angle of inclination of the ladder, how far will he climb before the ladder slips?

b). A man of weight 10W begins to ascend the ladder, how will he climb before the ladder slips?

908. A non uniform ladder AB of length 6m and mass 10kg has its centre of gravity at G, where AG = 4m. The ladder is inclined at 45° to the horizontal with its end B resting against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If a boy of mass 40kg can safely ascend 2m up this ladder before it slips, find the value of μ.

909. A non-uniform ladder AB is in equilibrium with A in contact with a horizontal floor and B in contact with a vertical wall. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is at G where $AG = \frac{2}{3}AB$. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the ladder makes an angle θ with the wall and the angle of friction between the ladder and the floor is λ.

c) Prove that $4\tan\theta = \tan 2\lambda$.

d) How far can a man of mass m ascend the ladder without the ladder slipping given that $\tan\theta = 45^{\circ}$ and the coefficient of friction between the ladder and the floor is $\frac{1}{2}$.

910. A uniform beam AB supported at an angle θ to the horizontal by a light string attached to end b, and with A resting on a rough horizontal ground (angle of friction, λ). The beam and the string lie in the same vertical plane and the beam rests in limiting equilibrium with the string at right angles to the beam. Prove that $\tan\lambda = \frac{\sin 2\theta}{3 - \cos 2\theta}$.

911. A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical and is held horizontally in equilibrium by a string which has one end attached to B and the other end attached to a point C on the wall, 4m above A. find the magnitude and the direction of the reaction at A.

912. A rod AB of length 0.6m and mass 10kg is hinged at A. its centre of mass is 0.5m from A. A light inextensible string inextensible string attached at B passes over a smooth fixed pulley 0.8m above A and

supports a mass M hanging freely. If a mass of 5kg is attached at b so as to keep the rod in a horizontal position, find;

a) The value of M

b) Reaction at the hinge.

913. A uniform beam AB of weight 30N is suspended by two strings at A and B. The beam is in equilibrium at 30^o and 60^o with the strings A and B respectively. Find the tensions in the strings.

CENTRE OF GRAVITY

914. OAB is a uniform triangular lamina with vertices O(0,0), A(9,0) and B(6,6).

a) Find the coordinates of the centre of gravity of the lamina

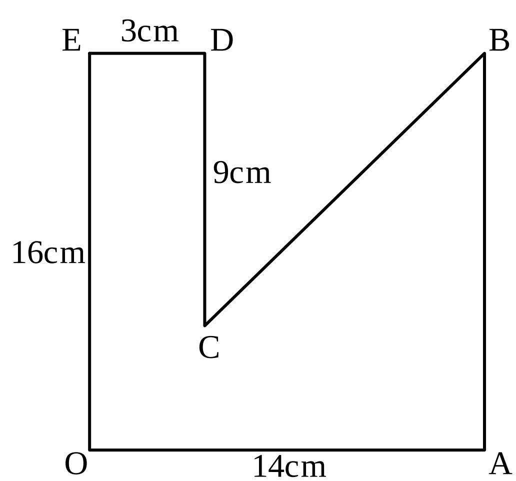
b) If the lamina is freely suspended at point O(0,0), calculate the angle the side OA makes with the vertical.

915. ABCD is a square lamina of side a from which a triangle ADE is removed E being a point on CD of distance t from C.

Show that the centre of mass of the remaining lamina is at a distance $\frac{a^2+at+t^2}{3(a+t)}$ from BC.

Hence show that if the lamina is placed in a vertical plane with CE on a horizontal table, equilibrium will not be possible if t is than $\frac{a(\sqrt{3}-1)}{2}$.

916. a) Find the centre of gravity of the uniform lamina shown below.



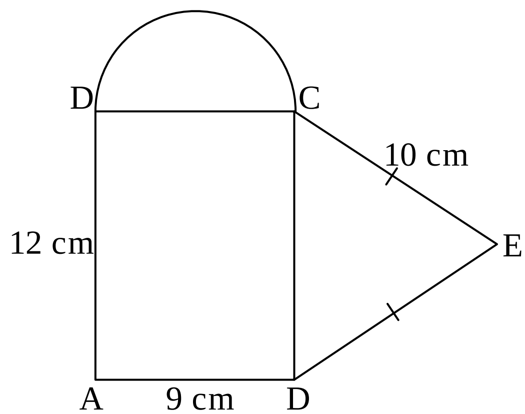
b) if the lamina is suspended from O, find the angle that OB makes with the vertical.

917. A sheet of paper is in the shape of a rectangle ABCD in which AB = 12cm, AD = 8cm. E is the midpoint of AD and the triangle CED is removed.

e) Find the distance of the centre of gravity of the remainder of the sheet from SD and AB.

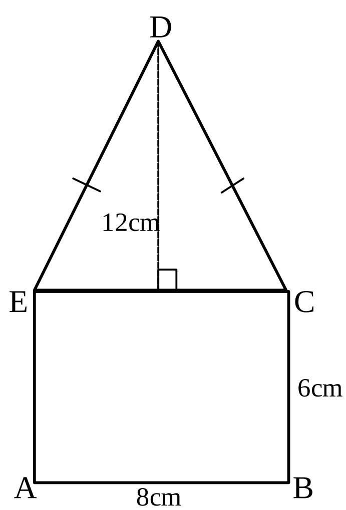
f) If the remainder of the sheet is hung by a light inextensible string through point A, find the angle AB makes with the vertical.

918. The figure below represents a lamina formed by welding together rectangular, semicircular and triangular metal sheets.



Find the position of the centre of gravity of the lamina from the sides AB and AD.

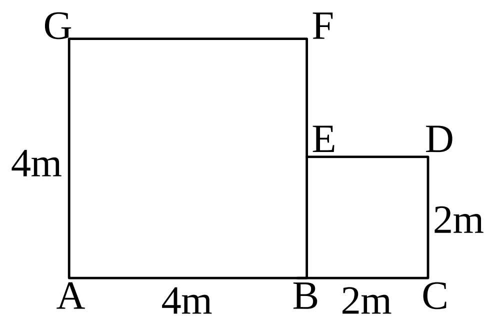
919. Two uniform laminae made of the same material are joined as shown in the figure below.



ABCE is a rectangle and DCE is an isosceles triangle with dimensions shown. Find the position of the centre of gravity of the complete lamina from AB.

920. Show that the centre of gravity of a solid cone of radius r and height h lies along its axis at a distance $\frac{h}{4}$ from the base.

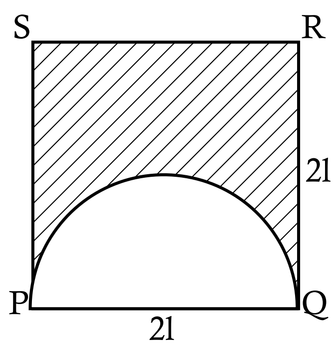
921. The diagram shows two uniform squares ABFG and BCDE joined together.



a) Find the centre of gravity of the composite body.

b) If the body is suspended freely at G, determine the angle AG makes with the horizontal.

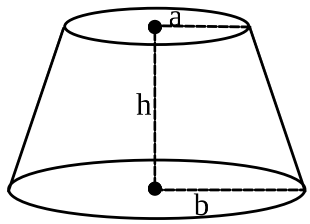
922. The figure below shows a uniform square lamina PQRS of side 2l with a semicircular cut off.



a) Show that the distance of the centre of gravity of the figure from PQ is $\frac{20l}{3(8-\pi)}$.

b) The figure is freely suspended from the point R. Find the angle that RS makes with the vertical.

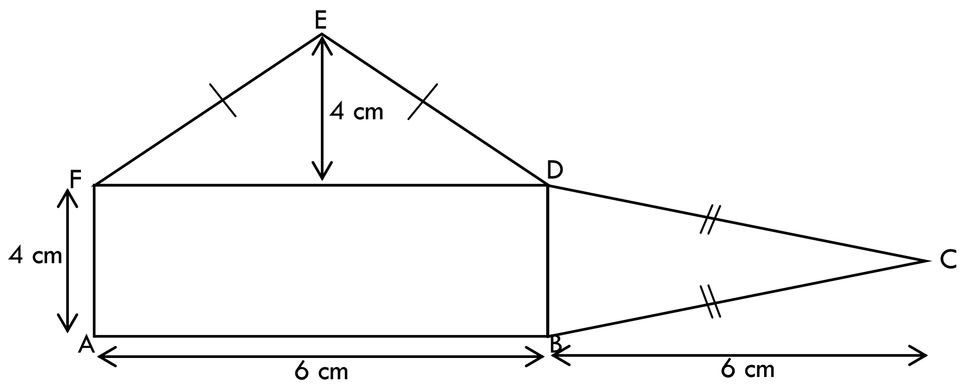
923. The figure below shows a solid conical frustum of height h and whose top and bottom radii being a and b respectively.



Show that the centre of gravity of this frustum lies along its axis at a distance $\frac{h\left(b^2+2ab+3a^2\right)}{4\left(b^2+ab+a^2\right)}$ from the bottom.

924. Four uniform rods AB,BC,CD and DA are each 4 metres in length and have masses of 2kg, 3kg, 1kg and 4kg respectively. If they are joined together to form a square frame work ABCD, find the position of its centre of gravity from AB.

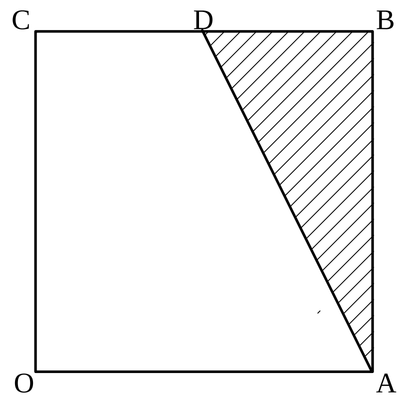
925. The figure ABCDEF is made up of three laminas that are as indicated in the diagram below.



a) Determine the centre of gravity from AB and AF taken as the x and y axes respectively (state it as a coordinate).

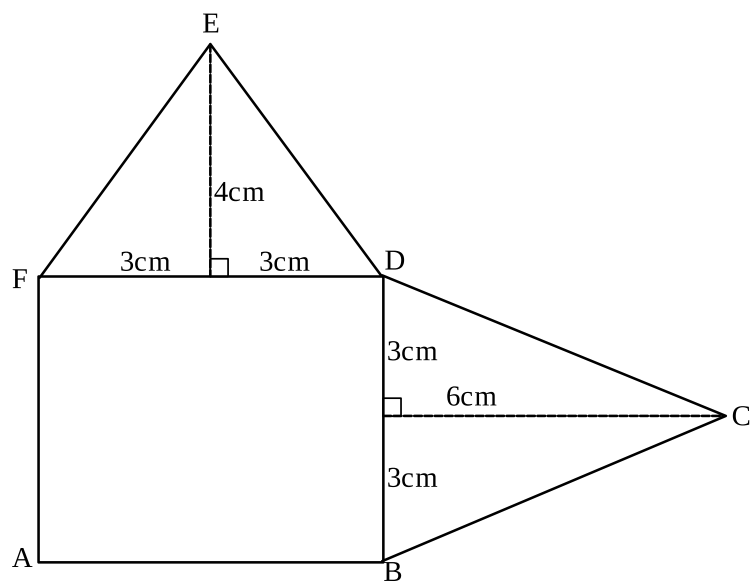
b) The lamina is freely suspended through a smooth pivot at A and hangs in equilibrium under its own weight, find the angle θ between the side AB and the vertical.

926. The figure below shows a uniform square lamina OABC of side a from which a triangular piece ABD has been cut. D is the midpoint of BD.



Find the position of the centre of mass of the lamina OADC.

927. The figure ABCDEF is made up of three laminas that are as indicated in the diagram below.



Determine the centre of gravity of the composite lamina ABCDEF from AF and AB taken as the y and x axes respectively. State it as a co-ordinate.

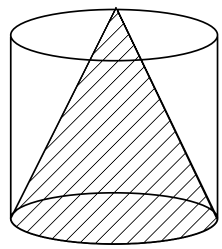
928. ABCD is a square lamina of side "a" from which a triangle ADE is removed. E being a point on CD at a distance "t" from C. Show that the centre of mass of the remaining lamina is at a distance of $\frac{a^2 + at + t^2}{3(a+t)}$ from BC.

929. A body consists of a solid hemisphere of radius r joined to a right circular cone of base radius r and perpendicular height h. the plane

surfaces of the cone and hemispheres coincide and both solids area made of the same uniform material.

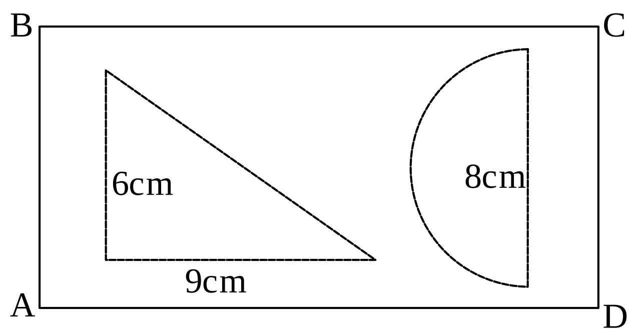
Show that the centre of gravity of the body lies on the axis of symmetry at a distance $\frac{3r^2 - h^2}{4(h+2r)}$ from the base of the cone.

930. A uniform cylindrical piece of metal of height h and radius r has a cone shape removed from it as shown in the diagram. The base of the cone is of radius r and its height of h.



Show that the centre of gravity of the resulting solid is at a distance of $\frac{3}{8}h$ from the point at the vertex of the cone.

931. Below is a uniform rectangular lamina of dimensions $10\text{cm} \times 16\text{cm}$ from which right angled triangular and semicircular pieces of dimensions shown have been cut out.



The straight side of the semicircular cut out is 1cm from the side CD and placed centrally between BC and AD. The perpendicular sides of the triangular cut-out are 2cm from the nearest sides of the lamina.

Find the position of the centre of gravity of the remaining lamina.

MOMENTUM & VECTOR MECHANICS AND RELATIVE MOTION

932. Three particles A,B and C lie close together on a smooth plane. A is connected to B and C by light inextensible strings. If B is set moving with velocity V across the plane. Find the:

a) First impulsive tension in the string AB
b) Initial velocity of A
c) Initial velocity of C

933. A hammer of mass 1kg, moving with a velocity of 6 ms^{-1} drives a nail of mass 30g, 2.5 cm into a fixed piece of wood downwards. Find the:
a) Common velocity of hammer and the nail just before impact.
b) Percentage loss in energy
c) Time of motion of the nail,
d) Force of resistance of the wood assuming it to be constant.

934. A boy wishes to swim across a river 100m wide as quickly as possible. The river flows at 3 kmh^{-1} and the body can swim at 4 kmh^{-1} in still water. Find the time it takes the boy to cross the river and how far down stream he travels.

935. A gun of mass 200kg fires horizontally a shell of mass 20kg. the gun's horizontal recoil is controlled by a constant force of 8000N which brings the gun to rest in 1.5seconds. Find the initial velocity of the shell.
a) Relative to the gun
b) In the air

936. Boat A is sailing due East at 18 kmh^{-1} and a second ship B sailing on a bearing of 030° at 12 kmh^{-1}. At a certain instant a third boat C appears to an observer on A to be sailing south and to an observer on ship b to be sailing on a bearing of 150°. find the speed of boat C and the bearing on which it is sailing.

937. A car of mass 300kg moving at 144kmh^{-1} collides with a stationary trailer of mass 900kg thereby losing its momentum by 15%. If the car decelerates at 6ms^{-2} after collision, calculate the;
a) trailer's velocity after collision,
b) distance the car would have to move before stopping.
c) deceleration force.

938. An aircraft capable of flying at 250 kmh^{-1} in still air is to be flown from airport A to airport B situated 300km from it on a bearing of 320°. If there is a wind of 50 kmh^{1} blowing from 030°, find the course that the pilot must set so as to reach B and the time taken to the nearest minute.

939. A gun of mass M fires a shell of mass m and recoils horizontally. Given that the shell travels along the barrel with speed v, find in terms of v, m and M, the speed with which the barrel begins to recoil if;

a) the barrel is horizontal
b) the barrel is inclined at an angle of 30° tot eh horizontal.

940. Two cyclists P and Q are travelling along straight roads which cross at an angle of 60° at point C. If their riding speeds towards C are 4 kmh^{-1} and $5kmh^{-1}$ and they are respectively 8km and 15km from C, find the:
a) Least distance between the cyclists
b) Time that elapses before the cyclists are closest
c) Distances of P and Q from C when they are nearest.

941. A force $(24ti - 12j)N$ acts on a particle of mass 2kg initially at rest at a point with position vector $(-4i + 3j)m$. Find the:
a) Velocity of the particle after t seconds.
b) Distance from the origin after 2 s.
c) Power exerted by the force at t = 2s.
d) Work done by the force between t = 1s and t = 2s.

942. Two particles A and B of equal mass are travelling along the same line with constant speed $4ms^{-1}$ and $3ms^{-1}$ respectively. If they collide and coalesce, find their common speed just after impact:
a) If they collide head on
b) If they were originally travelling in the same sense.

943. A bullet of mass 50g is fired horizontally into a freely suspended block of mass 2kg attached at the end of an elastic string of length 2m. Given that the bullet gets embedded in the block and the string is deflected through an angle of 60° to the vertical. Find the:
a) Maximum velocity of the block
b) Initial velocity of the bullet.

944. Two boats P and Q are sailing with respective speeds of $20kmh^{-1}$ and $19kmh^{-1}$. Initially P is 10km from Q on a bearing of 320° and is on a course of 200°. Find the:
a) Two possible courses Q can take in order to intercept P.
b) Time taken for interception to occur in each case.

945. A bullet of mass 50 grams travelling horizontally at 80 ms^{-1} hits a block of wood of mass 10 kg resting on a horizontal plane. If the bullet emerges with a speed of $50ms^{-1}$, find the speed with which the block moves.

946. a) At certain times, the position vector **r** and velocity vector **v** of two ships A and B are as follows.

$r_A = (-2i + 3j)km$, $V_A = (12i - 4j)kmh^{-1}$ at 11:45 am

$r_B = (8i + 7j)km$, $V_B = (2i - 14j)kmh^{-1}$ at 12:00 noon

If the ships maintain these velocities, find the:

i) Position vector of ship A at noon
ii) Time when the ships are closest
iii) Shortest distance between the ships
iv) Distance of ship A from the origin when the two ships are closest.
b) If instead ship B had a velocity had a velocity $V_B = (-2i - 14j)kmh^{-1}$, show that the ships will collide and find when and where the collision occurs.

947. A hammer of mass 6kg moving vertically downwards with a speed of 15 ms^{-1} strikes the top of the vertical post of mass 4kg without rebounding. If the two move together for 0.3s before coming to rest, what is the average resistance to motion?

948. A battleship and a patrol ship are initially 16km apart with the battleship on the bearing of 035^o from the patrol. The battleship sails at $14kmh^{-1}$ in the direction $S30^oE$ and the patrol ship at $17kmh^{-1}$ in the direction $N50^oE$.

a) Find the;
i. Shortest distance between the ships
ii. Time that elapses before the ships are closest
b) If the guns on the battleship have a range of up to 6km, find the time that elapses when the patrol ship is within range of these guns.

949. A boat travelling at a speed of 18 kmh^{-1} in the direction of $N30^oE$ in still water is blown by wind moving at a speed of $8ms^{-1}$ on a bearing of 150^o. Calculate the speed and the course the boat will be steered.

950. At 12:00 noon, the position vector R and velocity vector V of two ships A and B are as follows.

Ship	Position vector (r)	Velocity vector (v)
A	$r_A = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} km$	$V_A = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} kmh^{-1}$
B	$r_B = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} km$	$V_B = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} kmh^{-1}$

a) Determine the displacement of A relative to B.
b) If the two ships collide, determine the time and position of collision.

951. At 10:00 am, ship A and ship are 16 km apart. Ship A is on a bearing of $N35^oE$ from ship B. Ship A is travelling at 14 kmh^{-1} on a bearing $S29^oE$. Ship B is travelling at 17 kmh^{-1} on a bearing $N50^oE$, determine the;

a) Velocity of ship B relative to ship A.
b) Closest distance between the two ships and the time when it occurs.

952. At noon ship A is sailing due East at a const velocity of $20kh^{-1}$. At the same time ship B is sailing in a direction N 60^o E at a constant velocity of 15 km^{-1}. If they continue sailing with these velocities in these directions , determine the;
a) Time at shortest distance
b) Shortest distance

953. A particle with a position vector $2i+3k$ at $t=0\,s$ moves with a constant speed of 6 ms^{-1} in the direction $i-2j+2k$. Find the position vector of the particle after 2s.

954. To a motorist travelling due North at $40kmh^{-1}$, the wind appears to come from the direction $N60^oE$ at $50kmh^{-1}$.
a) Find the true velocity of the wind.
b) If the wind velocity remains constant but the speed of the motorist is increasing; find his speed when the wind appears to be blowing from the direction $N45^oE$.

955. A body moving with an acceleration $e^{2t}i-3\sin 2tj+4\cos 2tk$ is initially located at the point (1,-2,2)m and has a velocity of $4i-2j+4+k$ ms^{-1}. Find the speed of the body when $t=\frac{\pi}{4}s$.

956. A particle P of mass 0.5kg moves on a horizontal plane such that its velocity vector v ms^{-1} at time t seconds is given by $v = 12\cos(3t)\mathbf{i} - 5\sin(2t)\,\mathbf{j}$.
a) Find an expression for the force acting on P at time t seconds.
b) Given that $t=0$, P has a position vector $(4\mathbf{i} + 7\mathbf{j})$m relative to the origin O, find an expression for the position vector of P at a time t s.
c) Hence determine the distance of P from O at time $t=\frac{\pi}{2}$s.

957. A force $F = (t^2\mathbf{i} + 3t\,\mathbf{j} + 4\mathbf{k})$N acts on a body of mass 2kg. Initially the body is at rest at a $(2\mathbf{i} - \mathbf{j} + \mathbf{k})$. Find the:
a) Speed of the body after 5s.
b) Distance of the body from the origin after 2s.
c) Work done by the force after 4s.

958. A particle of mass 2 kg is acted upon by a force $F = 54t^3\mathbf{i} + 24t^3\,\mathbf{j} - 18t\,\mathbf{k}$ where t is time. Initially, the particle is located at a point with position vector (1, 0, 0) m and moving with velocity (1, 0, 1) ms^{-1}.
a) Determine its distance from the origin after 2s.
b) Determine the work done in the time interval $t = 1s$ to $t = 2s$.

959. A pile driver machine of mass 8 tonnes falls from a height of 500cm onto a pile of mass 2 tonnes. Given that the average resistance of the ground is 10^6N and that $g = 10\,ms^{-2}$, find the;

a) Speed at which the pile driver strikes the pile.
b) Common speed of the pile and the driver
c) Distance the pile penetrates into the ground.

960. A particle of mass 4kg starts from rest at a point $(2i - 3j + k)m$. It is acted upon by a force $F = (4i + 12tj - 3k)N$. Find the ;

a) Acceleration at any time,t.
b) Velocity at any time, t.
c) Work done by the force, F after 2 seconds.

961. A shell of mass 5kg is fired from a gun of mass 2,000kg, the shell leaves the gun with a speed of 400 ms^{-1}. Determine the;

a) Speed of recoil o f the gun.
b) Retardation when the retarding force of the gun is 4,000N.

962. A particle is initially at position (3,-1,4)m and has velocity $(v) = \left[(2t^3 + 16)i + \left(\frac{9}{2}t^2 - 4t + 15\right)j + \left(-\frac{3}{2}t^2 - 8\right)k\right]ms^{-1}$, determine the;

a) Acceleration and its magnitude at $t = 3s$.
b) Displacement and distance at $t = 2s$

963. The initial velocity of a particle moving with a constant acceleration is $(3i - 5j)ms^{-1}$. After 2 seconds the velocity of the particle is of magnitude $6ms^{-1}$ and parallel to $(i - j)$. Find the acceleration of the particle.

964. At time $t = 0$, two particles A and B have position vectors $(2i + 3j - 4k)m$ and $(8i + 6k)m$ respectively.

Particle A moves with constant velocity $(-i + 3j + 5k)ms^{-1}$ and B with constant velocity, $v\,ms^{-1}$. Given that when $t = 5$ seconds, B passes through the point A passed through one second earlier. Find v.

965. The velocity of a particle after t seconds is $12t^2i + (8t + 23)j\,ms^{-1}$. Calculate the average speed of the particle in the time interval $t = 1$ to $t = 3s$.

966. A particle of mass 2kg moves under the action of a force which depends on the time t given by force $F = 24t^2i + (36t - 16)j$ Newtons. Given that at $t = 0$ the particle is located at $3i - j$ and has velocity $6i + 15j$. Find the kinetic energy of the particle at $t = 2$.

967. A tourist vehicle is on a bearing of 050° from a lion. The vehicle is travelling at a constant speed of 10 ms^{-1} due south. The lion runs after the vehicle at a constant speed of 6 ms^{-1}.

a) In which direction should the lion run to get as close to the vehicle as possible?

b) What will be the least distance between the lion and the vehicle.

c) How long will the lion take to achieve the least distance?

968. Initially a particle is projected with a velocity $2i$ ms^{-1} from a point with position vector $(10i+90j)$m. Find the distance of the particle from the origin after 4 seconds.

969. A particle is acted upon by two forces F_1 and F_1 where;

$F_1 = 2i - j - tk$ and $F_2 = i - 4tj + 3tk$, at a time t. the particle is initially at rest. Find the momentum of the particle 5 seconds later.

970. An object P passes through a point whose position vector is $3i-2j$ with constant velocity $i+j$. At the same instant an object Q moving with a constant velocity $4i-2j$ passes through the point with position vector $i+4j$. Find the ;

a) the displacement of P relative to Q after t seconds.

b) the time when P and Q are closest together and the closest distance at that time.

971. A gun of mass 3000kg fires horizontally a shell at an initial velocity of 300 ms^{-1}. If the recoil of the gun is brought to rest by a constant opposing force of 9000N in two seconds, find the;

a) (i) initial velocity of the recoil gun

(ii) mass of the shell

(iii) gain in kinetic energy of the shell just after firing

b) (i) displacement of the gun

(ii) work done against the opposing force.

972. At 12 noon, ship A is 8km due east of ship A is moving due north at a constant speed of 10 km h^{-1} . Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible. Determine:

a) the bearing on which ship B moves.

b) the shortest distance between the two ships.

c) the time when the two ships are closest.

d) The distance travelled by each ship from its respective position at noon to the point of closest approach.

973. At noon, ships A and B are 65km apart with B due south of A. Ship A is sailing due East at a constant velocity of 20kmh^{-1}. At the same time ship B is sailing in the direction N60^{0}E at a constant velocity of 15 kmh^{-1}. If they continue sailing with these velocities in these directions, Determine the;

a) time at shortest distance

b) shortest distance.

974. At time 9:00am, the position vectors and velocity of two particles A and B are as follows;

$$R_A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} m, V_A = \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} ms^{-1}, R_B = \begin{pmatrix} 4 \\ -14 \\ 1 \end{pmatrix} m, V_B = \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix} ms^{-1}$$

Find;

a) the position vector of B relative to A at any time t seconds.

b) the value of t when A bad B are closest together.

c) the least distance between A and B.

CIRCULAR MOTION, ELASTICITY AND SIMPLE HARMONIC MOTION

975. a) A car travelling at 30 ms^{-1} has no tendency to side slip on a track of radius 250m banked at θ to the horizontal. Find the angle θ.

b) If the speed is increased to 40 ms^{-1}. It is about to slip upwards, determine the co-efficient of friction.

c) Hence find the minimum speed required for it to slide downwards.

976. It is rumored that Mbale clock tower round about whose radius is 8m makes a truck whose distance between the inner and outer wheels is 1.3 m and its centre of gravity 0.8 m above the ground to topple if the truck's speed exceeds 72 kmh^{-1}. Find the angle at which the roundabout must be inclined and the least coefficient of friction to avoid skidding of any form.

977. A light elastic spring has its upper end A fixed and a body of mass 0.6kg attached to its other end B. If the modulus of the spring is 4.5gN and natural length 1.5m.

c) Find the extension of the spring when the body hangs in equilibrium

d) The end B of the spring is pulled vertically downwards to C where BC = 10cm. Find the initial acceleration of the body when released from this position.

978. A particle executes simple harmonic motion about centre, O, with amplitude 5m and period $\frac{\pi}{2}$s. Find the distance it travels from O until when its speed is half the maximum value.

979. A particle moving in a straight line with simple harmonic motion between two points A and B has periodic time $\frac{\pi}{4}$ seconds. If $AB = 1m$. Find the;

a) velocity of the particle when at a distance of 25 cm from A.

b) maximum acceleration of the particle.

980. A particle is describing simple harmonic motion in a straight line directed towards a fixed point, O. when its distance from O is 3m, its velocity is 27 ms^{-1} and its acceleration is 81 ms^{-2}. Determine the amplitude of oscillation.

981. Two equal particles are connected by a string passing through a hole in a smooth table, one particle being on the table, the other underneath. How many revolutions per minute would the particle on the table have to perform in a circle of radius 15cm in order to keep the other particle at rest?

982. A particle executes simple harmonic motion. If it has speeds of 8ms^{-1} and 6 ms^{-1} at points at respective distances of 3m and 4m from the centre of motion; calculate the;

a) amplitude and period of motion

b) time the particle takes to move directly from A to B.

983. The work done in compressing a spring of natural length 3l to a length of 2l is twice as great as the work done in doubling the length of a string of natural length 2l. Show that their moduli of elasticity are in the ratio 12:1.

984. A particle is tied by an elastic string of length 30cm to a fixed point on a smooth horizontal table upon which the particle is describing a circle around the point at a constant speed. If the modulus of elasticity of the

string is equal to the weight of the particle and the number of revolution per minute is 20, show that the extension produced is nearly 5cm.

985. A particle is placed on the lowest point of a smooth spherical shell of radius $3am$ and is given a horizontal velocity of $\sqrt{13ag}\,ms^{-1}$. How high above the point of projection does the particle rise?

986. A body of mass 0.5kg is suspected from a fixed point A by a light elastic string of natural length 4cm and modulus of elasticity 19.6N. the particle is pulled vertically downwards to a point *d* cm below equilibrium position and released. If it just reaches the level of A, calculate the;

a) value of *d*.

b) kinetic energy of the particle when it passes the equilibrium position for the first time.

987. A particle of mass m is held on the surface of a fixed smooth solid sphere centre O and radius a at point P such that OP makes an angle $\cos^{-1}\left(\frac{3}{4}\right)$ with the upward vertical and then released. Prove that when OA makes an angle θ with the upward vertical , the velocity, v of the particle is given by $v^2 = \frac{1}{2}ga(3 - 4\cos\theta)$ provided that the particle remains on the surface of the sphere and find the normal reaction on the particle at this time.

988. A body lies on a smooth horizontal table and is connected to a point O on the table by a light elastic string of natural length 1.5m and modulus 24N. Initially the body is at p, 1.5m from o. the body is pulled directly away from O and held at a point Q 2m from O and then released. Find the:

a) Initial energy stored in the string when the particle is at P
b) Energy stored in the string when the body is held at Q
c) Kinetic energy of the body when the body passes through P after release from Q.

989. A particle of mass M hangs at rest from the end of a string of length L. The particle is projected horizontally with speed U and so starts to move in a vertical circle. Assuming that the particle continues to move in a

circle, show that the tension in the string when it makes an angle θ with its initial position is given by $T = \frac{Mu^2}{l} - 2gl + 3gl\cos\theta$.

990. A body of mass 2.5kg is attached to the end B of a light elastic string AB of natural length 2m and modulus 5gN. The mass is suspended vertically in equilibrium by the string whose other end A is attached to a fixed point. Find the;
   e) Depth below A of B when the body is in equilibrium.
   f) Distance through which the body must be pulled down vertically from its equilibrium position so that it will just reach A after release.

991. A and B are two points on the same horizontal level and 48cm apart. A light elastic string of natural length 40cm has one end attached to A and the other at B. A body of mass 200g is attached to the midpoint of the string and hangs in equilibrium at a point 7cm below level A and B. Find the modulus of the string.

992. An elastic string of natural length 120cm and modulus of elasticity 8N is stretched until the extending force is 6n. Calculate the
   a) Extension of the string
   b) Work done by the string.

993. A particle executing simple harmonic motion about point O has a velocity of $3\sqrt{3}ms^{-1}$ and $3ms^{-1}$ when at distances 100cm and 26.8cm respectively, from the end point. Calculate the amplitude of the motion.

994. A particle of mass 2kg is suspended from the end of a light elastic string of natural length 1.2m and modulus of elasticity 60N. the particle is then pulled vertically downwards through a distance of 20cm and released from rest.
   a) Find the extension in the string when the particle is
   b) at equilibrium point.
   c) Show that the subsequent motion is simple harmonic and find the periodic time.
   d) Find the distance of the particle from equilibrium point 2 seconds after being released.

995. A light spring of natural length 60cm and modulus 3gN has one end fixed and a body of mass 2kg is freely suspended from its other end.
   a) Find the extension of the spring.

b) What mass would cause the same length of extension when suspended to a spring of natural length 50cm and modulus 2gN?

996. A particle moving with simple harmonic motion has speeds of $5ms^{-1}$ and $8ms^{-1}$ when at distances of 16m and 12m respectively from its equilibrium position. Find the amplitude and period of the motion.

997. A body of mass M lies on a smooth horizontal surface is connected to a point O on the surface by a light elastic string of natural length l and modulus λ. When the body moves with a constant speed V around a horizontal circular path, centre O, the extension in the string is $\frac{1}{4}l$. Show that $\lambda = \frac{16MV^2}{5l}$.

998. A spring whose weight can be neglected is fixed in the vertical position and a weight W resting on it produces a compression a. Show that if the weight W is left to fall of the spring from a height of $\frac{3a}{2}$ above it, the maximum compression of the spring in the motion which follows is 3a.

999. A light elastic string is of natural length 50cm and modulus 147N. one end of the string is attached to a fixed point and a body of mass 3kg is freely suspended from the other end. Find;

a) the extension of the string in the equilibrium position.

b) the energy stored in the string.

1000. A particle is travelling in a straight line with SHM of period 4 seconds. If the greatest speed is $2ms^{-1}$, find the amplitude of the path and the speed of the particle when it is $\frac{3}{\pi}$ m from the centre.

SUCCESS