

PROPOSED MTC 2 UNEB GUIDE 2024.

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No. 1

$$P(A) = 0.4 \quad P(B) = 0.7 \quad P(A \cap B) = 0.35$$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$$

M₁

From the Contingency table.

$$P(A' \cap B') = P(B') - P(A \cap B')$$

$$= P(B) - P(A) + P(A \cap B)$$

$$= 0.3 - 0.4 + 0.35$$

$$= 0.25$$

M₁

B₁

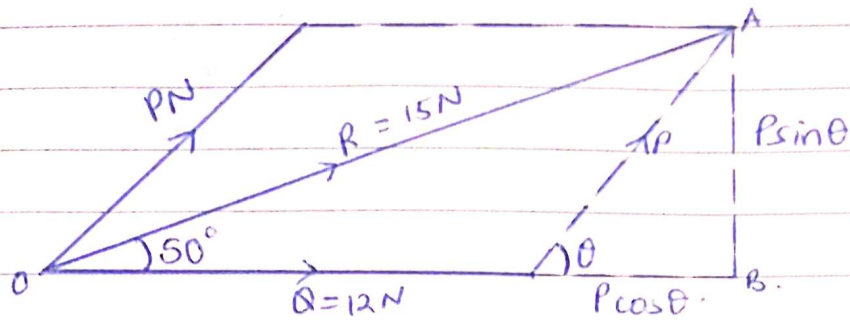
$$P(A'/B') = \frac{0.25}{0.3}$$

M₁

$$= \frac{5}{6} \text{ or } 0.8333.$$

A₁

No. 2.



Consider triangle OAB.

$$\cos 50^\circ = \frac{12 + P \cos \theta}{15}$$

$$P \cos \theta = 15 \cos 50^\circ - 12 \quad \text{--- (1)}$$

$$\text{Also } P \sin \theta = \frac{P \sin \theta}{15}$$

$$P \sin \theta = 15 \sin 50^\circ \quad \text{--- (2)}$$

Squaring and adding (1) and (2).

$$P^2 (\sin^2 \theta + \cos^2 \theta) = (15 \sin 50^\circ)^2 + (15 \cos 50^\circ - 12)^2$$

$$P = \sqrt{137.5964}$$

$$P = 11.73015 \text{ N}$$

$$P \sin \theta = 15 \sin 50^\circ$$

$$\sin \theta = \frac{15 \sin 50^\circ}{11.73015}$$

$$\theta = 78.4^\circ$$

No. 3.

(a).	$t(s)$	1	1.5	2
	$V(m\bar{s}^{-1})$	2	$\sqrt{\quad}$	7

$$\frac{V-2}{1.5-1} = \frac{7-2}{2-1}$$

$$\frac{V-2}{0.5} = \frac{5}{1}$$

$$V-2 = 5 \times 0.5$$

$$V = 2 + 2.5$$

$$V = 4.5 \text{ m}\bar{s}^{-1}$$

M₇

A₇

(b)

$t(s)$	3	4	t
$V(m\bar{s}^{-1})$	8	10	13

$$\frac{t-4}{13-10} = \frac{4-3}{10-8}$$

$$\frac{t-4}{3} = \frac{1}{2}$$

$$t-4 = 3 \times 0.5$$

$$t = 4 + 1.5$$

$$t = 5.5 \text{ seconds}$$

M₇

B₇

A₇

No. 4

$$\text{Weighted Price Index} = \frac{\sum \left(\frac{P_{2010}}{P_{2005}} \times W \right)}{\sum W} \times 100$$

$$= \frac{\left(\frac{3500 \times 25}{2500} \right) + \left(\frac{7000 \times 10}{5000} \right) + \left(\frac{2000 \times 50}{1500} \right) + \left(\frac{8000 \times 5}{5000} \right) + \left(\frac{1200 \times 50}{800} \right)}{25 + 10 + 50 + 5 + 50} \times 100$$

$$= \frac{35 + 14 + \frac{200}{3} + 8 + 75}{140} \times 100$$

$$= \frac{596}{3 \times 140} \times 100$$

$$= 141.905$$

Comment. There is an increase of 41.91% in the prices between 2005 and 2010.

No. 5

$$r = (t^3 \underline{i} + \sin t \underline{j}) \text{ m.}$$

$$\text{Acceleration } a = \frac{d^2 r}{dt^2}$$

$$a = \frac{d^2}{dt^2} \begin{pmatrix} t^3 \\ \sin t \end{pmatrix}.$$

$$= \frac{d}{dt} \begin{pmatrix} 3t^2 \\ \cos t \end{pmatrix}.$$

$$a = \begin{pmatrix} 6t \\ -\sin t \end{pmatrix}.$$

$$a = 6t \underline{i} - \sin t \underline{j} \text{ m s}^{-2}$$

$$\text{when } t = \frac{\pi}{3}.$$

$$a = 6\left(\frac{\pi}{3}\right) \underline{i} - \sin\left(\frac{\pi}{3}\right) \underline{j}$$
$$= 2\pi \underline{i} - \frac{\sqrt{3}}{2} \underline{j}.$$

$$a = \frac{1}{2} \left(4\pi \underline{i} - \sqrt{3} \underline{j} \right) \text{ m s}^{-2}$$

$$\text{From } F = Ma.$$

$$F = 4 \left(\frac{1}{2} (4\pi \underline{i} - \sqrt{3} \underline{j}) \right)$$

$$F = 8\pi \underline{i} - 2\sqrt{3} \underline{j} \text{ N.}$$

No. 6.

$$L = 2.7 \text{ m}$$

$$W = 4.80 \text{ m}$$

$$h = 3.281 \text{ m}$$

$$\Delta L = 0.05$$

$$\Delta W = 0.005$$

$$\Delta h = 0.0005$$

B₁

$$V = L \times W \times h$$

$$V_{\max} = L_{\max} \cdot W_{\max} \cdot h_{\max}$$

$$= (L + \Delta L) (W + \Delta W) (h + \Delta h)$$

$$= (2.7 + 0.05) \cdot (4.80 + 0.005) (3.281 + 0.0005)$$

$$= 2.75 \times 4.805 \times 3.2815$$

$$= 43.36092$$

$$\approx 43.361 \text{ (3dp)}$$

1
My

A₇

$$V_{\min} = L_{\min} \cdot W_{\min} \cdot h_{\min}$$

$$(L - \Delta L) (W - \Delta W) (h - \Delta h)$$

$$= (2.7 - 0.05) (4.80 - 0.005) (3.281 - 0.0005)$$

$$= 2.65 \times 4.795 \times 3.2805$$

$$= 41.68449$$

$$\approx 41.684 \text{ (3dp)}$$

My

A₇

No. 7.

$$P(X=x) = \begin{cases} \frac{1}{10} x & ; 1, 2, \dots, n \\ 0 & ; \text{otherwise} \end{cases}$$

From $\sum_{\text{all } x}^n P(X=x) = 1$

$$\frac{1}{10} (1+2+3+\dots+n) = 1$$

$$1+2+3+\dots+n = 10$$

$$\frac{n(n+1)}{2} = 10$$

$$n(n+1) = 20 \quad \text{--- (1)}$$

$$E(X) = \sum_{\text{all } x}^n x P(X=x)$$

$$3 = \frac{1}{10} (1 + 2^2 + 3^2 + \dots + n^2)$$

$$1 + 2^2 + 3^2 + \dots + n^2 = 30$$

$$\frac{n(n+1)(2n+1)}{6} = 30$$

$$n(n+1)(2n+1) = 180 \quad \text{--- (2)}$$

Sub (1) in (2).

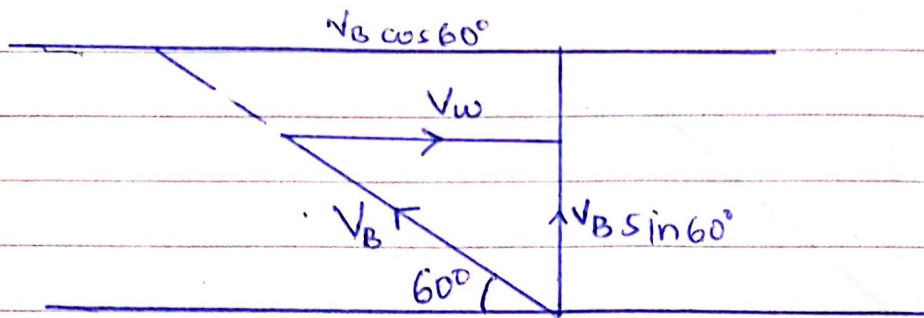
$$20(2n+1) = 180$$

$$2n+1 = 9$$

$$2n = 8$$

$$n = 4$$

No. 8



$$\text{Time taken} = \frac{AB}{V_B \sin 60^\circ}$$

$$= \frac{50}{2.5 \sin 60^\circ}$$

$$= 23.09 \text{ seconds}$$

$$\text{Resultant velocity } V = V_B \sin 60^\circ$$
$$= 2.5 \sin 60^\circ$$

$$= 2.1651 \text{ m s}^{-1}$$

B₇

M₁

A₇

M₁

A₇

No. 9

Time (min)	f	c	x	fx	f.d
0-10	20	10	5.0	100	2
10-15	18	5	12.5	225	3.6
15-30	60	15	22.5	1350	4
30-45	45	15	37.5	1687.5	3
45-55	50	10	50.0	2500	5
55-60	30	5	57.5	1725	6
60-80	60	20	70.0	4200	3
80-90	10	10	85.0	850	1
	$\Sigma f = 293$			$\Sigma fx = 12,637.5$	

B₁ Σf B₁ B₁ Midpoint
xB₁ Σfx B₁ frequency
density

$$\begin{aligned}
 \text{(a) Mean } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{12,637.5}{293} \\
 &= 43.1314 \text{ minutes.}
 \end{aligned}$$

M₁A₁

b) (i) Using a graph paper.

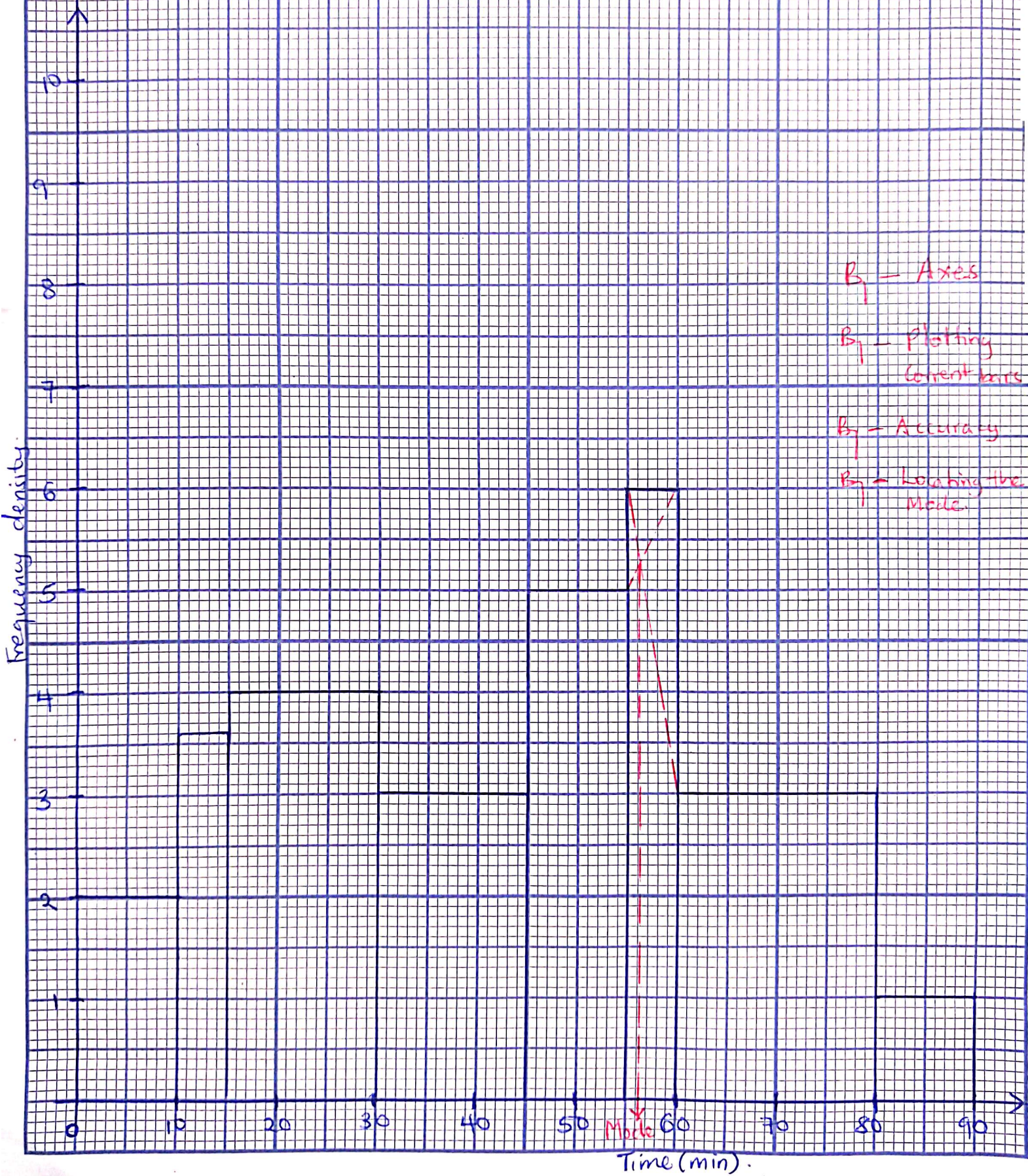
$$\text{ii) Mode} = 55 + (1.5 \times 1)$$

$$= 55 + 1.5$$

$$= 56.5 \text{ minutes}$$

M₁A₁

A Histogram



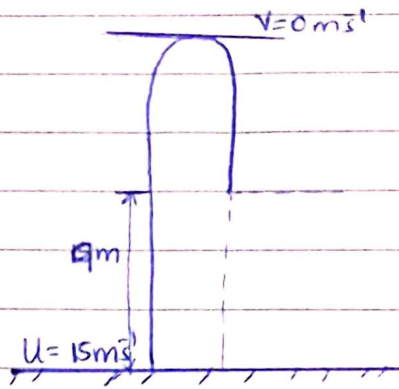
B₁ - Axes

B₁ - Plotting
Content bars

B₁ - Accuracy

B₁ - Locating the
Mode

10(9)



From $S = Ut - \frac{1}{2}gt^2$.

$$9 = 15t - \frac{1}{2}(9.8)t^2$$

$$9 = 15t - 4.9t^2$$

$$4.9t^2 - 15t + 9 = 0$$

$$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(4.9)(9)}}{2 \times 4.9}$$

$$t = \frac{15 \pm \sqrt{48.6}}{9.8}$$

$$t_1 = 2.242, \quad t_2 = 0.8192$$

Time taken before the ball was caught is 2.242 seconds.

At max point, $v = 0$.

$$0 = 15 - 9.8t$$

$$t = 1.5325$$

$$V = U + gt$$

$$V = 0 + 9.8(2.242 - 1.532)$$

$$V = 6.972 \text{ m/s}$$

M₁

M₁

B₁

A₁

B₁

M₁

A₁

(b)

$$a = \frac{1}{5} (2\hat{i} + 3\hat{j} - 4\hat{k}) \text{ m/s}^2$$

$$u = 11\hat{i} - 8\hat{j} + 3\hat{k}$$

$$r_0 = (-2\hat{i} + \hat{j}) \text{ m}$$

$$r(t=5) = r_0 + ut + \frac{1}{2} at^2$$

$$r(t=5) = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 11 \\ -8 \\ 3 \end{pmatrix} (5) + \frac{1}{2} \begin{pmatrix} 2/5 \\ 3/5 \\ -4/5 \end{pmatrix} (5^2)$$

M₁

$$= \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 55 \\ -40 \\ 15 \end{pmatrix} + \begin{pmatrix} 5 \\ 7.5 \\ -10 \end{pmatrix}$$

M₁

$$= \begin{pmatrix} 58 \\ -31.5 \\ 5 \end{pmatrix}$$

$$r(t=5) = 58\hat{i} - 31.5\hat{j} + 5\hat{k}$$

A₇

$$\text{Distance} = |r(t=5)|$$

$$= \sqrt{58^2 + (-31.5)^2 + 5^2}$$

M₁

$$= \sqrt{3364 + 992.25 + 25}$$

$$= \sqrt{4381.25}$$

$$\approx 66.2 \text{ m}$$

The distance covered is 66.2 m.

A₇

$$\text{Let } y_n = x_n e^{(x_n^2+1)}$$

$$h = \frac{1-0}{6-1} = \frac{1}{5} = 0.2$$

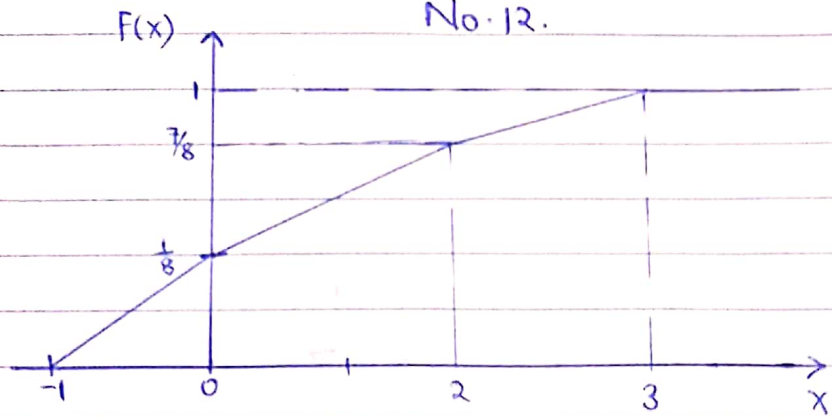
n	x_n	y_0, y_5	y_1, \dots, y_4
0	0.0	0.0000	
1	0.2		0.5658
2	0.4		1.2760
3	0.6		2.3377
4	0.8		4.1241
5	1.0	7.3891	
Total		7.3891	8.3038

$$\begin{aligned} \int_0^1 x e^{(x^2+1)} dx &\approx \frac{0.2}{2} [7.3891 + 2(8.3038)] \\ &\approx 2.3996 \\ &\approx 2.400 (3 \text{ dp}). \end{aligned}$$

$$\begin{aligned} \text{Percentage error} &= \frac{|\text{actual value} - \text{Approximate}|}{\text{actual value}} \times 100 \\ &= \frac{|2.335 - 2.400|}{2.335} \times 100 \\ &= \frac{0.065}{2.335} \times 100 \\ &= 2.770 \end{aligned}$$

(c). By increasing on the number of ordinates with in the given interval.

No. 12.



Consider interval $-1 \leq x < 0$

$(-1, 0)$ $(0, \frac{1}{8})$.

$$\frac{0 - (-1)}{\frac{1}{8} - 0} = \frac{F(x) - 0}{x - (-1)}$$

$$\frac{1}{8} = \frac{F(x)}{x+1}$$

$$F(x) = \frac{1}{8}(x+1).$$

M₇

For interval $0 \leq x < 2$.

$(0, \frac{1}{8})$ $(2, \frac{7}{8})$.

$$\frac{\frac{7}{8} - \frac{1}{8}}{2 - 0} = \frac{F(x) - \frac{1}{8}}{x - 0}$$

$$\frac{3}{8} = \frac{F(x) - \frac{1}{8}}{x}$$

$$F(x) = \frac{3}{8}x + \frac{1}{8}$$

$$F(x) = \frac{1}{8}(3x+1).$$

M₇

For interval $2 \leq x < 3$.

$(2, \frac{7}{8})$ $(3, 1)$

$$\frac{1 - \frac{7}{8}}{3 - 2} = \frac{F(x) - 1}{x - 3}$$

$$\frac{1}{8} = \frac{F(x) - 1}{x - 3}$$

$$F(x) - 1 = \frac{1}{8}(x - 3).$$

$$F(x) = \frac{1}{8}x - \frac{3}{8} + 1$$

$$F(x) = \frac{1}{8}(x + 5)$$

M₇

For interval $x \geq 3$
 $F(x) = 1$

M₇

$$F(x) = \begin{cases} 0 & ; x < -1 \\ \frac{1}{8}(x+1) & ; -1 \leq x < 0 \\ \frac{1}{8}(3x+1) & ; 0 \leq x < 2 \\ \frac{1}{8}(x+5) & ; 2 \leq x < 3 \\ 1 & ; x \geq 3. \end{cases}$$

A₇

11, $P(1 < X < 2.5) = F(2.5) - F(1)$.

$$= \frac{1}{8}(2.5+5) - \frac{1}{8}(3(1)+1)$$

M₇

$$= \frac{7.5}{8} - \frac{4}{8}$$

$$= \frac{7}{16} \text{ or } 0.4375$$

A₇

b) (1) P.d.f, $f(x)$.

$$f(x) = \frac{d}{dx} F(x).$$

$$\text{For } -1 \leq x < 0. \quad F(x) = \frac{1}{8}(x+1)$$

$$f(x) = \frac{d}{dx} \left(\frac{1}{8}(x+1) \right).$$

$$f(x) = \frac{1}{8}$$

$$\text{For } 0 \leq x < 2. \quad F(x) = \frac{1}{8}(3x+1)$$

$$f(x) = \frac{d}{dx} \left(\frac{1}{8}(3x+1) \right)$$

$$f(x) = \frac{3}{8}$$

$$\text{For } 2 \leq x < 3$$

$$F(x) = \frac{1}{8}(x+5).$$

$$f(x) = \frac{d}{dx} \left(\frac{1}{8}(x+5) \right).$$

$$f(x) = \frac{1}{8}$$

$$f(x) = \begin{cases} \frac{1}{8} & ; \quad -1 < x < 0 \\ \frac{3}{8} & ; \quad 0 \leq x < 2 \\ \frac{1}{8} & ; \quad 2 \leq x < 3 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$$E(x) = \int_{\text{all } x} x f(x) dx.$$

$$= \int_{-1}^0 \frac{1}{8} x dx + \int_0^2 \frac{3}{8} x dx + \int_2^3 \frac{1}{8} x dx.$$

$$= \frac{1}{8} \left[\frac{x^2}{2} \right]_{-1}^0 + \frac{3}{8} \left[\frac{x^2}{2} \right]_0^2 + \frac{1}{8} \left[\frac{x^2}{2} \right]_2^3$$

$$= \frac{1}{8} (0 - (\frac{1}{2})) + \frac{3}{8} (2 - 0) + \frac{1}{8} (\frac{9}{2} - 2)$$

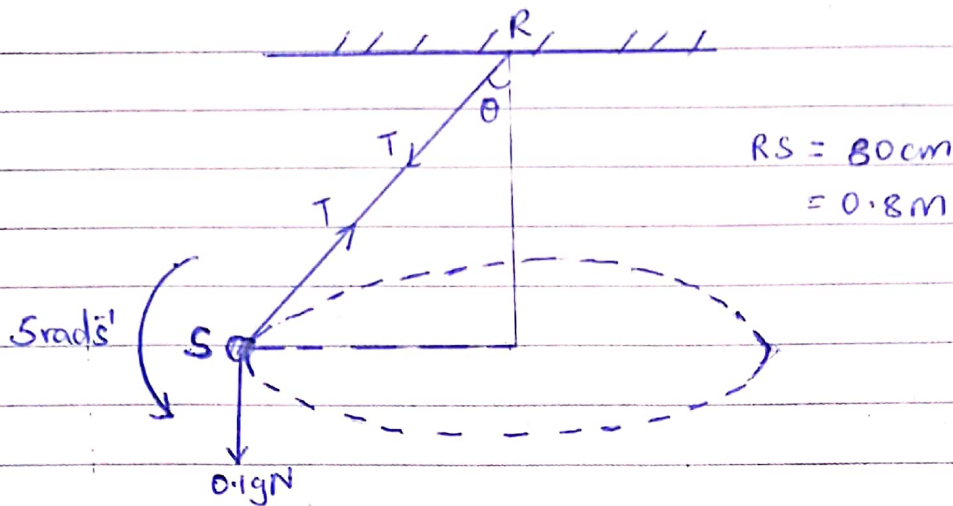
$$= -\frac{1}{16} + \frac{3}{4} + \frac{5}{16}$$

$$E(x) = 1$$

M₁

M₁

A₇



B4 Correct
diagram

Applying Newton's law horizontally gives

$$T \sin \theta = m r \omega^2$$

$$T \sin \theta = m \times 0.8 \sin \theta \times \omega^2$$

$$T = m l \omega^2$$

$$T = 0.1 \times 0.8 \times 5^2$$

$$T = 2 \text{ N}$$

M1
B7

M7

A7 Answer

b)

Resolving vertically.

$$T \cos \theta = mg$$

$$2 \cos \theta = 0.1(9.8)$$

$$\cos \theta = \frac{0.1(9.8)}{2}$$

$$\theta = 60.66^\circ$$

From $r = l \sin \theta$.

$$r = 0.8 \sin 60.66$$

$$r = 0.6974 \text{ m}$$

M7 Resol

M7 Subs

A7 Corre

A7

The radius of the horizontal circle = 69.74 cm.

A7 Corre

No. 14.

$$2x^3 - 4x + 3 = 0.$$

$$\therefore y = 2x^3 - 4x + 3.$$

x	-2.0	-1.75	-1.50	-1.25	-1.0
y	-5	-0.72	2.25	1.91	5.0

B₁ minimum of 5 points with 100% dp

From the graph, Root $x_0 \approx -1.68$

$$b) f(x) = 2x^3 - 4x + 3.$$

$$f'(x) = 6x^2 - 4$$

B₁ Derivative

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(2x_n^3 - 4x_n + 3)}{6x_n^2 - 4}$$

M₁ Substitution

$$= \frac{6x_n^3 - 4x_n - 2x_n^3 + 4x_n - 3}{6x_n^2 - 4}$$

$$x_{n+1} = \frac{4x_n^3 - 3}{6x_n^2 - 4}$$

$$x_0 = -1.68$$

$$x_1 = \frac{4(-1.68)^3 - 3}{6(-1.68)^2 - 4} = -1.6983$$

M₁, B₁ Substitution and value of x_1

$$x_2 = \frac{4(-1.6983)^3 - 3}{6(-1.6983)^2 - 4} = -1.6981$$

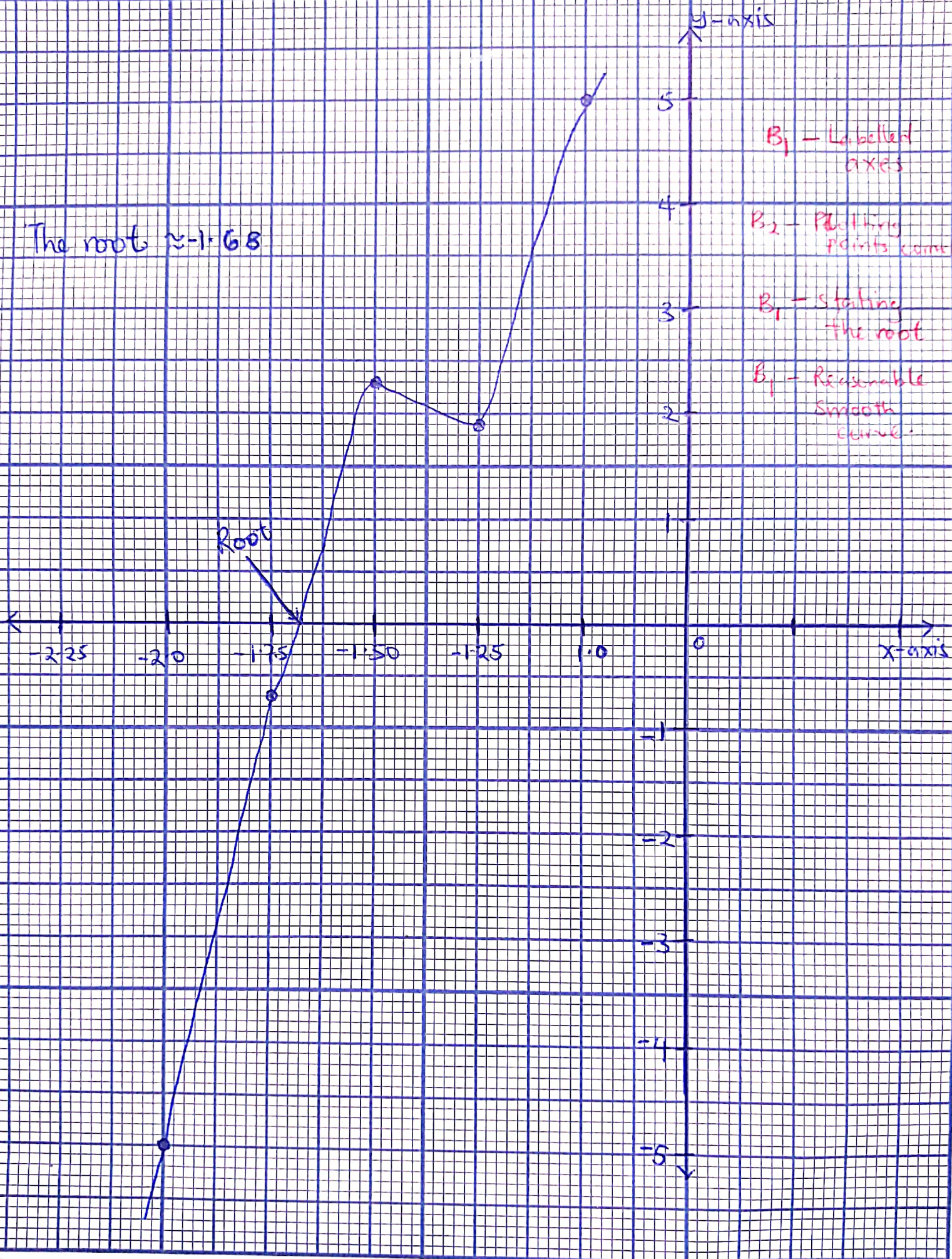
B₁ Value of x_2

$$|x_2 - x_1| = |-1.6981 - -1.6983| = 0.0003$$

\therefore The root is -1.698 (3.d.p).

B₁ Required root to 3dp

The root ≈ -1.68



No. 15

$$n = 120$$

$$p = \frac{1}{4}$$

$$q = \frac{3}{4}$$

$$\begin{aligned} \mu &= np \\ &= 120 \times \frac{1}{4} \end{aligned}$$

$$\mu = 30$$

$$\begin{aligned} \sigma &= \sqrt{npq} \\ &= \sqrt{120 \times 0.25 \times 0.75} \\ &= \sqrt{\frac{45}{2}} \\ &= 4.7434. \end{aligned}$$

B₁ Mean and Standard deviation

$X \sim$ number of correct options.

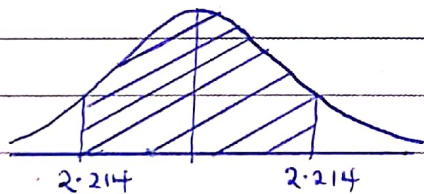
$$P(20 \leq X \leq 40) = P(19.5 < X < 40.5).$$

M₁ Continuity Correction

$$= P\left(\frac{19.5 - 30}{\sqrt{\frac{45}{2}}} < Z < \frac{40.5 - 30}{\sqrt{\frac{45}{2}}}\right)$$

M₂ Standardising

$$= P(-2.214 < Z < 2.214)$$



$$\begin{aligned} P(-2.214 < Z < 2.214) &= \Phi(2.214) + \Phi(+2.214) \\ &= 0.4867 + 0.4867 \\ &= 0.9734 \end{aligned}$$

M₁ Addition

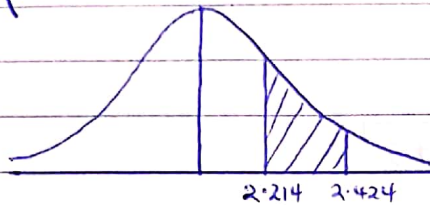
B₁ Table value

A₁

$$P(X=41) = P(40.5 < X < 41.5)$$

$$= P\left(\frac{40.5-30}{\sqrt{22.5}} < Z < \frac{41.5-30}{\sqrt{22.5}}\right)$$

$$= P(2.214 < Z < 2.424)$$



$$\begin{aligned} P(2.214 < Z < 2.424) &= \Phi(2.424) - \Phi(2.214) \\ &= 0.4924 - 0.4867 \\ &= 0.0057 \end{aligned}$$

M₁ Continuity

M₁ Subtraction

A₇ Output

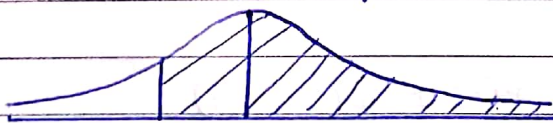
b)

$$P(X \geq X_1) = 0.8$$

$$P(X > X_1 + 0.5) = 0.8$$

$$P\left(Z > \frac{X_1 - 0.5 - 30}{\sqrt{22.5}}\right) = 0.8$$

$$P\left(Z > \frac{X_1 - 30.5}{\sqrt{22.5}}\right) = 0.8$$



$$\Phi\left(\frac{X_1 - 30.5}{\sqrt{22.5}}\right) = 0.3$$

$$\frac{X_1 - 30.5}{\sqrt{22.5}} = -0.842$$

$$X_1 = 30.5 + \sqrt{22.5}(-0.842) = 26.506$$

The passmark is 26.5

M₁ Continuity

M₁ Standardisation

M₁ Table value

M₁ Subtraction

A₇ Output

No. 16

Let W be the weight per unit Area.

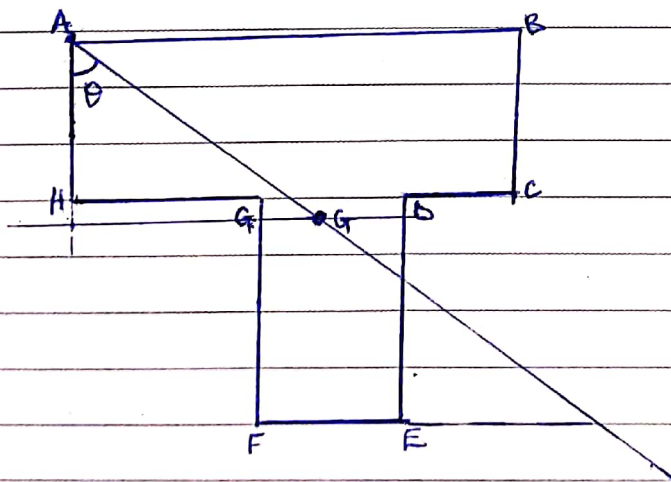
Lamina	Area	Weight	Distance of C.O.G from		
			AH	AB	
ABCH	6 m^2	$6W$	2.5	0.6	M_1
GDFE	3 m^2	$3W$	2.5	2.7	M_1
Composite	9 m^2	$9W$	\bar{x}	\bar{y}	M_1

C.O.G from AH: $9W\bar{x} = 2.5(3W) + 2.5(6W)$
 $9\bar{x} = 7.5 + 15$
 $\bar{x} = 2.5 \text{ m}$

M_1
 M_1
 M_1

Centre of gravity from AB: $9W\bar{y} = 2.7(3W) + 0.6(6W)$
 $9\bar{y} = 11.7$
 $\bar{y} = 1.3 \text{ m}$

M_1
 M_1



$\tan \theta = \frac{2.5}{1.3}$
 $\theta = \tan^{-1}\left(\frac{2.5}{1.3}\right) = 62.5^\circ$

M_1
 M_1
 M_1

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The solutions in this guide are according to my opinion. I accept to own any mistake detected.